Shortcuts to Adiabaticity

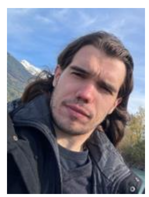
Adolfo del Campo
University of Luxembourg & DIPC

Carnot Workshop on Quantum Thermodynamics and Open Quantum Systems

25-27 November 2024, Dijon, France

Group at Luxembourg

Current members

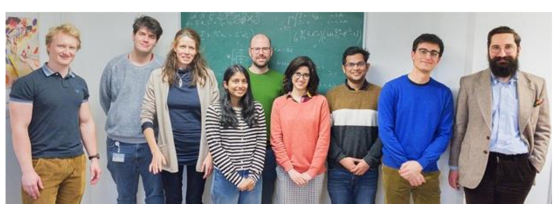
    

K. Takahashi A. Grabarits S. Shinn K. R. Swain M. Massaro

Collaborator Aurelia Chenu

AURELIA'S QUANTUM LAB

uni.lu UNIVERSITÉ DU LUXEMBOURG



Outline

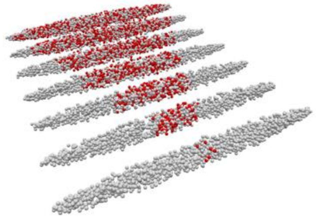
- Part I: Shortcuts To Adiabaticity

- Part II: Many-particle Quantum Machines

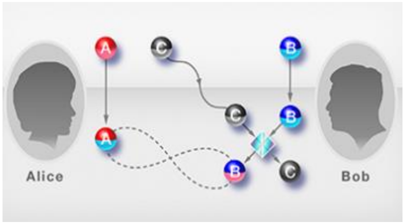
Part I: Shortcuts to Adiabaticity

Fast and robust driving protocols reducing excitations

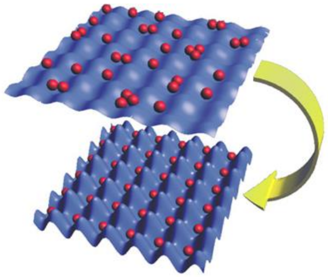
Far-from-equilibrium dynamics



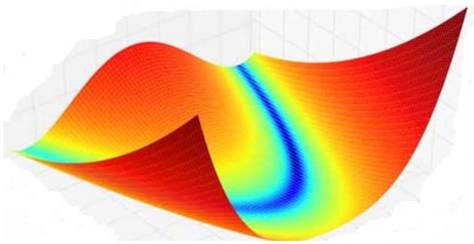
Quantum information and computation



Strongly correlated systems



Optimal control theory



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Counterdiabatic driving

Counterdiabatic driving

Consider driving a system Hamiltonian

$$\hat{H}_0(t)|n(t)\rangle = E_n(t)|n(t)\rangle$$

Write the adiabatic approximation

$$|\psi_n(t)\rangle = \exp\left[-\frac{i}{\hbar}\int_0^t E_n(s)ds - \int_0^t \langle n(s)|\partial_s n(s)\rangle ds\right]|n(t)\rangle$$

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Is there a Hamiltonian for which the adiabatic approximation is exact?

$$i\hbar\partial_t|\psi_n(t)\rangle = \hat{H}(t)|\psi_n(t)\rangle$$

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Is there a Hamiltonian for which the adiabatic approximation is exact?

$$i\hbar\partial_t|\psi_n(t)\rangle = \hat{H}(t)|\psi_n(t)\rangle$$

Yes, indeed!

$$\hat{H}(t) \equiv \hat{H}_0(t) + \hat{H}_1(t)$$

$$\hat{H}_1(t) = i\hbar\sum_n \left(|\partial_t n\rangle\langle n| - \langle n|\partial_t n\rangle|n\rangle\langle n|\right)$$

Counterdiabatic driving

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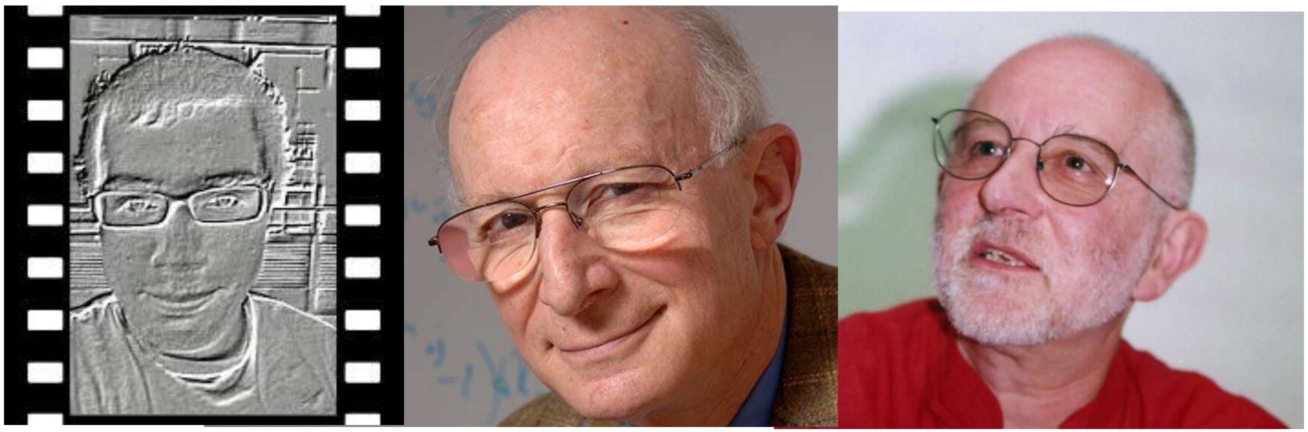
Yes, indeed!

$$\hat{H}(t) \equiv \hat{H}_0(t) + \hat{H}_1(t)$$

$$\hat{H}_1(t) = i\hbar\sum_{n\neq m}\sum_m \frac{|m\rangle\langle m|\partial_t \hat{H}_0|n\rangle\langle n|}{E_n(t) - E_m(t)}$$

Counterdiabatic driving



Theory: Demirplak & Rice 2003, 2005, 2008; = M. V. Berry 2009 "Transitionless quantum driving"
CD inspired experiment for TLS: Morsch's group Nature Phys. 2012; NVC: Suter's group PRL 2013

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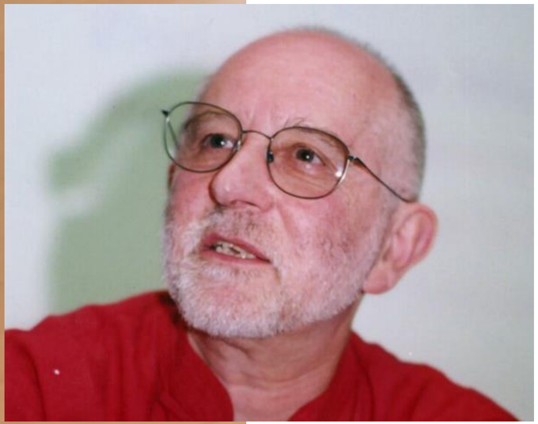
From this cup on the 18th of May
2018 Sir Michael Berry drank
(three times) coffee (espresso +
a bit cold milk) prepared by our
Collaboratrice administrative sur projet,
Yoanna Koleva who also provided
him some cookies. Michael was happy!
Dr. Dmitry Fedorov
22.05.2018
SPUERKEESS
www.bcee.lu
BUTLERS



Counterdiabatic driving: applications

Counterdiabatic terms are often **nonlocal**

RAP in two-level system (spin flip) $\hat{H}_1 \propto \sigma_y$ $\hat{H}_1' \propto \sigma_z$

Demirplak & Rice 2003 Bason et al 2012

Time-dependent harmonic oscillator $\hat{H}_1 \propto (xp + px)$ $\hat{H}_1' \propto x^2$

Muga el at 2010, Jarzynski 2013 Ibáñez et al 12, AdC 13

Transport of matter waves $\hat{H}_1 \propto p$ $\hat{H}_1' \propto x$

Deffner-Jarzynski-AdC 14

Search for experimentally-realizable local Unitarily equivalent Hamiltonians

$$\hat{H}' = U\hat{H}U^\dagger - i\hbar U\partial_t U^\dagger$$

Torrontegui et al. Adv. At. Mol. Opt. Phys. 62, 117 (2013)
Guery-Odelin et al. Rev. Mod. Phys. 91, 045001 (2019)

CD Experiments: TLS and TDHO

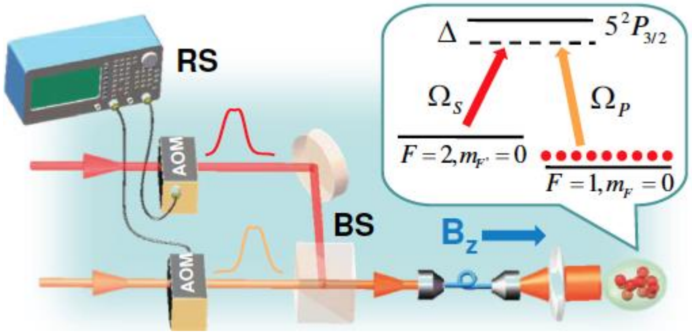
nature COMMUNICATIONS

ARTICLE

Received 28 Jan 2016 | Accepted 6 Jul 2016 | Published 11 Aug 2016 DOI: 10.1038/ncomms12479 OPEN

Experimental realization of stimulated Raman shortcut-to-adiabatic passage with cold atoms

Yan-Xiong Du[1], Zhen-Tao Liang[1], Yi-Chao Li[2], Xian-Xian Yue[1], Qing-Xian Lv[1], Wei Huang[1], Xi Chen[2], Hui Yan[1] & Shi-Liang Zhu[1,3,4]



CD for 2 & 3 Level systems

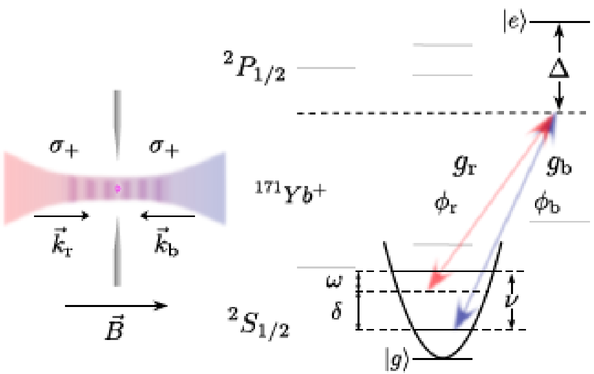
nature COMMUNICATIONS

ARTICLE

Received 29 Feb 2016 | Accepted 19 Aug 2016 | Published 27 Sep 2016 DOI: 10.1038/ncomms12999 OPEN

Shortcuts to adiabaticity by counterdiabatic driving for trapped-ion displacement in phase space

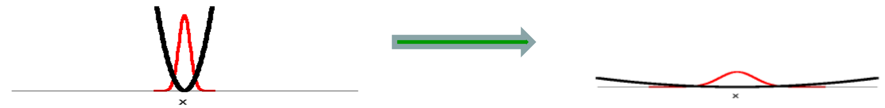
Shuoming An[1], Dingshun Lv[1], Adolfo del Campo[2] & Kihwan Kim[1]



CD for systems with Continuous Variables

CD Experiments: Ultracold Gases

Family of interacting quantum fluids



$$\hat{H} = \sum_{i=1}^{N} \left[-\frac{\hbar^2}{2m} \nabla^2 + \frac{1}{2} m \omega(t)^2 \mathbf{r}_i^2 \right] + \sum_{i<j} V(\mathbf{r}_i - \mathbf{r}_j)$$

Scaling-invariant dynamics when $V(\gamma \mathbf{r}) = \gamma^{-2} V(\mathbf{r})$

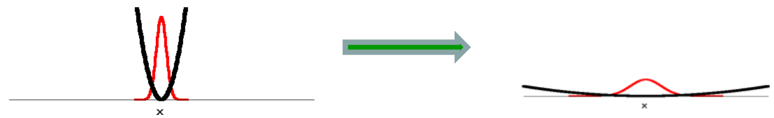
Shortcut to adiabaticity = Fast motion video of adiabatic dynamics

Auxiliary Counterdiabatic Control => harmonic trap

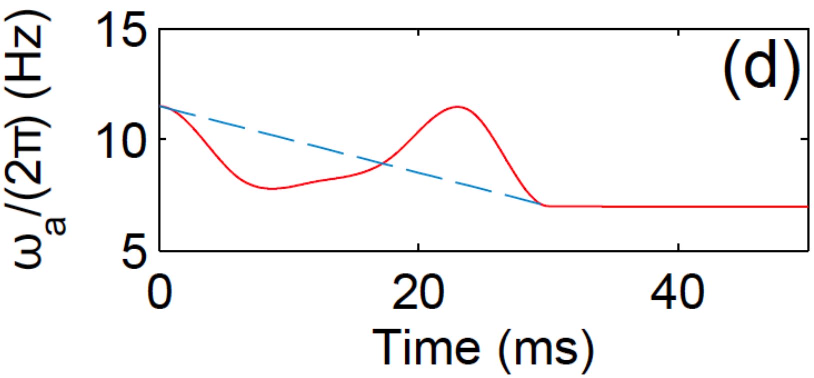
$$\omega(t)^2 \rightarrow \Omega^2(t) = \omega^2(t) - \frac{3}{4}\frac{\dot{\omega}^2}{\omega^2} + \frac{1}{2}\frac{\ddot{\omega}}{\omega}.$$

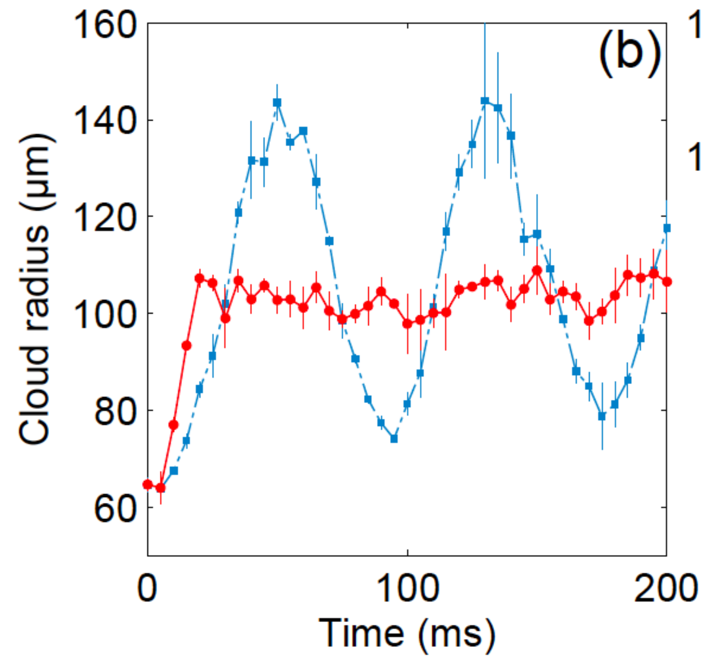
A. del Campo, PRL **111**, 100502 (2013)

