Reassessing Quantum-Thermodynamic Enhancements in Continuous Thermal Machines

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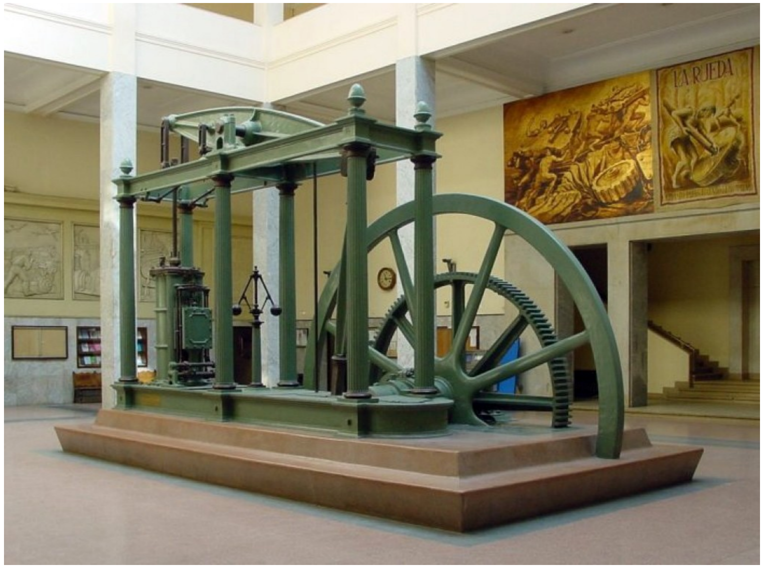
Carnot Workshop on Quantum Thermodynamics and Open Quantum Systems

25-27 November 2024, Dijon, France

Macroscopic (classical) heat engines



(Watt's steam engine, 1832
D. Napier & Son, London)

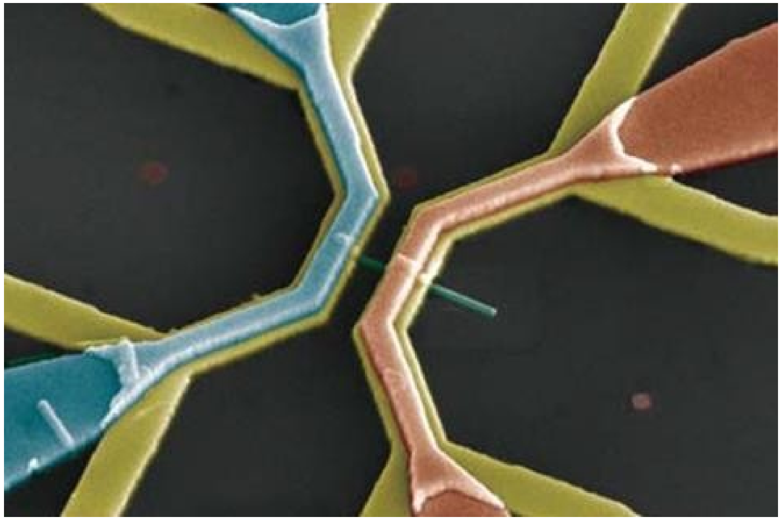
+ Large number of degrees of freedom

+ Fluctuations become negligible

+ Classical thermodynamics

"Réflexions sur la Puissance Motrice du Feu" S.Carnot

Microscopic (quantum) heat engines



(Quantum-dot engine, 2018
H. Linke group, Sweden)

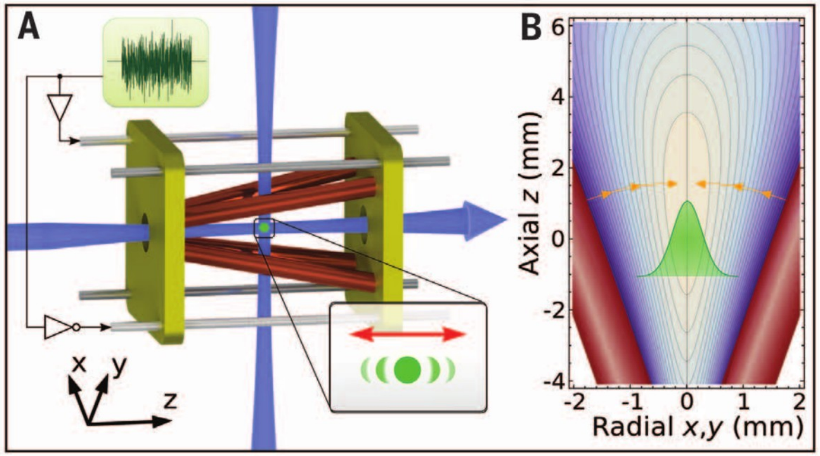
+ Small systems (micro or nano scale)