Experiments: Thermal cloud, BEC and 1D Bose gas



Shortcut vs standard expansion

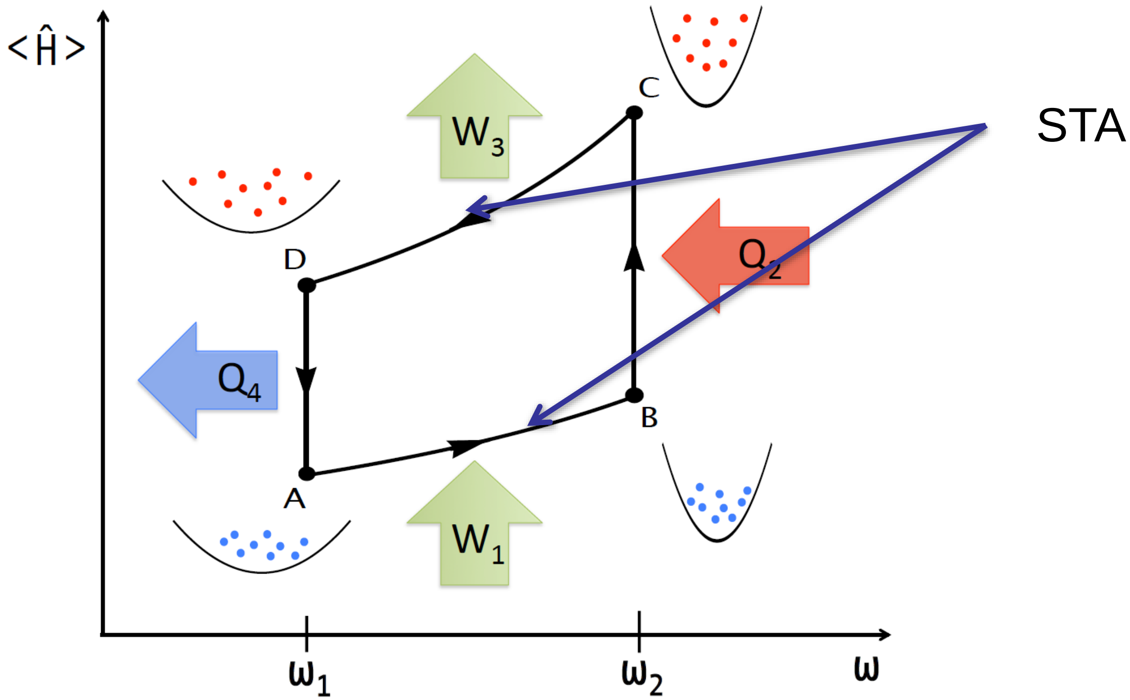




Experiments: 1D Bose gas
Rohringer et al. Sci. Rep. **5**, 9820 (2015)
Experiments: mean-field BEC
J.-F. Schaff et al. EPL **93**, 23001 (2011)
Experiments: single-particle
J.-F. Schaff et al. Phys. Rev. A **82**, 033430 (2010)

Theory (quantum fluids)
AdC PRA **84**, 031606(R) (2011)
AdC & Boshier, Sci. Rep. **2**, 648 (2012)
AdC PRL **111**, 100502 (2013)

Superadiabatic QHE strokes (e.g. in Otto Cycle)

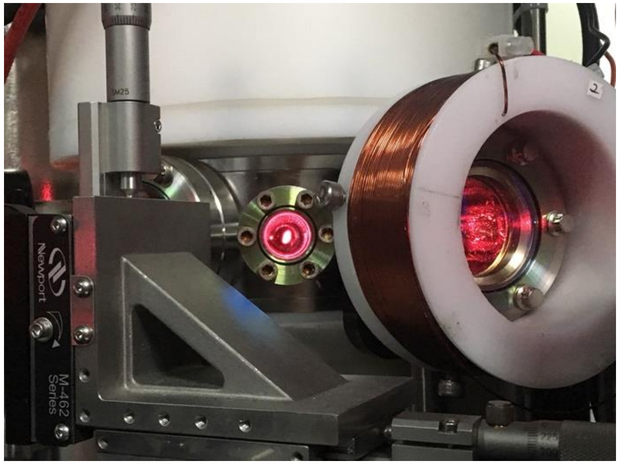


$$\eta \leq 1 - Q^* \frac{\omega_1}{\omega_2}$$

$$\eta_{\rm STA} = \eta_{\max} = 1 - \frac{\omega_1}{\omega_2}$$

- AdC, J. Goold, M. Paternostro, Sci. Rep. **4**, 6208 (2014); arXiv:1305.3223
- J. Deng et al., Phys. Rev. E **88**, 062122 (2013); arXiv:1307.4182
- M. Beau, J. Jaramillo, AdC, Entropy **18**, 168 (2016) (many-particle QHE)

Superadiabatic Many-particle QHE Strokes

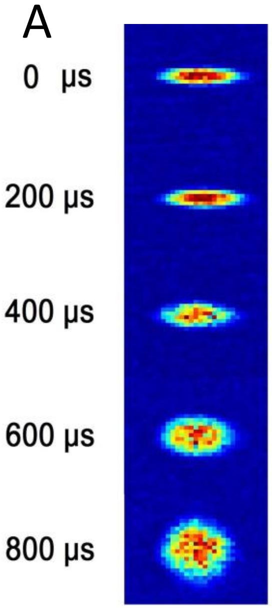


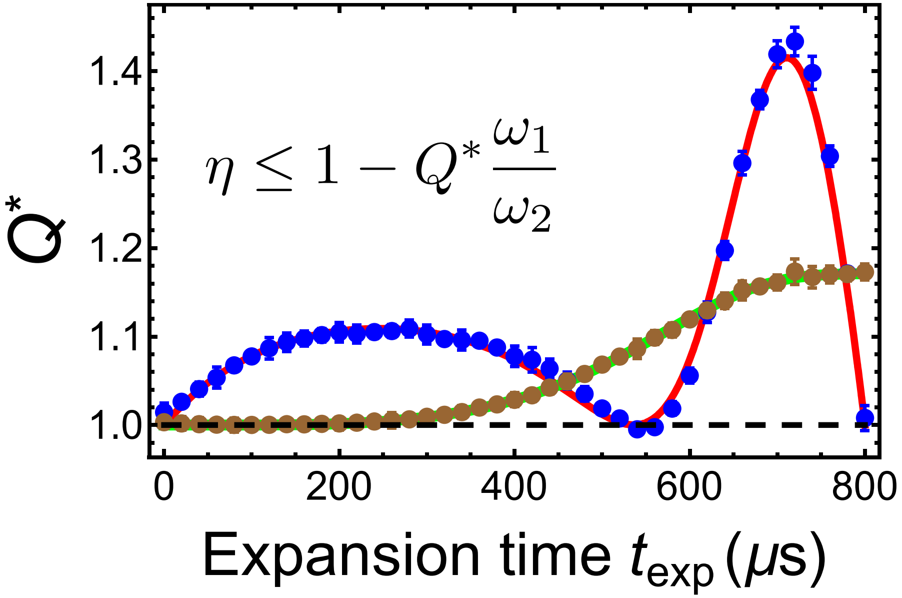
SCIENCE ADVANCES | RESEARCH ARTICLE

PHYSICS

Superadiabatic quantum friction suppression in finite-time thermodynamics

Shujin Deng,[1] Aurélia Chenu,[2] Pengpeng Diao,[1] Fang Li,[1] Shi Yu,[1] Ivan Coulamy,[3,4] Adolfo del Campo,[3] Haibin Wu[1,5]*





Counterdiabatic driving: Open systems

Alipour et al. Quantum 4, 336 (2020)

Counterdiabatic driving: Open systems

Prescribed a quantum state trajectory

$$\varrho(t) = \frac{1}{Z_0(t)} e^{-\beta H_0(t)} \qquad H_0(t) = \sum_n E_n |n_t\rangle\langle n_t|$$

Find equation of motion

$$\partial_t \varrho(t) = -i[H_{\rm CD}(t), \varrho(t)] + \sum_n \partial_t \lambda_n(t) |n_t\rangle\langle n_t|$$

$$H_{\rm CD}(t) = H_0(t) + i \sum_n \left(|\partial_t n_t\rangle\langle n_t| - \langle n_t|\partial_t n_t\rangle |n_t\rangle\langle n_t| \right)$$

Physical scenarios?

I) Non-Hermitian systems (Balanced gain and loss)

II) Non-Markovian systems (standard QOS)

Alipour et al. Quantum 4, 336 (2020)

Scenario I: Non-Hermitian systems

Non-Hermitian Hamiltonian

$$H(t) = H_{\rm CD}(t) - i\Gamma(t) = H_{\rm CD}(t) + \frac{i}{2}\sum_n \frac{\partial_t \lambda_n(t)}{\lambda_n(t)} |n_t\rangle\langle n_t|$$

Equation of motion

$$\partial_t \varrho(t) = -i\left[H(t)\varrho(t) - \varrho(t)H^\dagger(t)\right] = -i\left[H_{\rm CD}(t), \varrho(t)\right] - \left\{\Gamma(t), \varrho(t)\right\}$$

Not trace-preserving

$$\partial_t {\rm Tr}(\varrho(t)) = -2{\rm Tr}(\Gamma(t)\varrho(t)) = \sum_n \partial_t \lambda_n(t)$$

Balanced gain and loss

Brody Graefe PRL 109, 230405 (2012)

$$\partial_t \varrho = -i(H\varrho - \varrho H^\dagger) - \partial_t {\rm Tr}(\varrho)\,\varrho = -i\left[H_{\rm CD}, \varrho\right] + \left(2\langle\Gamma\rangle\varrho - \{\Gamma, \varrho\}\right)$$

Alipour et al. Quantum 4, 336 (2020)

Scenario II: Non-Markovian QOS

New term as dissipator

$$\mathcal{D}_{\rm CD}(\varrho) = \sum_n \partial_t \lambda_n(t) |n_t\rangle\langle n_t|$$

Introduce Lindblad operators

$$L_{mn}(t) = |m_t\rangle\langle n_t| \qquad \gamma_{mn}(t) = \frac{\partial_t \lambda_m(t)}{r\lambda_n(t)}$$

Lindblad-like form master equation

$$\partial_t \varrho = -i[H_{\rm CD}, \varrho] + \sum_{mn} \gamma_{mn} \Big(L_{mn} \varrho L_{mn}^\dagger - \tfrac{1}{2} \{ L_{mn}^\dagger L_{mn}, \varrho \} \Big)$$

Generally non-Markovian

Alipour et al. Quantum 4, 336 (2020)

Counterdiabatic driving: Open systems

I) Non-Hermitian evolution (balanced gain and loss)

II) Non-Markovian master equation

Alipour et al. Quantum 4, 336 (2020)

Thermalizing a TDHO (open)

Rotating frame

$$\tilde{\varrho}(t) = U_x \varrho(t) U_x^\dagger \qquad U_x = e^{i\frac{m}{2\hbar}\alpha_t \hat{x}^2} \qquad \alpha_t = \frac{\dot{u}_t}{1-u_t^2} - \frac{\dot{\omega}_t}{\omega_t}$$

STA in Open TDHO simply via dephasing

$$\partial_t \tilde{\varrho} = \frac{1}{i\hbar}\left[\frac{\hat{p}^2}{2m} + \frac{1}{2}m\tilde{\omega}_{\rm CD}^2\hat{x}^2, \tilde{\varrho}\right] - \gamma_t[\hat{x},[\hat{x},\tilde{\varrho}]]$$

Frequency

$$\tilde{\omega}_{\rm CD}^2 = \left[\omega_t^2 - \frac{3}{4}\left(\frac{\dot{\omega}_t}{\omega_t}\right)^2 + \frac{\ddot{\omega}_t}{2\omega_t}\right] - \Omega_t^2 - \dot{\Omega}_t + \Omega_t\frac{\dot{\omega}_t}{\omega_t} \qquad \Omega_t = -\frac{1}{2}\frac{\dot{\omega}_t}{\omega_t} + \frac{\dot{u}_t}{1-u_t^2}$$

Dephasing

$$\gamma_t = \frac{m\omega_t}{\hbar}\frac{\dot{u}_t}{(1-u_t)^2}$$

Alipour et al. Quantum 4, 336 (2020)

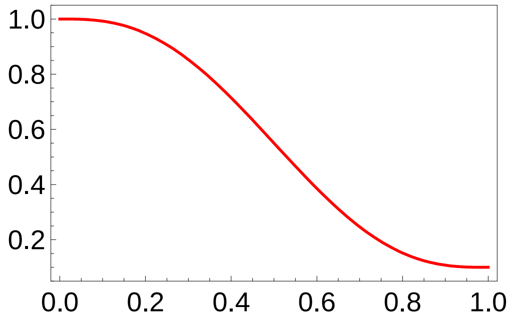
Thermalizing a TDHO

Prescribed trajectory

$$(\beta_t, \omega_t)$$

$$\partial_t \tilde{\varrho} = \frac{1}{i\hbar}\left[\frac{\hat{p}^2}{2m} + \frac{1}{2}m\tilde{\omega}_{\rm CD}^2\hat{x}^2, \tilde{\varrho}\right] - \gamma_t[\hat{x}, [\hat{x}, \tilde{\varrho}]]$$

Heating



$\tilde{\omega}_{\rm CD}^2/\omega_0^2$

ω_t^2/ω_0^2

Cooling

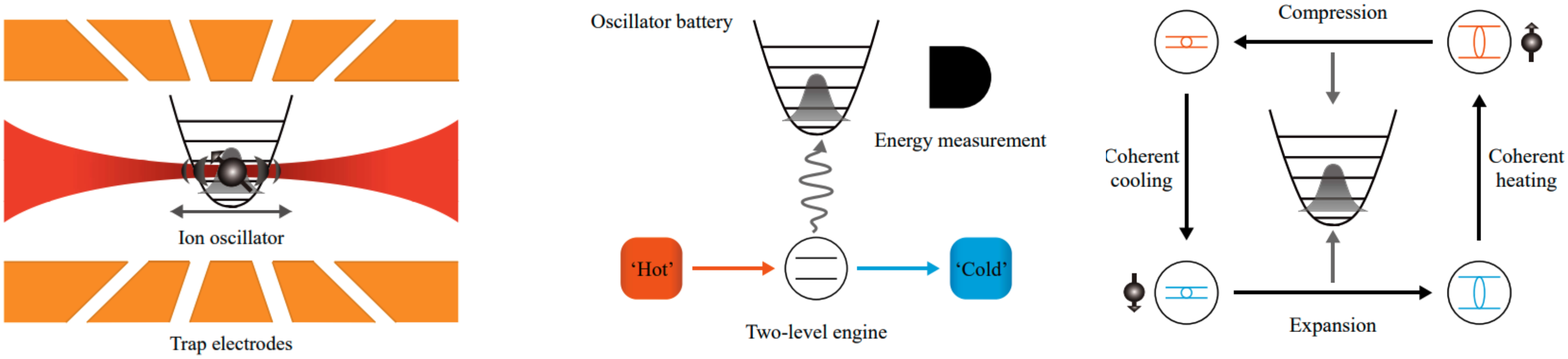
Alipour et al. Quantum 4, 336 (2020)

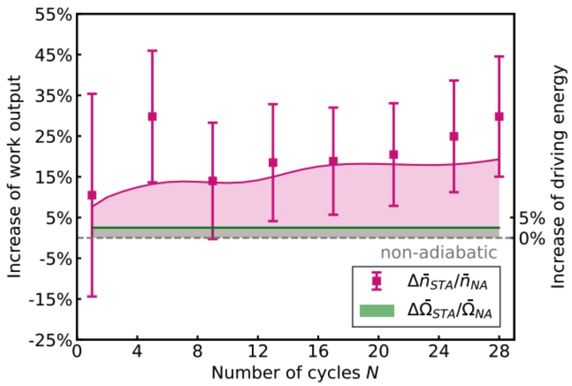
STA in QHE

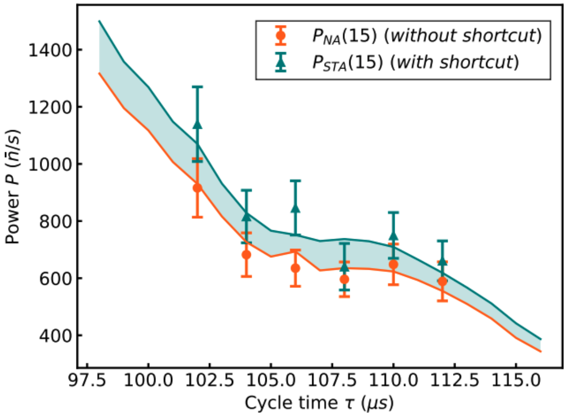
arXiv:2404.15075 [pdf, other]

An energy efficient quantum-enhanced machine

Waner Hou, Xingyu Zhao, Kamran Rehan, Yi Li, Yue Li, Eric Lutz, Yiheng Lin, Jiangfeng Du







Quantifying cost

Norm of counterdiabatic term

[Demirplak & Rice J. Chem. Phys. 129, 154111 (2008); Campbell & Deffner, PRL118, 100601 (2017)]

$$\|\hat{H}_1\|^2 = \frac{\hbar^2}{2\tau^2}\sum_n \mathrm{tr}\dot{\hat{P}}_n^2$$

Energy fluctuations [AdC et al. PRL 109, 115703 (2012); AdC, PRL 126, 180603 (2021)]

$$\Delta H_{\rm CD}^2 = \langle \hat{H}_{\rm CD}^2\rangle - \langle \hat{H}_{\rm CD}\rangle^2 = \sum_n p_n^0 \langle n(t)|\hat{H}_1^2(t)|n(t)\rangle$$

$$\langle n(t)|\hat{H}_1^2(t)|n(t)\rangle = \dot{\lambda}^2\chi_f^{(n)}(\lambda) = \dot{\lambda}^2\sum_{m\neq n}\frac{|\langle m(\lambda)|\frac{d}{d\lambda}\hat{H}_0|n(\lambda)\rangle|^2}{|\varepsilon_m-\varepsilon_n|^2}$$

Work fluctuations [Funo et al. PRL 118, 100602 (2017)]

$$\langle W(t)\rangle = \langle W(t)\rangle \mathrm{ad} \qquad \mathrm{Var}[W(t)] - \mathrm{Var}[W(t)]_{\rm ad} = \hbar^2\sum_n p_n^0 g_{\mu\nu}^{(n)}\dot{\lambda}^\mu\dot{\lambda}^\nu$$

$$Q_{\mu\nu}^{(n)} = g_{\mu\nu}^{(n)} + i\sigma_{\mu\nu}^{(n)} \qquad Q_{\mu\nu}^{(n)} := \langle \partial_\mu n(t)|[1-|n(t)\rangle\langle n(t)|]|\partial_\nu n(t)\rangle$$

Quantum speed limits refine the time-energy uncertainty relation

Guy Chenu

History of Quantum Speed Limits

Landau

Krylov

1945 Mandelstam and Tamm "MT"

$$\tau \geq \frac{\pi}{2}\frac{\hbar}{\Delta H}$$

1967 Fleming

1990 Anandan, Aharonov

1992 Vaidman, Ulhman

1993 Uffnik

1998 Margolus & Levitin "ML"

$$\tau \geq \frac{\hbar\pi}{2E}$$

2000 Lloyd

2003 Giovannetti, Lloyd, Maccone: MT & ML unified

2003 Bender: no bounds in PT-symmetric QM

2009 Levitin,Toffoli

2013 QSL for Open Quantum Systems

2017 Decoherence times

2019 QSL for Classical Systems

Seminal results

Mandelstam-Tamm (1945)

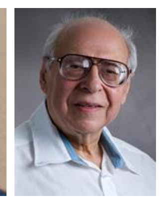


Heisenberg EOM + definition of time = TEUR

$$\frac{d}{dt}\hat{A} = \frac{1}{i\hbar}[\hat{A}, \hat{H}] \qquad \tau(A) = \frac{\Delta A}{\frac{d}{dt}\langle \hat{A} \rangle}$$

$$\tau(A)\Delta H \geq \frac{\hbar}{2}$$

Margolus-Levitin (1998)

Survival amplitude, vanishing real and imaginary parts

$$|\Psi(t)\rangle = \sum_n c_n e^{-iE_n t/\hbar}|n\rangle \quad a(t) := \langle \psi(0)|\psi(t)\rangle \quad \mathrm{Re}(a) \geq 1 - \frac{2E}{\pi\hbar}t + \mathrm{Im}(a)$$

$$\tau \geq \frac{\hbar\pi}{2E}$$

Distance travelled

Bures angle/length

$$\mathcal{L}(t) = \mathcal{L}(|\Psi(0)\rangle, |\Psi(t)\rangle) = \arccos\sqrt{F(t)}$$

$$F(t) = F[\rho_0, \rho_t] = \left(\mathrm{Tr}\sqrt{\sqrt{\rho_0}\rho_t\sqrt{\rho_0}}\right)^2$$

Quantum Cramer-Rao bound

$$\mathcal{L}(\tau) \leq \int_0^\tau ds\sqrt{I_Q(t)/4}$$

Quantum Fisher information

$$I_Q(t) = \frac{4}{\hbar^2}[\langle\Psi(t)|H(t)^2|\Psi(t)\rangle - \langle\Psi(t)|H(t)|\Psi(t)\rangle^2]$$

$$= \frac{4}{\hbar^2}\mathrm{var}_{\rho(t)}[H(t)]$$

Energy fluctuations
(unitary dynamics)

Anandan & Aharonov, PRL 65, 1697 (1990)
Uhlmann, Phys. Lett. A 161, 329 (1992)

Quantum Speed Limit

Mandelstam-Tamm QSL for driven systems

Time for something to happen bounded by inverse of average energy fluctuations

$$\tau \geq \tau_{\rm QSL} = \frac{\hbar \mathcal{L}(\tau)}{\overline{\Delta H}}$$

Anandan & Aharonov, PRL 65, 1697 (1990)
Uhlmann, Phys. Lett. A 161, 329 (1992)

Quantum Brachistochrone and Counterdiabatic driving

Time-Optimal Quantum Evolution

Alberto Carlini, Akio Hosoya, Tatsuhiko Koike, and Yosuke Okudaira
Phys. Rev. Lett. **96**, 060503 – Published 15 February 2006

- Time-optimal evolution and optimal Hamiltonian with a given initial and final states and bounded energy fluctuations

- Believed to be equivalent to counter-diabatic driving

PAPER
How fast and robust is the quantum adiabatic passage?
Kazutaka Takahashi[1]
Published 17 July 2013 • © 2013 IOP Publishing Ltd
Journal of Physics A: Mathematical and Theoretical, Volume 46, Number 31

"The result here shows transitionless quantum driving [...] can be derived from the QB equation"

Test with scale-invariant dynamics

For an initial eigenstate $$H[\omega(0)]\Psi(0) = E(0)\Psi(0) \qquad \omega(0) \Longrightarrow \omega(t)$$

Solutions of the TDSE after modulating the trap frequency

$$\Psi(t) = \frac{1}{b^{\frac{DN}{2}}} \exp\left[i\frac{m\dot{b}}{2\hbar b}\sum_{i=1}^{N}\vec{r}_i^{\ 2} - i\int_0^t \frac{E(0)}{\hbar b(t')^2}dt'\right]\Psi\left(\frac{\vec{r}_1}{b}, \ldots, \frac{\vec{r}_N}{b}, t=0\right)$$

Scaling factor given by Ermakov equation

$$\ddot{b} + \omega(t)^2 b = \omega_0^2/b^3$$

AdC, PRL 126, 180603 (2021)

QSL for counter-diabatic driving

Driving Hamiltonian and quantum state

$$H_{\rm CD} = H(t) + H_1(t) \qquad H_1(t) = \frac{\dot{b}}{b}C = \frac{\dot{b}}{b}\frac{1}{2}\sum_{i=1}^{N}\{\vec{r}_i, \vec{p}_i\}$$

$$\Psi\left(t\right) = \frac{e^{i\alpha_t}}{b^{\frac{DN}{2}}}\ \Psi\left(\frac{\vec{r}_1}{b}, \dots, \frac{\vec{r}_N}{b}, t=0\right)$$

AdC, PRL 126, 180603 (2021)

QSL for counter-diabatic driving

Driving Hamiltonian and quantum state

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$$\Psi\left(t\right)=\frac{e^{i\alpha_t}}{b^{\frac{DN}{2}}}\ \Psi\left(\frac{\vec r_1}{b},\dots,\frac{\vec r_N}{b},t=0\right)$$

Speed of evolution and distance traveled

$$\Delta H_{\rm CD}^2=\left(\frac{\dot b}{b}\right)^2\langle C^2(t)\rangle=\left(\frac{\dot b}{b}\right)^2\hbar^2\sigma^2$$

$$\gamma(\tau)=\int_0^\tau dt\frac{\Delta H_{\rm CD}(t)}{\hbar}=\sigma\alpha\log b(\tau)=\log\left(\frac{\omega(\tau)}{\omega_0}\right)^{-\alpha\frac{\sigma}{2}}$$

AdC, PRL 126, 180603 (2021)

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$$\gamma(\tau) = \int_0^\tau dt \frac{\Delta H_{\rm CD}(t)}{\hbar} = \sigma\alpha\log b(\tau) = \log\left(\frac{\omega(\tau)}{\omega_0}\right)^{-\alpha\frac{\sigma}{2}}$$

Bures angle (geodesic)

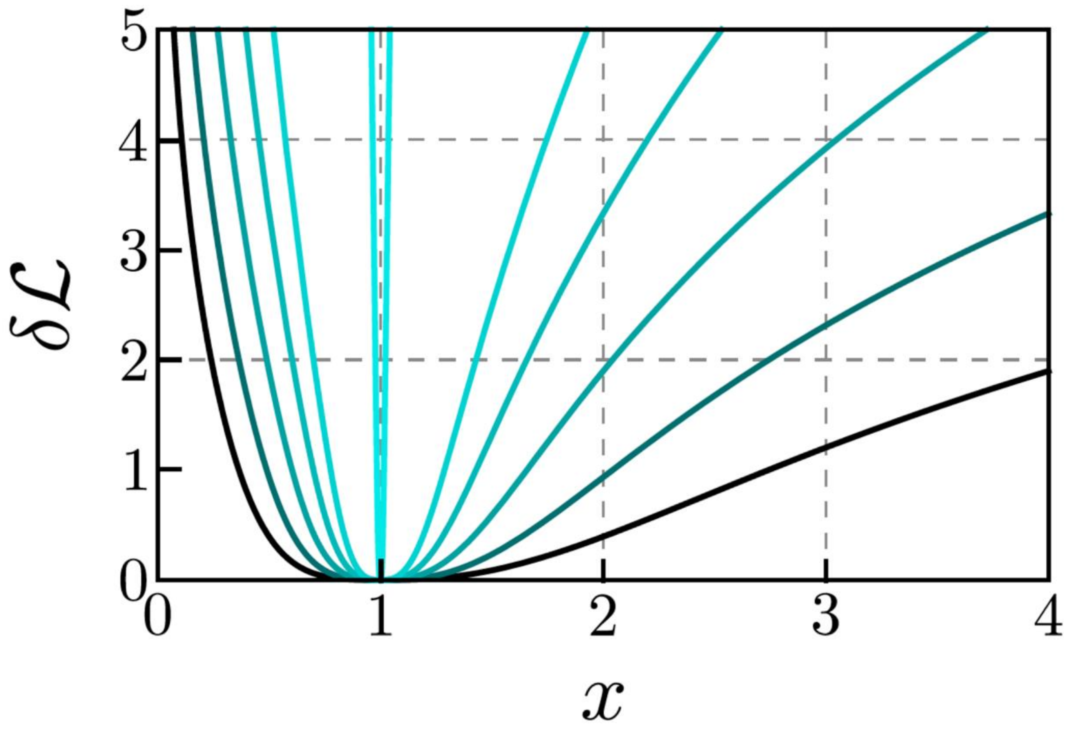
$$\mathcal{L}(\tau) = \arccos\sqrt{F(\tau)} = \arccos\left[\frac{\omega_0}{4\omega(\tau)}\left(1+\frac{\omega(\tau)}{\omega_0}\right)^2\right]^{-\frac{\sigma^2}{2}}$$

AdC, PRL 126, 180603 (2021)

CD does not saturate the MT QSL

$$\delta\mathcal{L}(\tau) = \int_0^\tau \Delta H(t)dt - \mathcal{L}(\tau) \geq 0$$

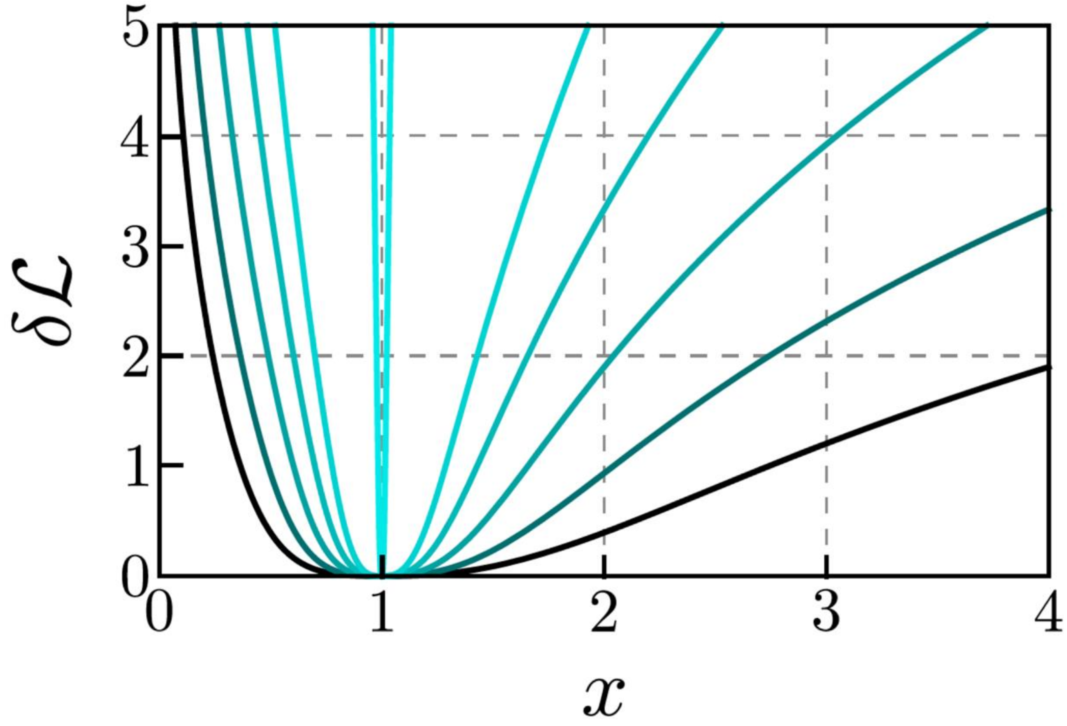
$$\delta\mathcal{L}(\tau) = -\alpha\frac{\sigma}{2}\log x - \arccos\left[\left(\frac{1+x}{2\sqrt{x}}\right)^{-\sigma^2}\right] \qquad x = \omega(\tau)/\omega_0$$