+ Fluctuations are important

+ Stochastic and quantum thermodynamics

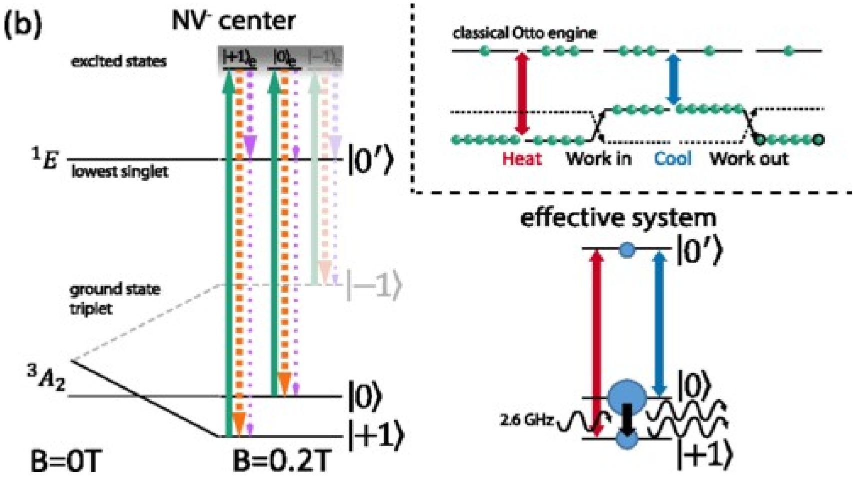
Motivation

Single-ion cyclic quantum engine



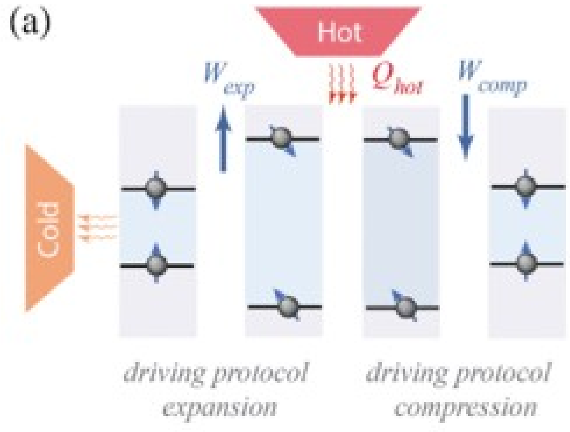
Roßnagel *et al.* Science (2016)

Continuous engine with NV centers



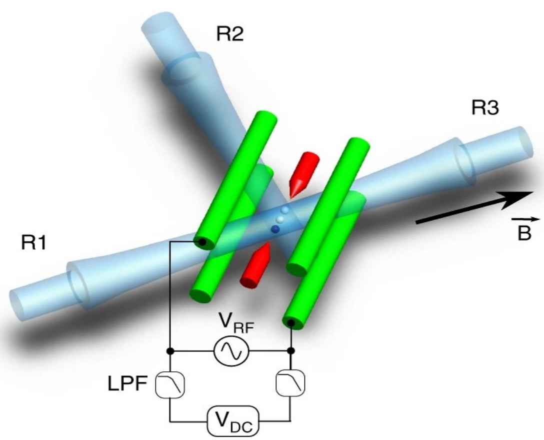
Klatzow *et al.* Phys. Rev. Lett **122** (2019)

Spin Otto cycle with NMR techniques



Peterson *et al.* Phys. Rev. Lett **123** (2019)