AdC, PRL 126, 180603 (2021)

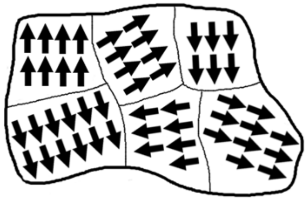
CD does not saturate the MT QSL

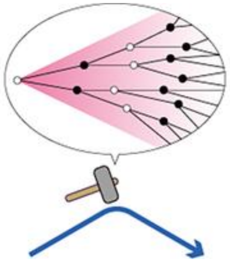
Evolution never saturates the speed limit

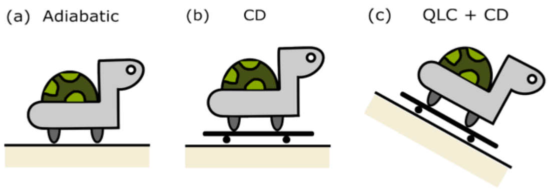


AdC, PRL 126, 180603 (2021)
Bukov, Sels, Polkovnikov, PRX 9, 011034 (2019)
Geometric Speed Limit of Accessible Many-Body State Preparation

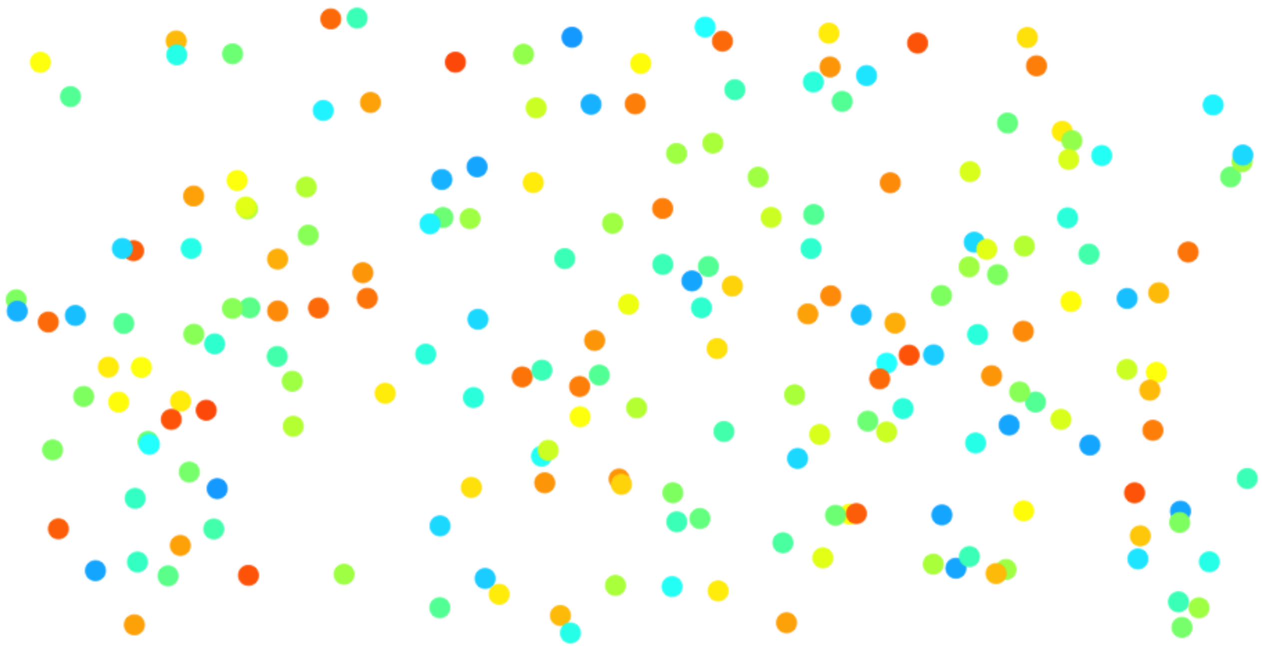
History of Shortcuts to Adiabaticity







Part II: Many-particle Quantum Thermodynamics



Part II: Many-Particle Quantum Heat Engines

- Harnessing interactions
- Harnessing quantum statistics
- Harnessing quantum criticality

I) YY Chen, G Watanabe, YC Yu, XW Guan, AdC, npj Quantum Information 5, 88 (2019)

II) G Watanabe, BP Venkatesh, P Talkner, MJ Hwang, AdC, Phys. Rev. Lett. 124, 210603 (2020)

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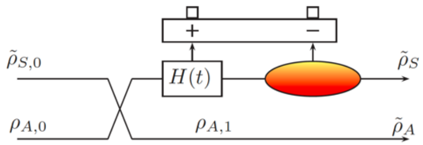
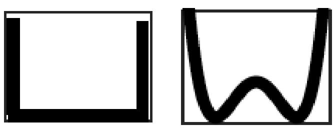
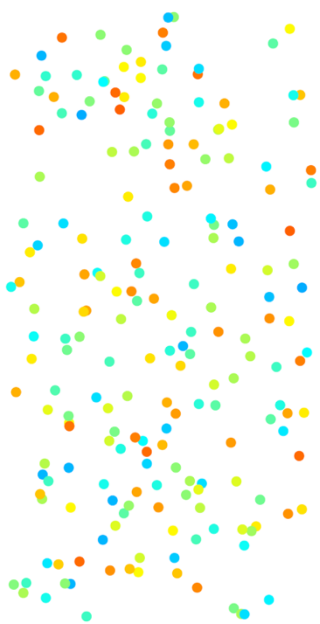
Towards many-particles QHE







Two particles (Bosons/Fermions) in quantum Szilard engine
[Kim et al. Phys. Rev. Lett.106 07040 (2011)
Bengtsson et al, Phys. Rev. Lett. 120, 100601 (2018)]

Quantum-information engines with many-body states
[Diaz de la Cruz & Martin-Delgado, Phys. Rev. A 89, 032327 (2014)]

Nointeracting particles in a nonharmonic trap
[Zheng & Poletti, Phys. Rev. E 92, 012110 (2015)]

Super-radiant quantum heat engine
[Hardal & Müstecaplıoğlu, Scientific reports 5, 12953 (2015)]

Quantum supremacy of many-particle-thermal machines
[Jaramillo, Beau, AdC, New J. Phys. 18, 075019 (2016)]

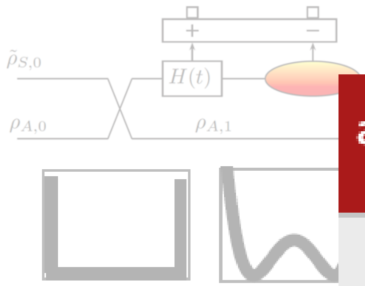
The power of a critical heat engine
[Campisi & Fazio, Nat. Commun. 7, 11895 (2016)]

Scaling-up QHE efficiently via shortcuts to adiabaticity
[Beau, Jaramillo, AdC, Entropy 18, 168 (2016)]

Cooperative many-body enhancement of quantum thermal machine power
[Niedenzu & Kurizki, NJP 20, 113038 (2018)]

Adolfo del Campo: adolfo.delcampo@uni.lu

Towards many-particles QHE



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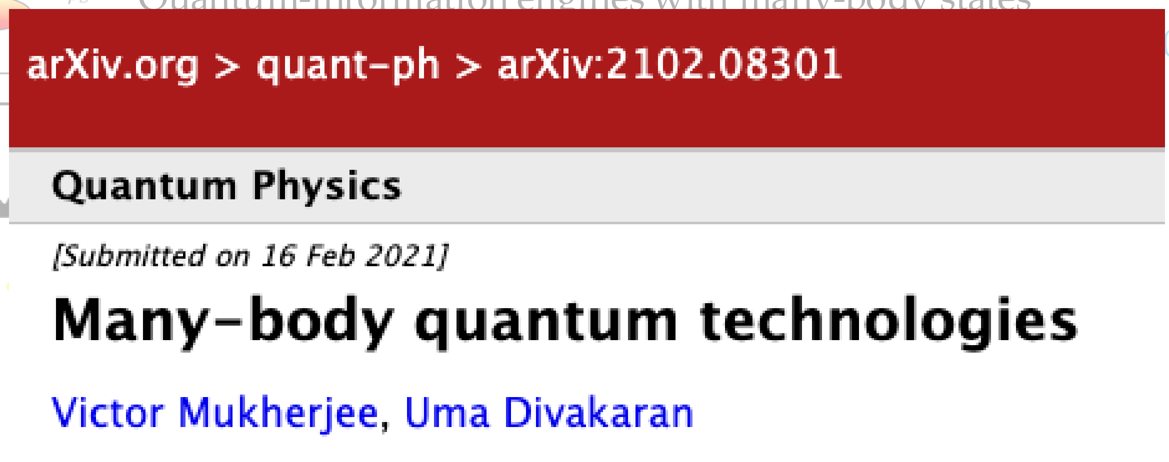
arXiv.org > quant-ph > arXiv:2102.08301

Quantum Physics

[Submitted on 16 Feb 2021]

Many-body quantum technologies

Victor Mukherjee, Uma Divakaran

The power of a critical heat engine
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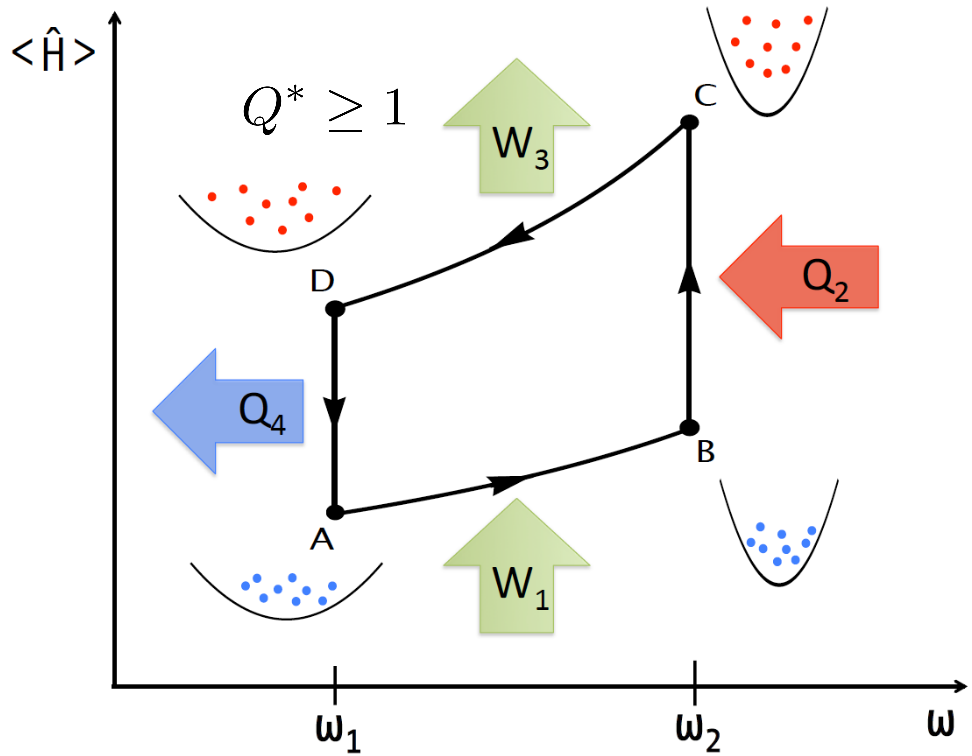
Finite-time Many-particle QHE (Otto cycle)



Finite time bound

$$\eta = \frac{W}{Q_{\rm in}} \leq 1 - Q^* \frac{\omega_1}{\omega_2} \leq 1 - \frac{\omega_1}{\omega_2}$$

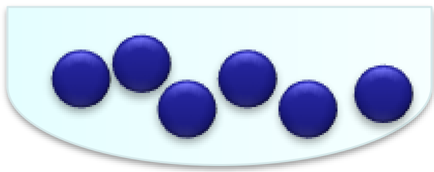
Working medium (self-similar)

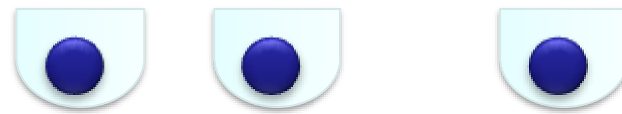
$$\hat{H} = \sum_{i=1}^{N} \left[-\frac{\hbar^2}{2m}\nabla^2 + \frac{1}{2} m\omega(t)^2 \mathbf{r}_i^2 \right] + \sum_{i<j} V(\mathbf{r}_i - \mathbf{r}_j)$$

Jaramillo et al NJP 18, 075019 (2016); Beau et al Entropy 18, 168 (2016)

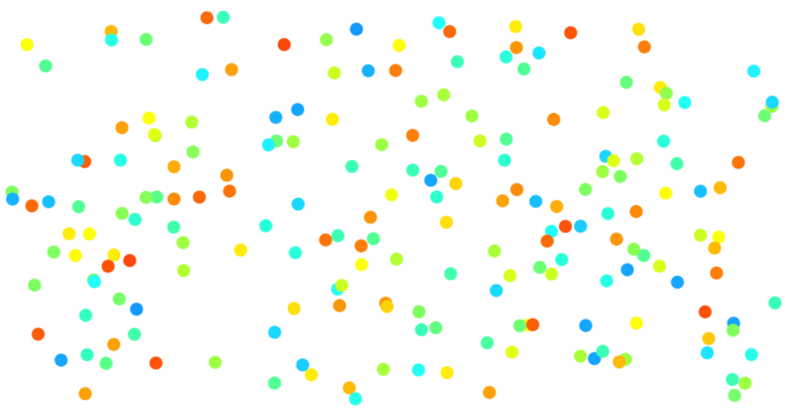
Many-particle QHE

Single N-particle engine vs N single-particle engines?





What substance is optimal as working medium?



Quantum supremacy of many-particle thermal machines

J Jaramillo[1,2], M Beau[1,3] and A del Campo[1,4]

Jaramillo et al NJP 18, 075019 (2016); Beau et al Entropy 18, 168 (2016)

Adolfo del Campo: adolfo.delcampo@uni.lu

Many-Particle Quantum Heat Engines

- ◆ Harnessing interactions
- ◆ Harnessing quantum statistics
- ◆ Harnessing quantum criticality

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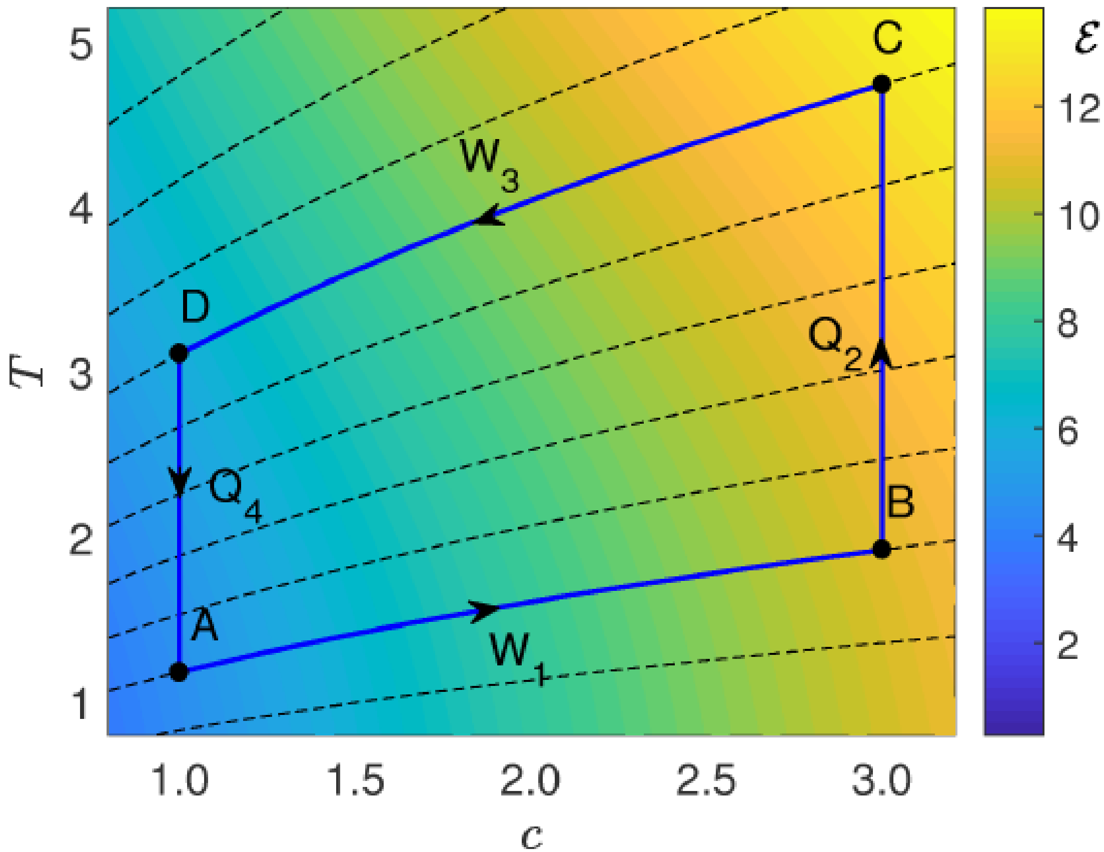
Interaction-driven QHE



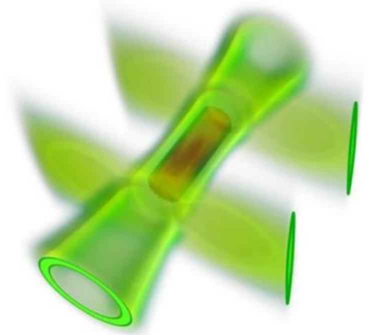
- Multi stroke cycle
- Thermal reservoirs
- Interaction strength c

2 isentropes
(interaction ramps up/down)
+
2 isochores
(heating & cooling)

Interparticle interaction strength

YY Chen et al npj Quantum Information 5, 88 (2019)

Working medium: many-body, interacting



- Ultracold bosons confined in tight-waveguide
- Effective Lieb-Liniger model: 1D bosons with contact interactions

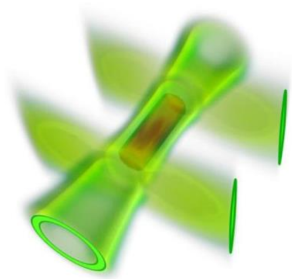
$$\hat{H}_{\rm LL} = -\sum_{j=1}^{N} \partial_{x_j}^2 + \sum_{1\leq j<\ell\leq N} 2c\delta(x_j - x_\ell)$$

- Tunable interactions via Feshbach & confinement-induced resonance
- Universal low-energy behaviour: Luttinger liquid

YY Chen et al npj Quantum Information 5, 88 (2019)

Working medium: many-body, interacting

- 1D Interacting Bose gas: Thermodynamics from coordinate Bethe-ansatz
- Finite-temperature, finite size, box trap
- Bethe Ansatz Eqs



$$\Psi_{\{\epsilon_i k_i\}}(x_1, \cdots, x_N) = \sum_P A(k_{P_1}, \cdots, k_{P_N})\, e^{\mathrm{i}\sum_{j=1}^{N} k_{P_i} x_i}$$

YY Chen et al npj Quantum Information 5, 88 (2019)

Working medium: many-body, interacting

- 1D Interacting Bose gas: Thermodynamics from coordinate Bethe-ansatz
- Finite-temperature, finite size, box trap
- Spectrum from Bethe Ansatz Eqs [M. Gaudin, Phys. Rev. A 4, 386 (1971)]

$$Lk_i = \pi I_i - \sum_{j \neq i} \left(\arctan \frac{k_i - k_j}{c} + \arctan \frac{k_i + k_j}{c} \right)$$

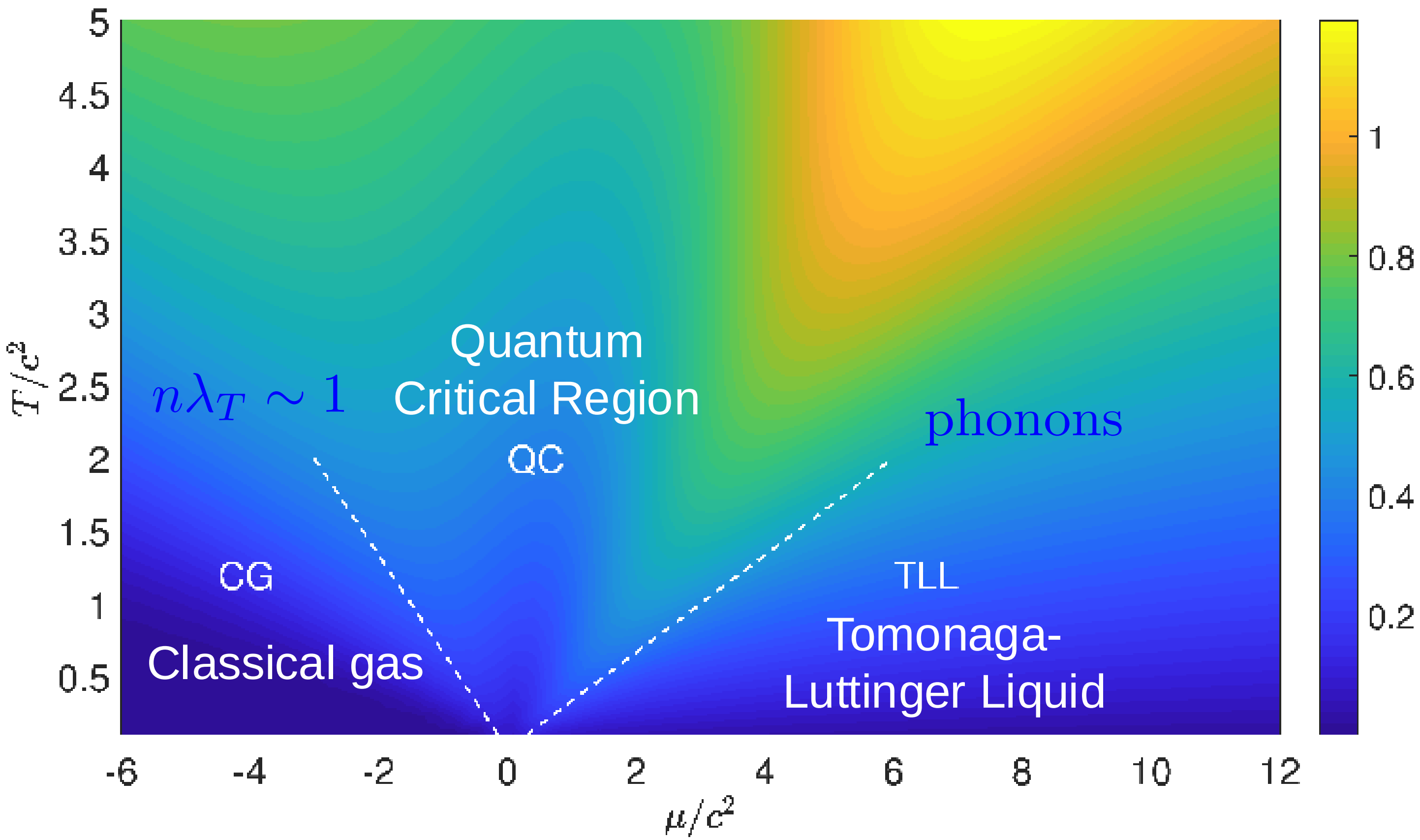
Set of Integer quantum numbers label an energy eigenstate

$$\mathcal{I}_n = \{I_i^{(n)}\} \quad \Longrightarrow \quad \{k_{n,i}\} \quad \Longrightarrow \quad \epsilon_n = \sum_i k_{n,i}^2$$

Numerically tractable

YY Chen et al npj Quantum Information 5, 88 (2019)

Working medium: phase diagram



Selected for a Viewpoint in *Physics*
PRL **119,** 165701 (2017) PHYSICAL REVIEW LETTERS week ending 20 OCTOBER 2017

Quantum criticality and the Tomonaga-Luttinger liquid in one-dimensional Bose gases

Bing Yang,[1,3] Yang-Yang Chen,[2] Yong-Guang Zheng,[1,3] Hui Sun,[1,3] Han-Ning Dai,[1,3]
Xi-Wen Guan,[2,4,*] Zhen-Sheng Yuan,[1,3,5,6,†] and Jian-Wei Pan[1,3,5,6,‡]

Working medium: Strong coupling

- Large interactions/system size L

$$\epsilon_n(c) \approx \frac{\pi^2 \lambda_c}{L^2} \sum_{i=1}^{N} I_i^{(n)2} \qquad \lambda_c = 1 - \frac{4(N-1)}{cL} + \frac{12(N-1)^2}{c^2 L^2}$$

- Emergent scale invariance

$$\frac{\epsilon_n(c)}{\epsilon_n(c')} = \frac{\lambda_c}{\lambda'_c}$$

- At large coupling only, ID-QHE = generalized Otto cycle

$$W = Q_2 - Q_4 = [1 - (\lambda_{c_A}/\lambda_{c_B})]Q_2 \quad \Longrightarrow \quad \boxed{\eta = 1 - \frac{\lambda_{c_A}}{\lambda_{c_B}}}$$

YY Chen et al npj Quantum Information 5, 88 (2019)

Universality

- Low-temperature: excitations are phonons

- Universal Tomonaga-Luttinger Liquid (TLL)

Free energy density $$\mathcal{F} = \mathcal{E}_0 - \frac{\pi T^2}{6 v_s}$$

$$N, L \to \infty \quad n = N/L$$

YY Chen et al npj Quantum Information 5, 88 (2019)

Universality

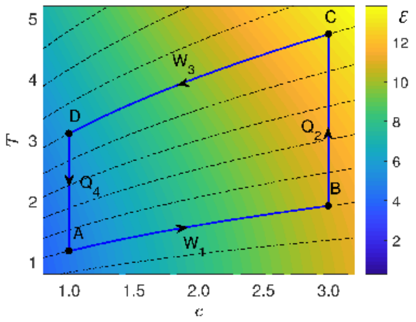
- Low-temperature: excitations are phonons

- Universal Tomonaga-Luttinger Liquid (TLL)

Free energy density
$$\mathcal{F} = \mathcal{E}_0 - \frac{\pi T^2}{6 v_s}$$

Entropy density
$$s = -\frac{\partial \mathcal{F}}{\partial T} = \frac{\pi T}{3 v_s}$$

Heat exchange
$$Q_2 = L \int_{s_B}^{s_C} T ds = \frac{\pi L}{6 v_s^B}(T_C^2 - T_B^2)$$
$$Q_4 = L \int_{s_A}^{s_D} T ds = \frac{\pi L}{6 v_s^A}(T_D^2 - T_A^2)$$



YY Chen et al npj Quantum Information 5, 88 (2019)

Adolfo del Campo: adolfo.delcampo@uni.lu

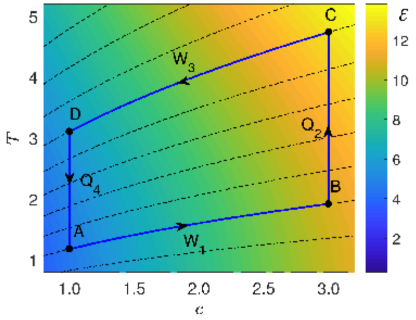
Working medium: Universality

- Low-temperature: excitations are phonons

- Universal Tomonaga-Luttinger Liquid (TLL)

Isentropes

$$\xi \equiv \frac{T_A}{T_B} = \frac{T_D}{T_C} = \frac{v_s^A}{v_s^B}$$



YY Chen et al npj Quantum Information 5, 88 (2019)

Working medium: Universality

- Low-temperature: excitations are phonons
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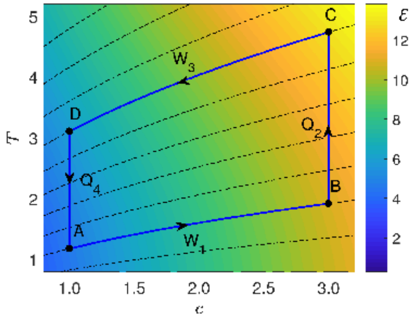
Isentropes

$$\xi \equiv \frac{T_A}{T_B} = \frac{T_D}{T_C} = \frac{v_s^A}{v_s^B}$$

Universal performance

Work output

$$W_{\mathrm{TLL}} = \frac{\pi L T_C^2}{6 v_s^B}(1-\xi)\left(1 - \frac{\kappa^2}{\xi^2}\right)$$



$\kappa = T_A/T_C$

YY Chen et al npj Quantum Information 5, 88 (2019)