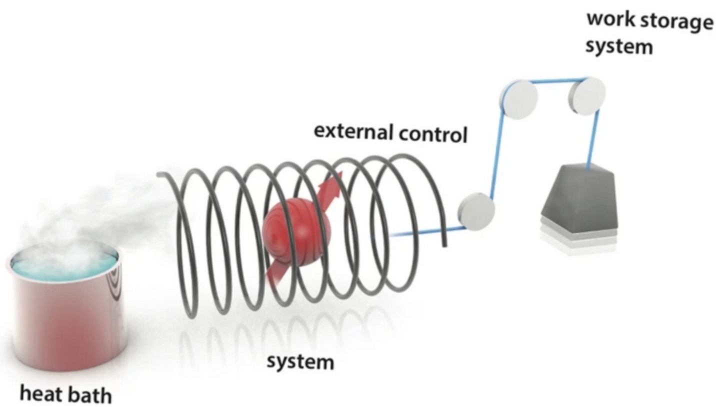
Quantum absorption refrigerator

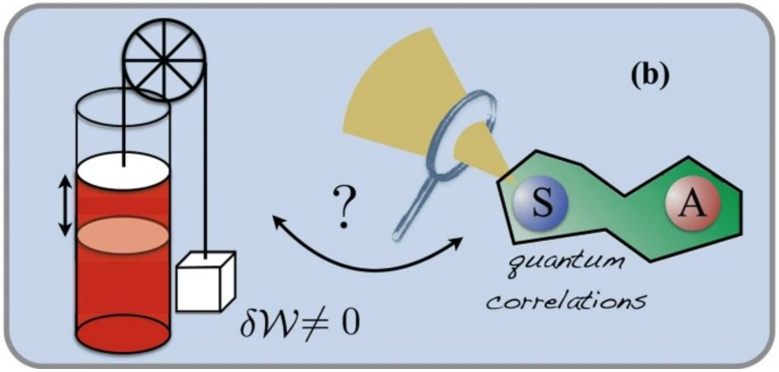


Maslennikov *et al.* Nat Commun. (2019)

Quantum-thermodynamic advantage?

Quantum coherence and quantum correlations are thermodynamic resources that contain **free energy** (and **ergotropy**) that can be extracted as work





Kammerlander & Anders, Sci. Rep. **6**, 22174 (2016)

Korzekwa et al. New J. Phys **18**, 023045 (2016)

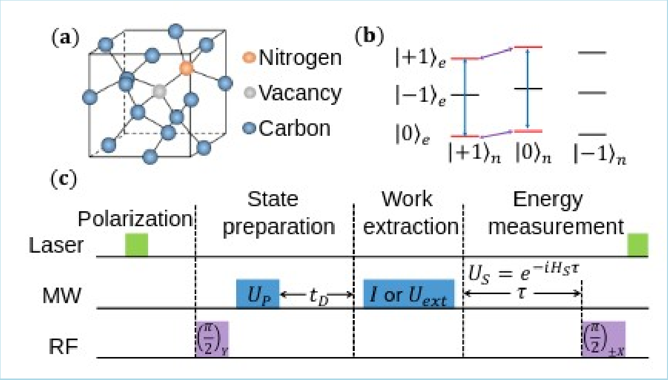
Skrzypczyk et al. Nat. Commun. **5**, 4185 (2014)

Francica et al. npj Quantum Information **3**, 12 (2017)

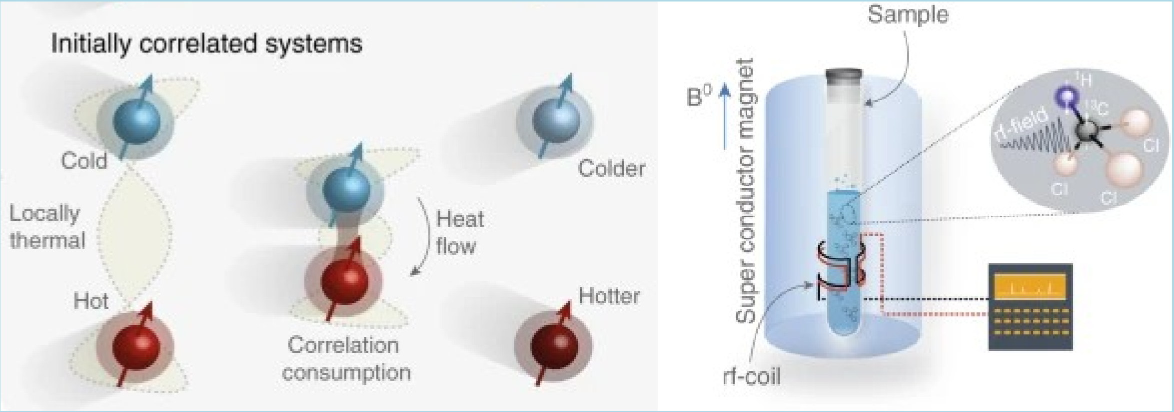
Manzano et al. Phys. Rev. Lett. **121**, 120602 (2018)