Working medium: Universality

- Low-temperature: excitations are phonons
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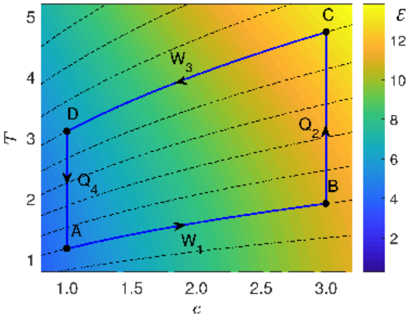
Isentropes

$$\xi \equiv \frac{T_A}{T_B} = \frac{T_D}{T_C} = \frac{v_s^A}{v_s^B}$$

Universal performance

Efficiency

$$\eta_{\mathrm{TLL}} = 1 - \frac{v_s^A}{v_s^B} = 1 - \xi$$



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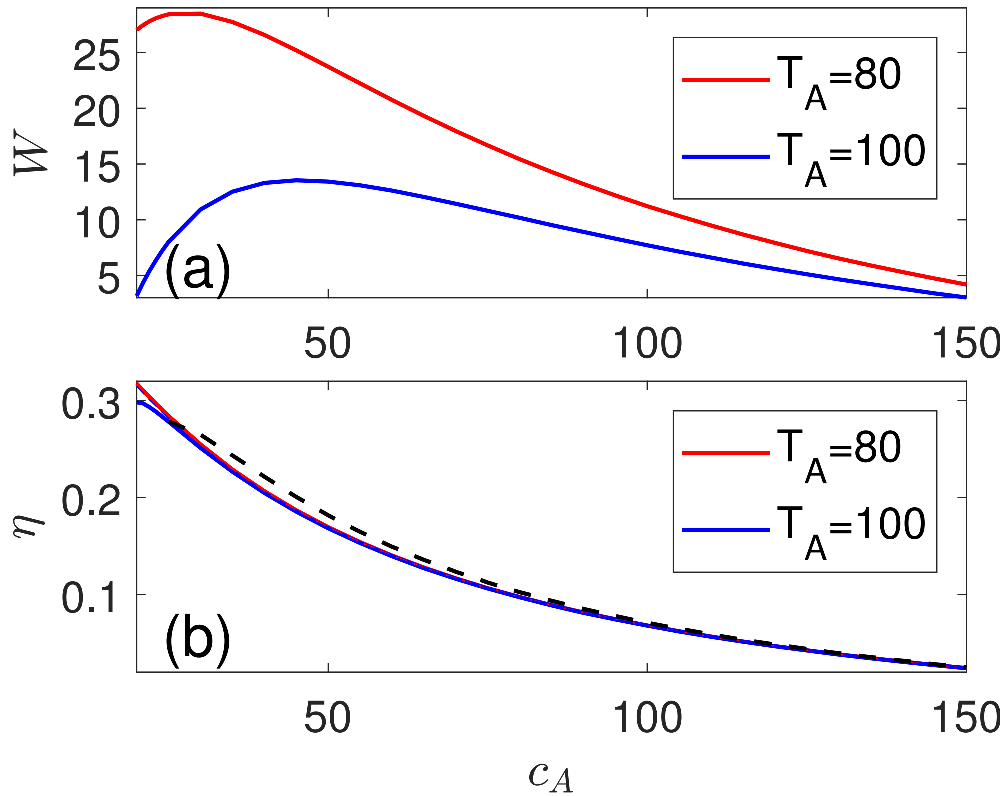
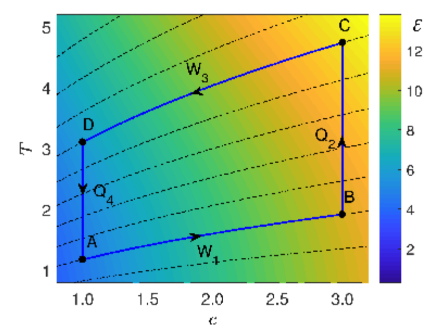
Work output and efficiency



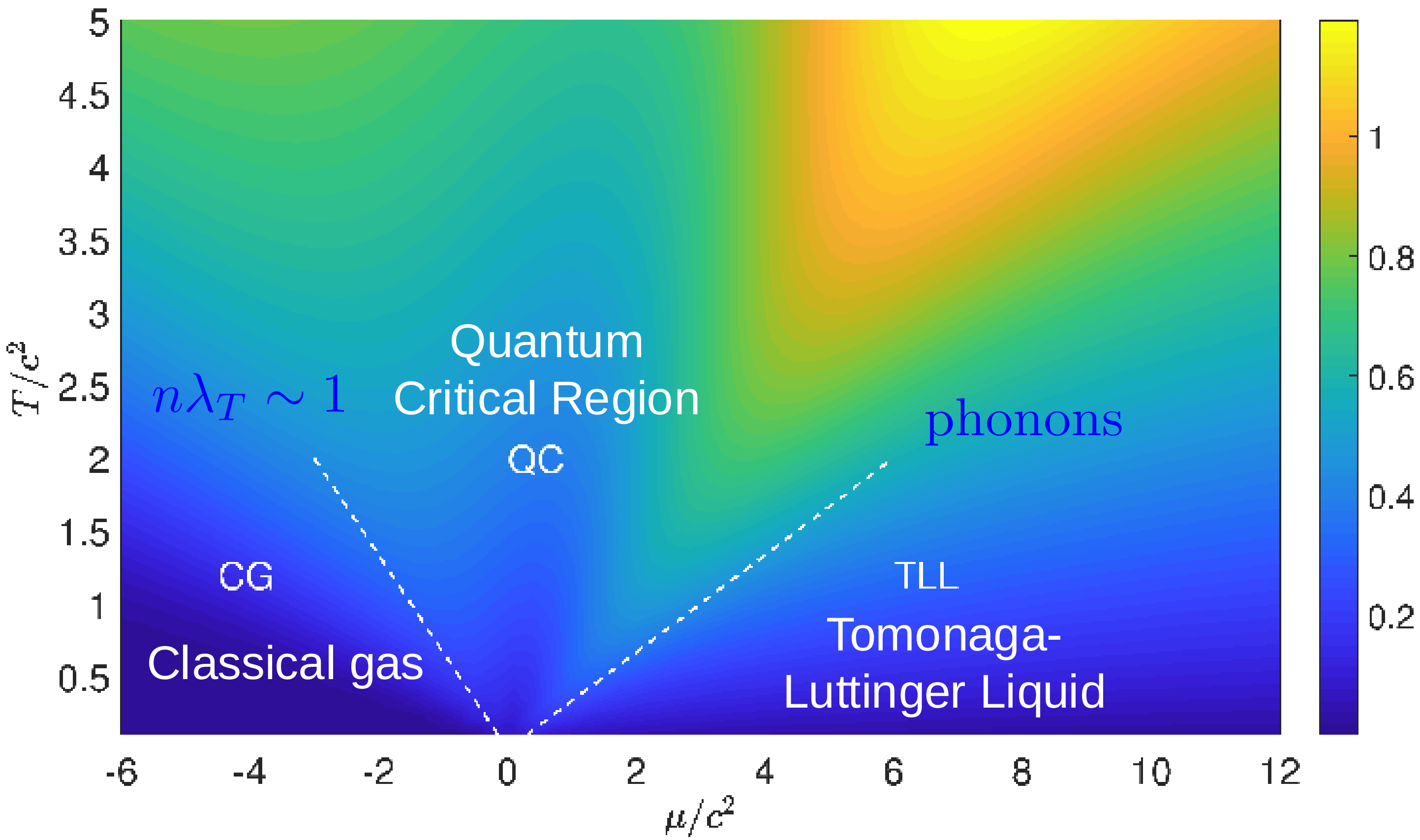


Max work

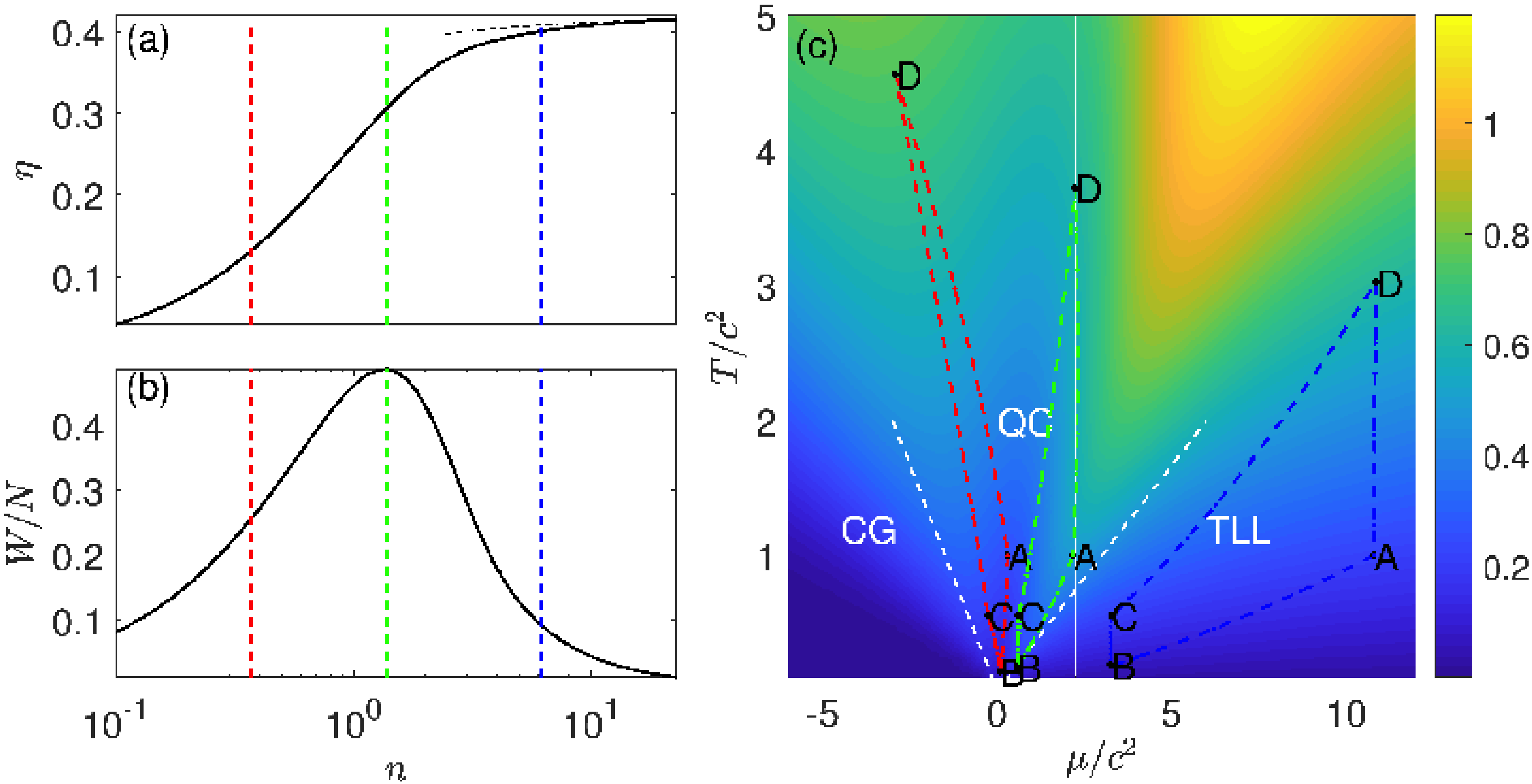
$$\xi_c \simeq (2\kappa^2)^{\frac{1}{3}} [1 - (\kappa/2)^{\frac{2}{3}}/3], \quad \kappa = T_A/T_C$$

YY Chen et al npj Quantum Information 5, 88 (2019)

Working medium: phase diagram



Interaction-driven QHE



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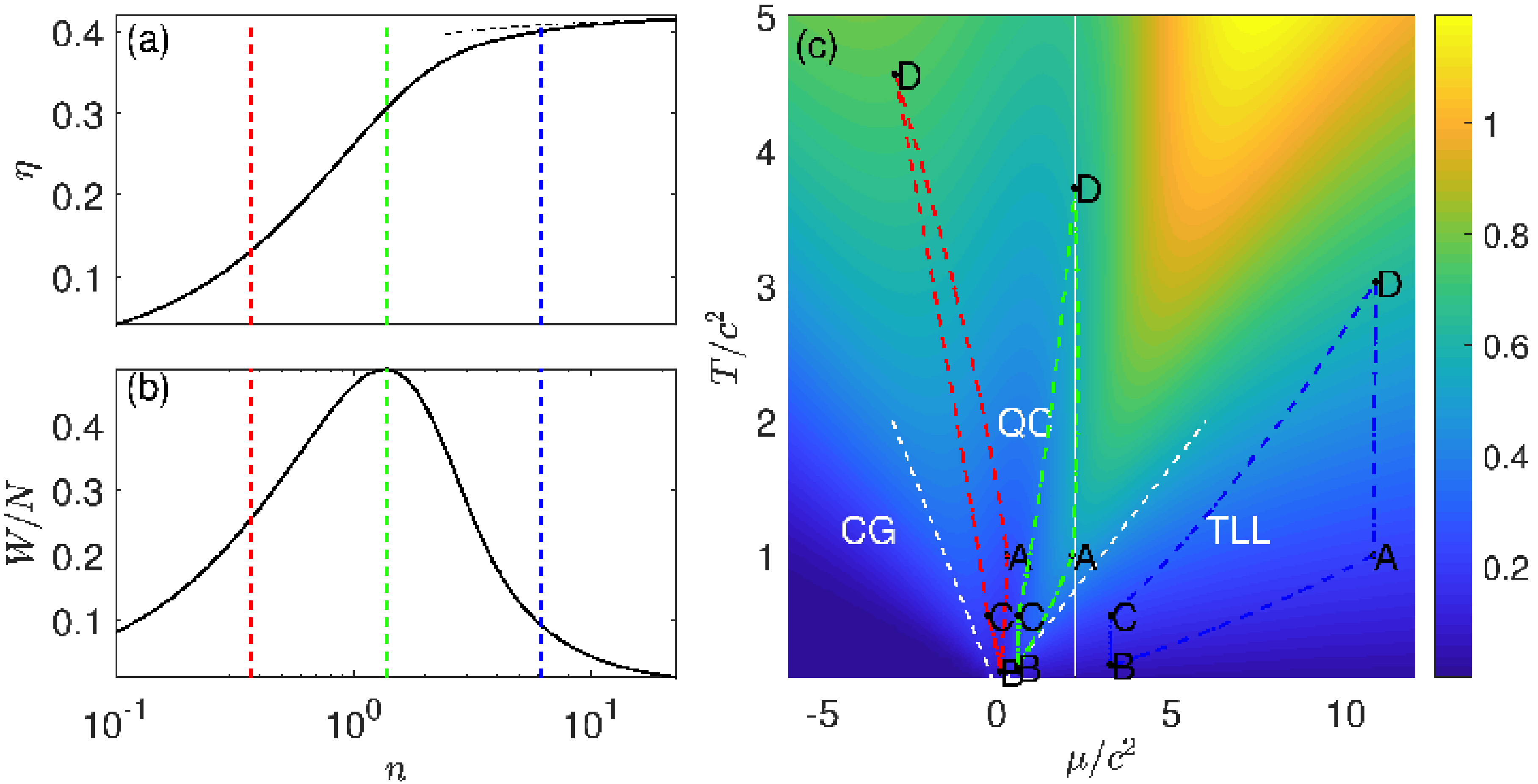

Optimal performance at quantum criticality!

YY Chen et al npj Quantum Information 5, 88 (2019)

Many-Particle Quantum Heat Engines

- Harnessing interactions

- Harnessing quantum statistics

- Harnessing quantum criticality

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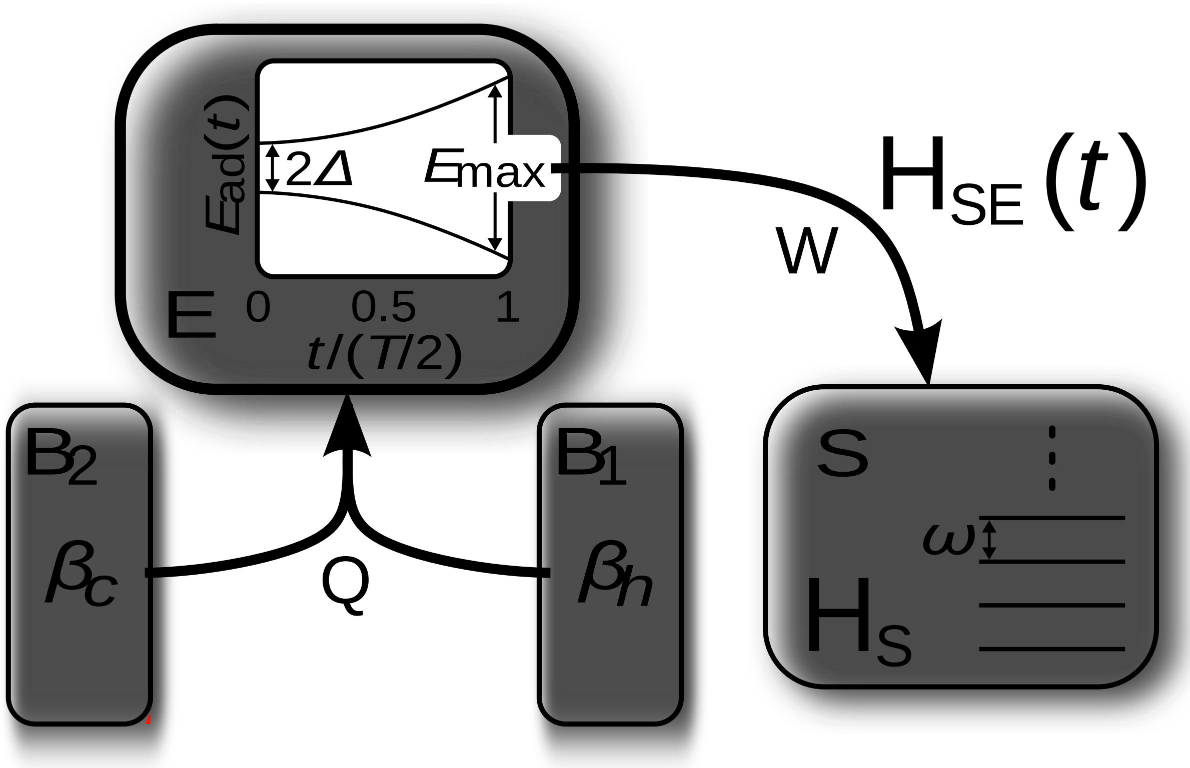
Quantum performance over n-cycles

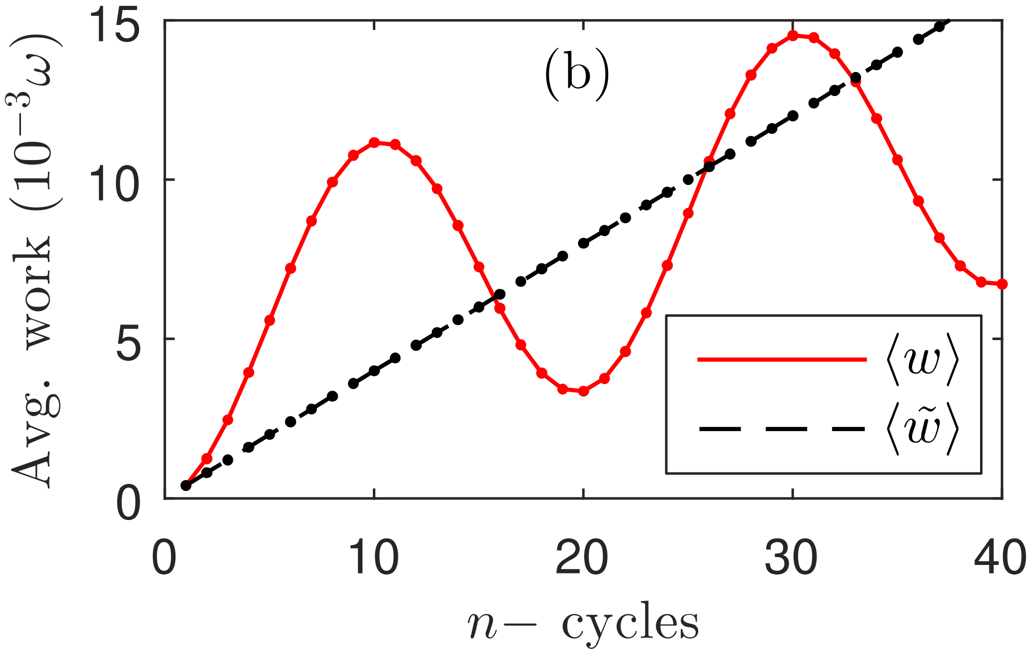


Measure work after 1 cycle
Measure work after n cycles

Watanabe, Venakatesh, Talkner, AdC, PRL 118, 050601 (2017)

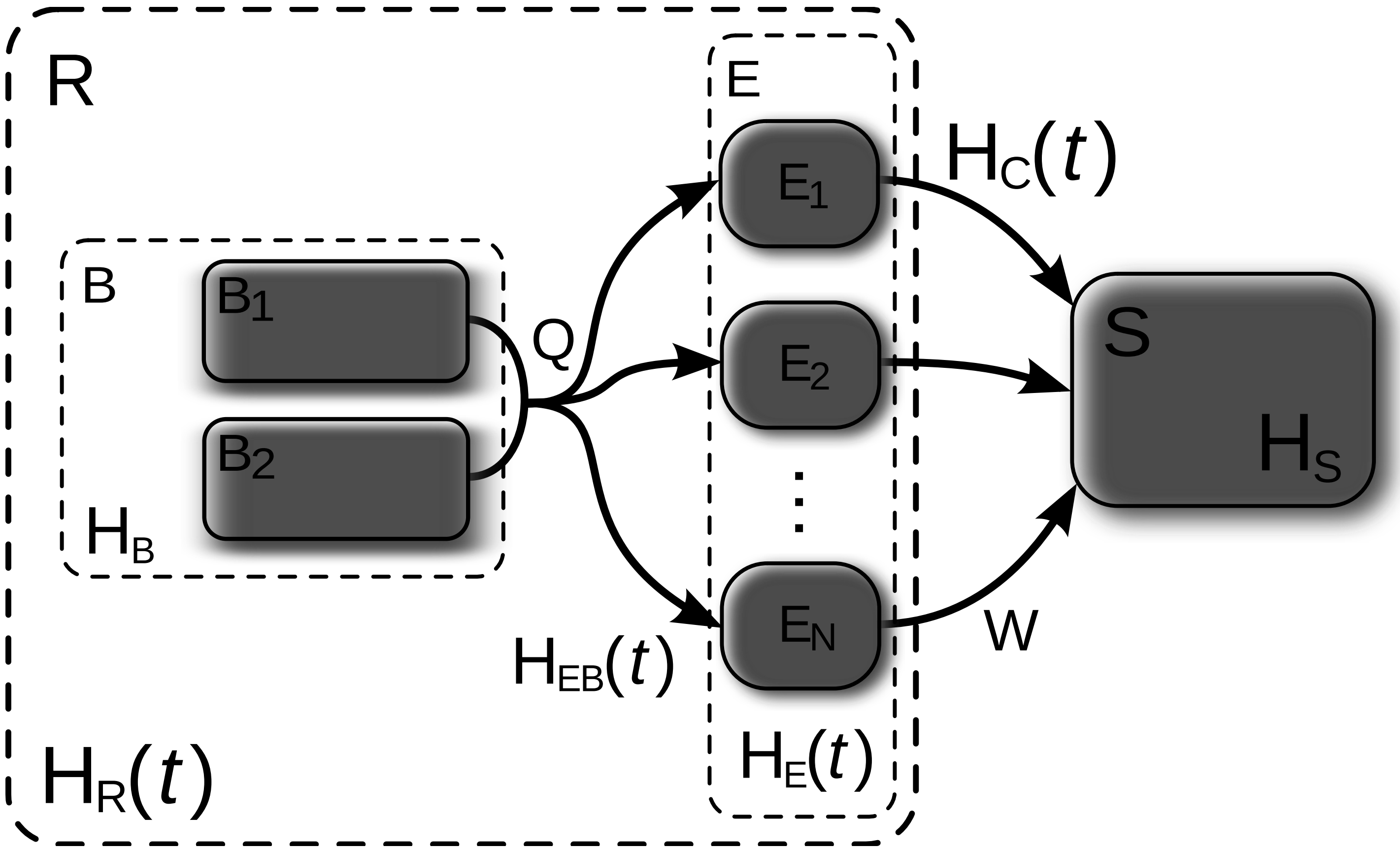
Quantum performance over many cycles

Efficiency over one cycle does not carry over many cycles



Watanabe, Venakatesh, Talkner, AdC, PRL 118, 050601 (2017)

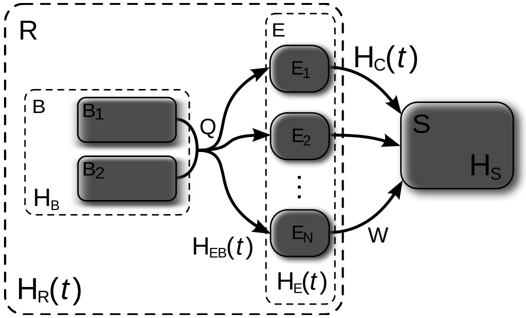
QHE Ensemble



Watanabe et al. PRL 124, 210603 (2020)

QHE Ensemble

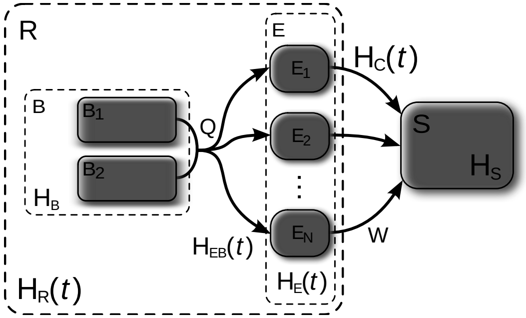
- Each QHE runs Otto cycle
- Couples to work storage

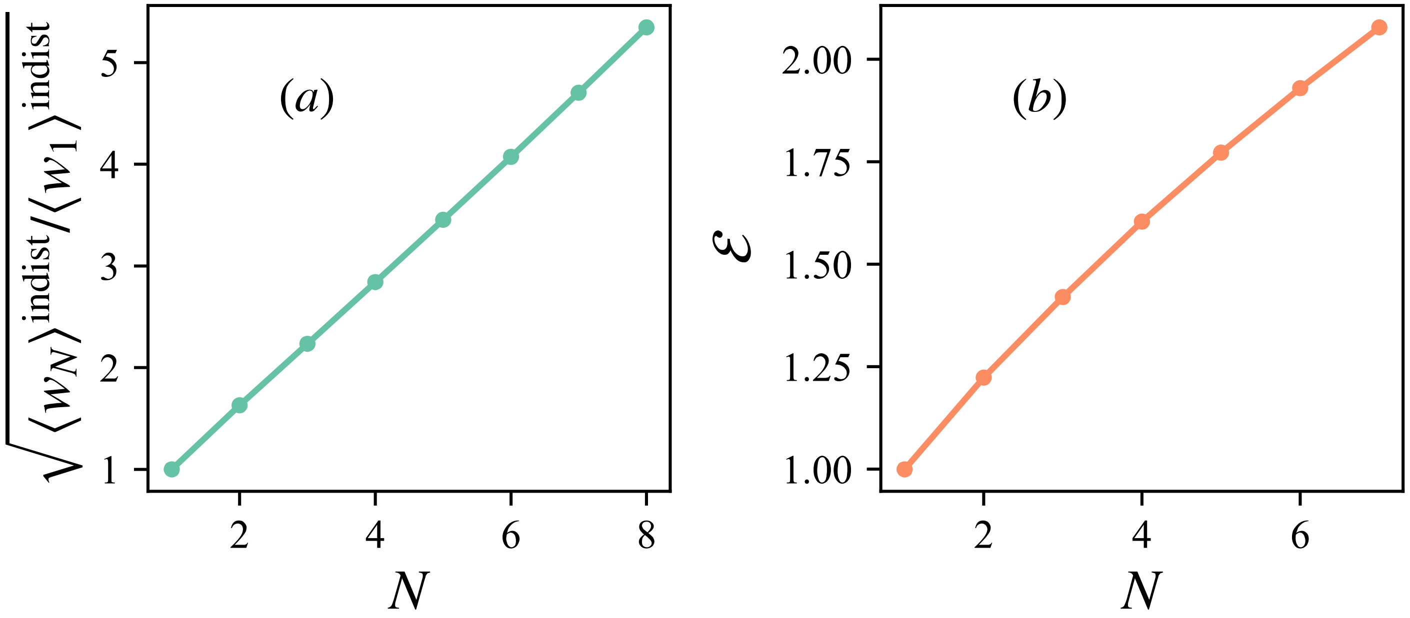


Watanabe et al. PRL 124, 210603 (2020)

Bosonic QHE Ensemble: Work output

- Each QHE runs Otto cycle
- Couples to work storage

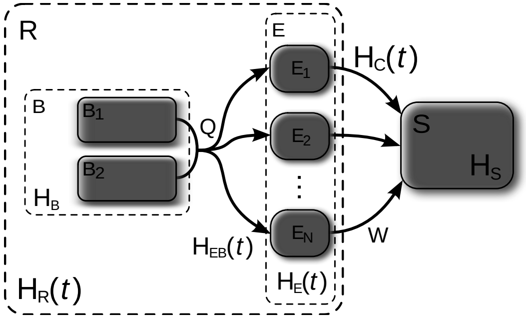


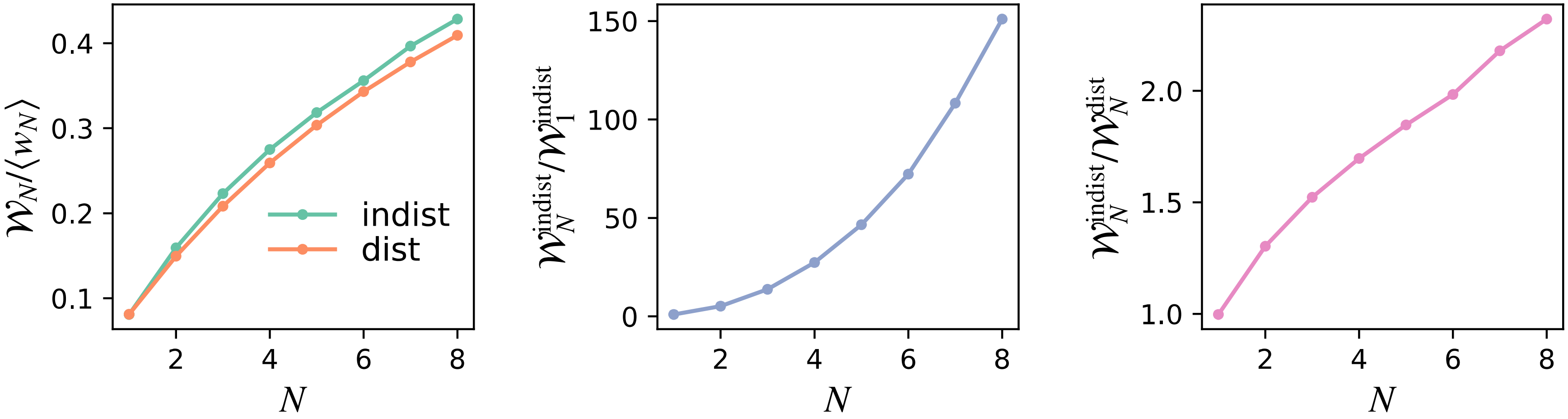


Watanabe et al. PRL 124, 210603 (2020)