Perarnau-Llobet et al. PRX **5**, 041011 (2015)

Niu et al. Phys. Rev. Lett. **133**, 180401 (2024)



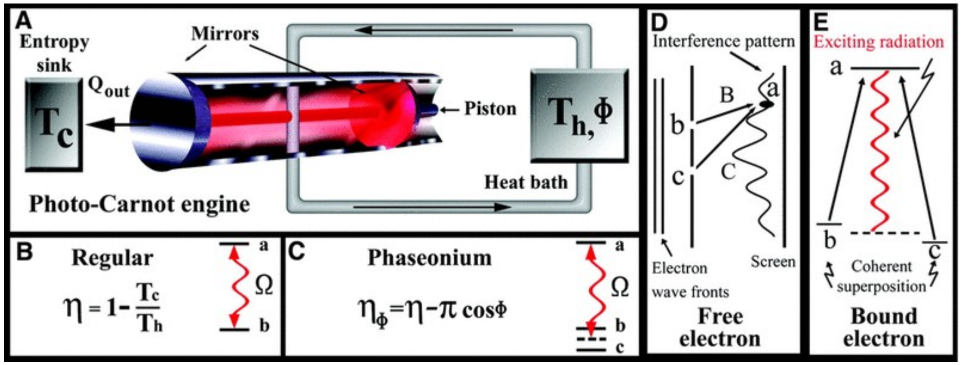
Micadei et al. Nat. Commun. **10**, 2456 (2019)