Bosonic QHE Ensemble: ergotropy

- Each QHE runs Otto cycle
- Couples to work storage

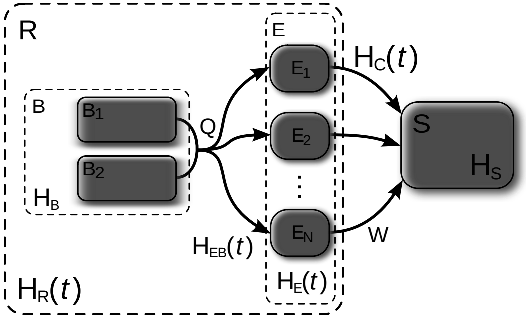


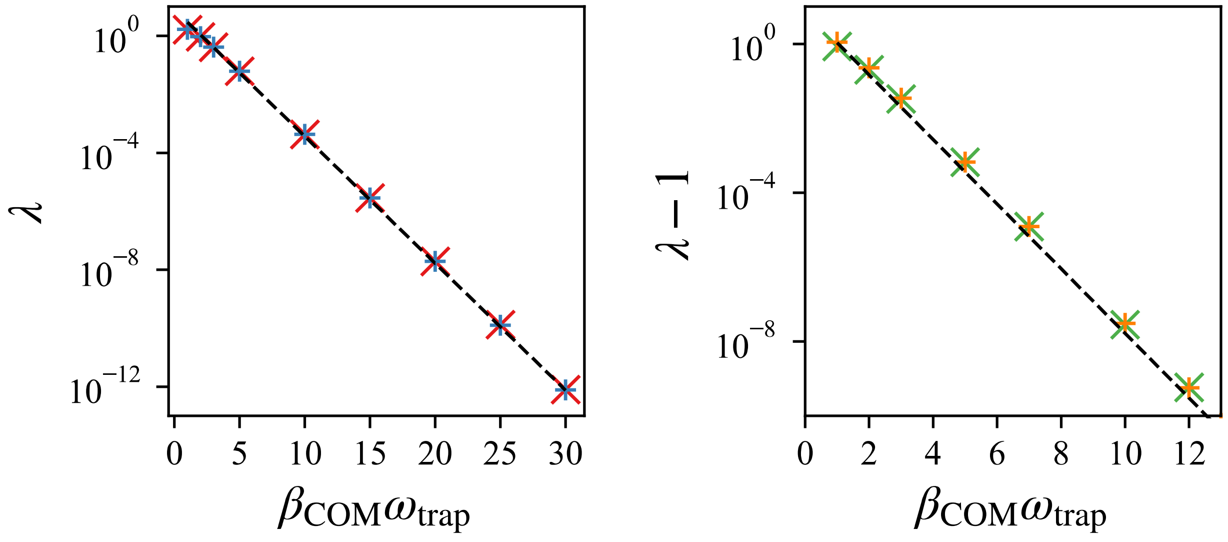


Watanabe et al. PRL 124, 210603 (2020)

Fermionic QHE Ensemble: don't

$$\lambda = \frac{\langle w_N \rangle}{\langle w_1 \rangle} \simeq 8 \exp\left(-\beta_{\rm COM}\omega_{\rm trap}\right) \qquad (\text{even} \quad N)$$
$$\simeq 1 + 8 \exp\left(-2\beta_{\rm COM}\omega_{\rm trap}\right) \qquad (\text{odd} \quad N)$$





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Many-Particle Quantum Heat Engines

- ◆ Harnessing interactions
- ◆ Harnessing quantum statistics
- ◆ Harnessing quantum criticality

I) YY Chen, G Watanabe, YC Yu, XW Guan, AdC, npj Quantum Information 5, 88 (2019)

II) G Watanabe, BP Venkatesh, P Talkner, MJ Hwang, AdC, Phys. Rev. Lett. 124, 210603 (2020)

III) Revathy B S, V Mukherjee, U Divakaran, AdC, Phys. Rev. Research 2, 043247 (2020)

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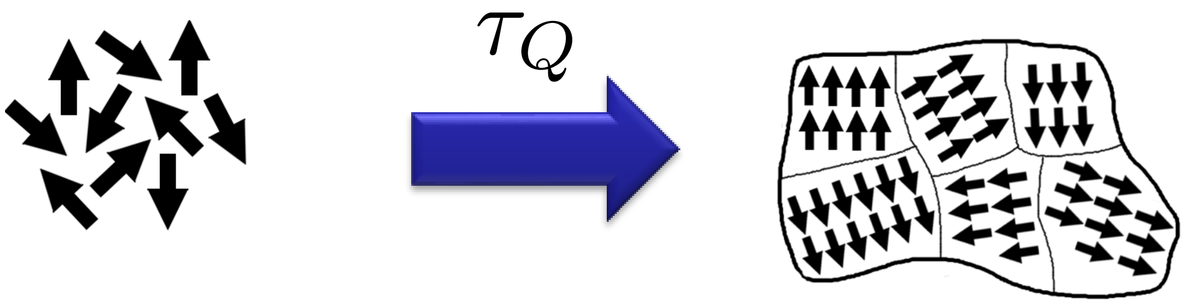
Kibble-Zurek Mechanism



Domain size: **universal** power law scaling with quench time

$$\hat{\xi} = \xi(\hat{t}) = \xi_0 \left(\frac{\tau_Q}{\tau_0} \right)^{\frac{\nu}{1+z\nu}}$$

T. W. B. Kibble, JPA 9, 1387 (1976)
T. W. B. Kibble, Phys. Rep. 67, 183 (1980)
W. H. Zurek, Nature (London) 317, 505 (1985)
W. H. Zurek, Acta Phys. Pol. B. 1301 (1993)

KZM in quantum systems: Ising chain

Ising chain is equivalent to ensemble of two-level systems

$$\mathcal{H} = -J\sum_{m=1}^{N}(\sigma_m^z\sigma_{m+1}^z + g\sigma_m^x) = \sum_k E_k(\gamma_k^\dagger\gamma_k - 1/2)$$

Kink number operator: number of kinks = number of excited two-level systems

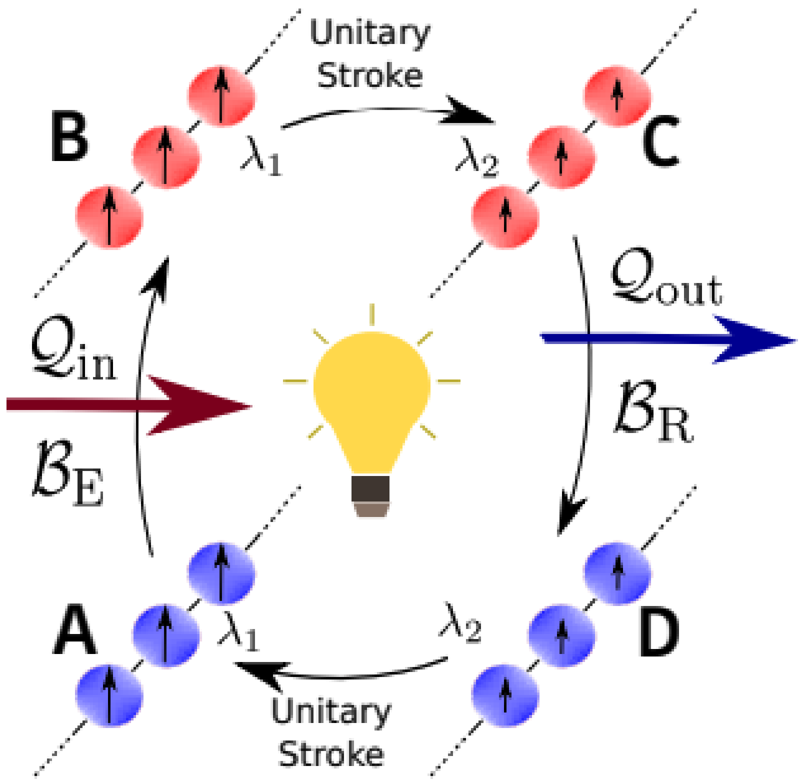
$$\hat{\mathcal{N}} = \frac{1}{2}\sum_{m=1}^{N}(1-\sigma_m^z\sigma_{m+1}^z) = \sum_k \gamma_k^\dagger\gamma_k$$

Sweep from paramagnet to ferromagnet: mean kink number

$$g(t) = -\frac{t}{\tau_Q} \qquad \langle n\rangle = Nd = N\frac{1}{2\pi}\sqrt{\frac{\hbar}{2J\tau_Q}}$$

2005
Polkovnikov
Damski
Dziarmaga
Zurek, Dorner, Zoller

Harnessing quantum criticality



Working substance

$$H = \sum_k \Psi_k^\dagger \tilde{H}_k \Psi_k$$

$$\tilde{H}_k = (\lambda + a_k)\sigma^z + b_k\sigma^+ + b_k^*\sigma^-$$

Quantum Critical Point

$$\Delta = 2\sqrt{(\lambda + a_k)^2 + |b_k|^2}$$

$$\lim_{\lambda\to\lambda_c} \Delta\Big|_{k_c} = 0$$

Fermionic baths

$$\rho(t) = \bigotimes_k \rho_k(t)$$

$$\frac{d\rho_k}{dt} = -i[H_k, \rho_k] + \mathcal{D}_k[\rho_k]$$

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Harnessing quantum criticality

Finite-time thermodynamics

$$\eta = \frac{\mathcal{Q}_{\rm in} + \mathcal{Q}_{\rm out}}{\mathcal{Q}_{\rm in}} = -\frac{\mathcal{W}}{\mathcal{Q}_{\rm in}} \qquad \mathcal{P} = -\frac{\mathcal{Q}_{\rm in} + \mathcal{Q}_{\rm out}}{\tau_{\rm total}}$$

Universal when working substance is cooled near its ground state

$$\mathcal{P} = \frac{\mathcal{W}}{\tau_{\rm total}} \approx \frac{\mathcal{W}_\infty}{\tau_2} + R\tau_2^{-\frac{\nu d + x\nu z + 1}{\nu z + 1}}$$

Optimal finite-time thermodynamics at maximum power

$$\tau_{\rm opt} = \left[\frac{R\,(\nu d + x\nu z + 1)}{|\mathcal{W}_\infty|\,(\nu z + 1)}\right]^{(\nu z+1)/[\nu d + (x-1)\nu z]}$$

$$\hat{\eta} = -\frac{\mathcal{W}_\infty + \mathcal{E}_{\rm ex,A}(\tau_{\rm opt})}{\mathcal{E}_{\rm B} - \mathcal{E}_{\rm A}^{\rm G} - \mathcal{E}_{\rm ex,A}(\tau_{\rm opt})}$$

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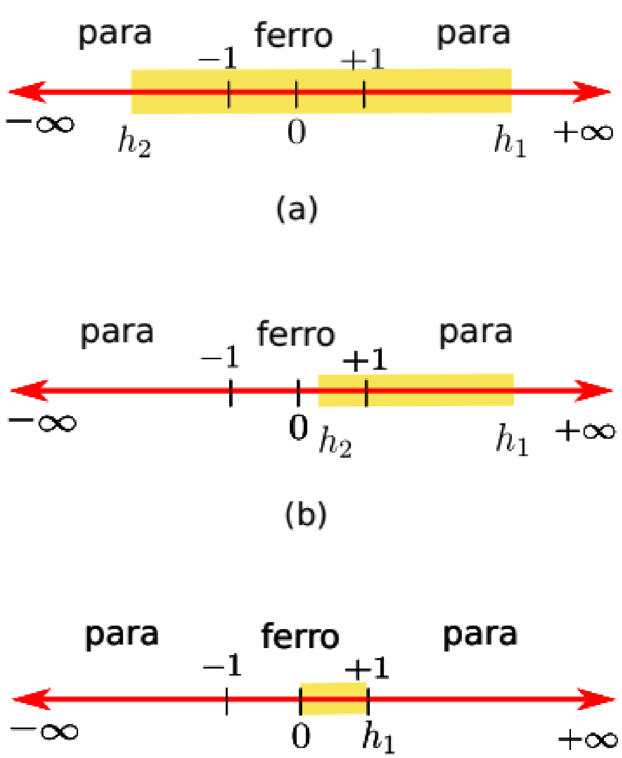
Harnessing quantum criticality

1d Quantum Ising chain



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Harnessing quantum criticality

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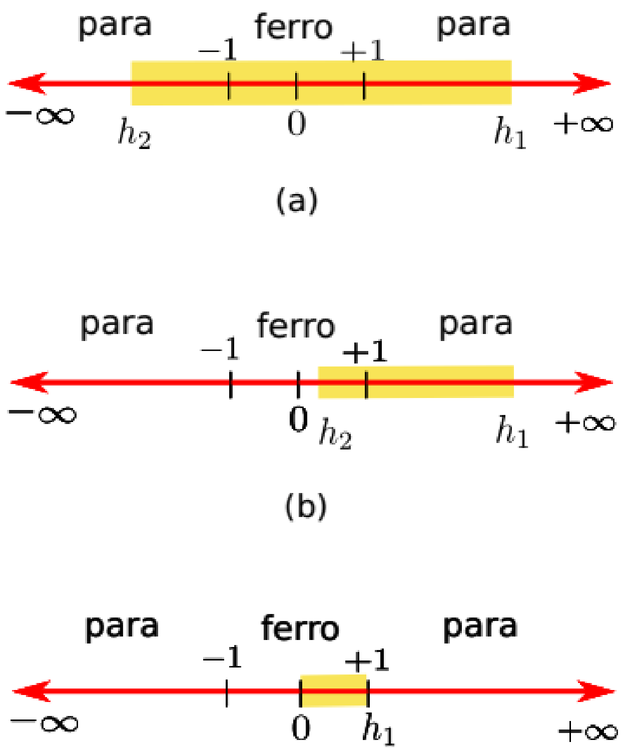

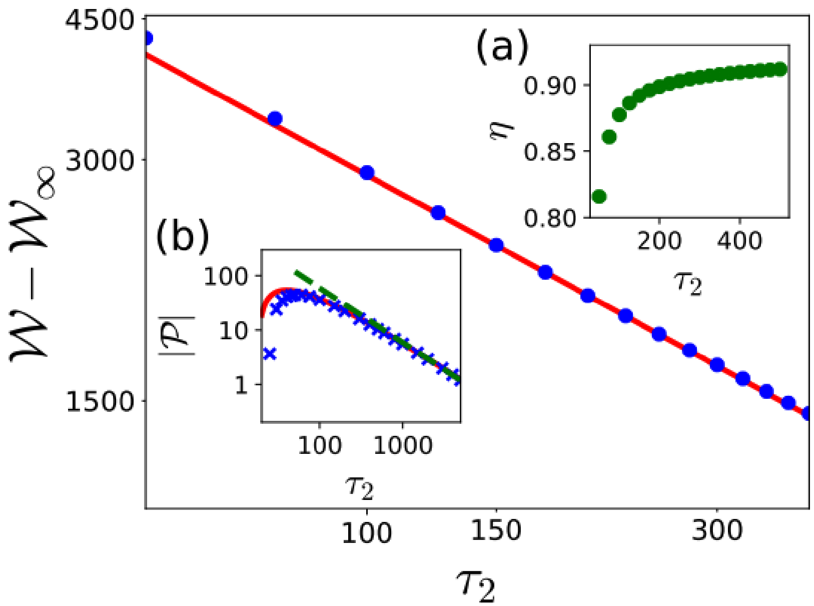

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