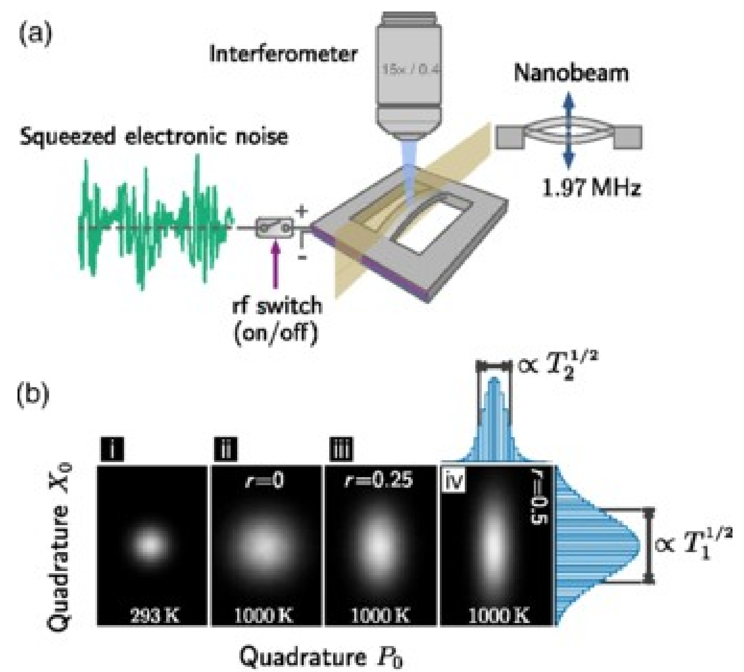
Quantum resources for thermal operation

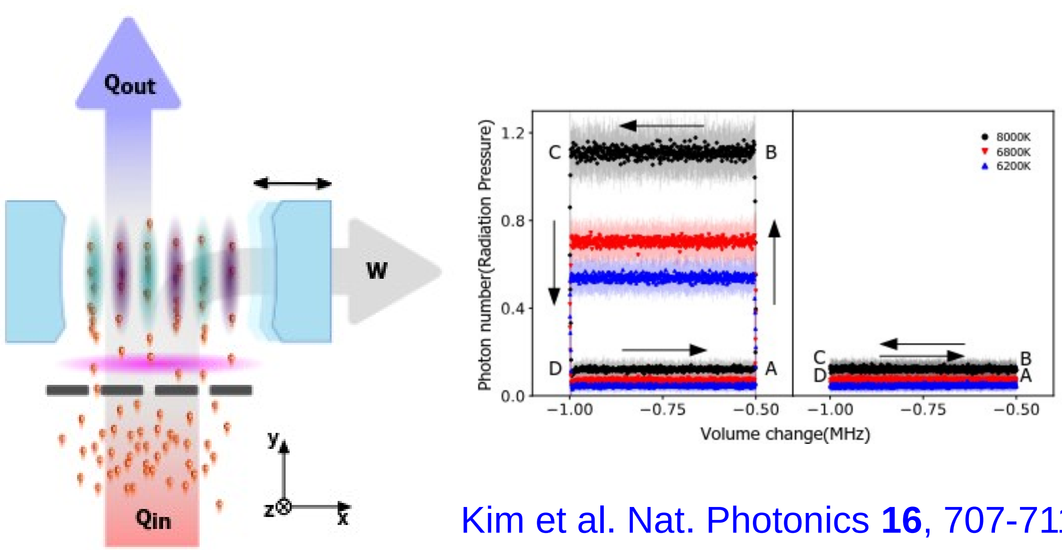
Quantum thermal machines may benefit and be powered by quantum resources

(… in contrast to classical thermal machines).



Scully et al. Science **299**, 862 (2003)



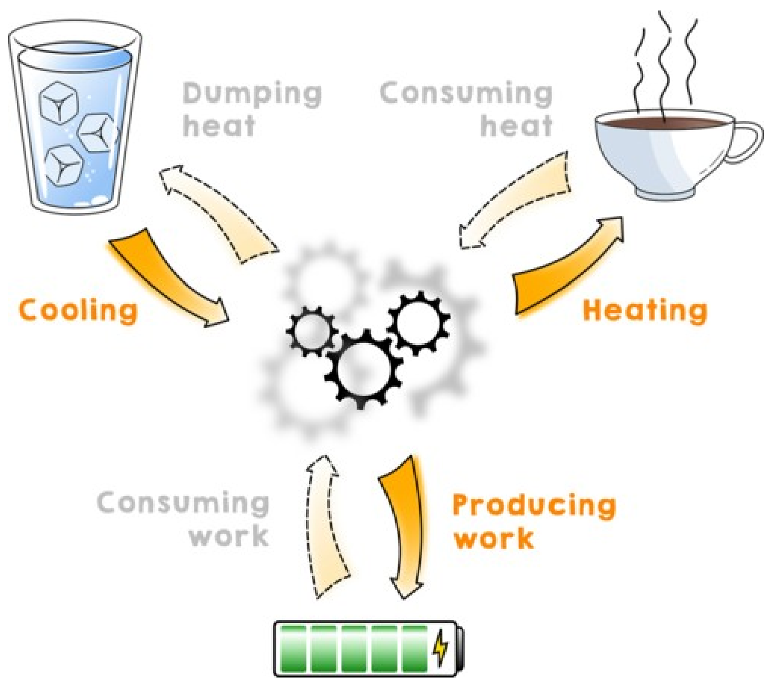


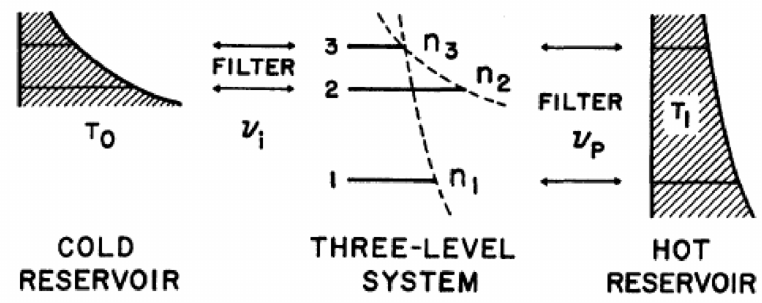
Klaers et al. Phys. Rev. X **7**, 031044 (2017)

Kim et al. Nat. Photonics **16**, 707-711 (2022)

Can quantum engines outperform classical ones using the same amount of resources?

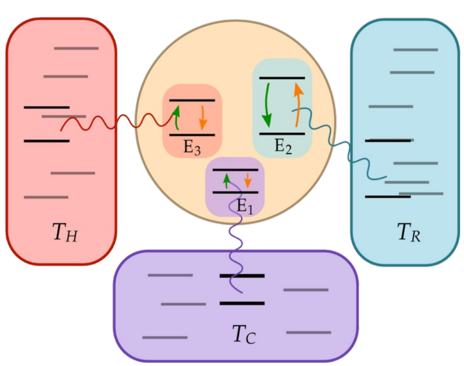
Focus on continuous (steady-state) thermal machines:



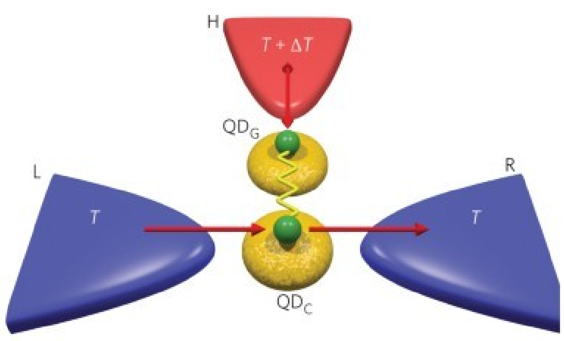


[Scovil and Schulz-Dubois, *PRL* ***2*** *(1959)*]

Linden et al. PRL (2010)



Thierschmann et al. Nat. Nano (2015)



Reviews: Kosloff & Levy, *Annu. Rev. Phys. Chem.* ***65*** *(2014)*, Mitchison, Contemporary Physics **60**, 164 (2019)

Many models of thermal machines are **based on quantum effects** (e.g. tunneling) or even show an **intrinsic quantum dynamics**, leading e.g. to entanglement, but...

How can we ensure a Quantum-thermodynamic advantage?

TUR

Classical Markovian Models in NESS $\Longrightarrow$

$$\frac{\mathrm{Var}[J(t)]}{\langle J(t)\rangle^2} \geq \frac{2k_B}{\langle \dot{S}_{\mathrm{tot}}\rangle}$$

Entropy production (EP)

Arbitrary current

Barato and Seifert, PRL (2015), Gingrich *et al.* PRL (2016), ...

+ Breaking of classical nonequilibrium inequalities such as TUR as a witness

No TUR $\Longrightarrow$ **No Classical Markovian model in NESS** $\Longrightarrow$ **Quantum advantage**

Agarwalla & Segal PRB (2018), Ptaszyński PRB (2018), Kalaee et al. PRE (2021)

Rignon-Bret et al. PRE (2021), Souza et al. PRE (2022), Manzano & López PRR (2023)

However, breaking of TUR is only a witness...

TUR $\not\Longrightarrow$ **Classical Markovian model**

- **Can we have quantum-thermodynamic advantage even if TUR is not broken?**

- **Can some types of quantum engines outperform any other classical engine using the same resources?**

- **Moreover, may these advantages occur in relevant regimes of operation?**

Our work

Reassessing quantum-thermodynamic enhancements in continuous thermal machines

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[1] *Institute for Cross-Disciplinary Physics and Complex Systems IFISC (UIB-CSIC), Campus Universitat Illes Balears, E-07122 Palma de Mallorca, Spain*

(Dated: November 22, 2024)

Quantum coherence has been shown to impact the operational capabilities of quantum systems performing thermodynamic tasks in a significant way, and yet the possibility and conditions for genuine coherence-enhanced thermodynamic operation remain unclear. We show that for steady-state quantum thermal machines —both autonomous and externally driven— that interact weakly with thermal reservoirs and work sources, the presence of coherence induced by perturbations in the machine Hamiltonian guarantees a genuine thermodynamic advantage. Such advantage applies both for the cases in which the induced coherence is between levels with different energies or between degenerate levels. On the other hand, we show that engines subjected to noise-induced coherence, can be outperformed by classical stochastic engines using exactly the same set of (incoherent) resources. We illustrate our results with three prototypical models of heat engines and refrigerators and employ multi-objective optimization techniques to characterize quantum-enhanced regimes in connection with the thermodynamic uncertainty relation and beyond it.

arXiv: 2403.19280 [quant-ph] (v2)

Continous thermal machines

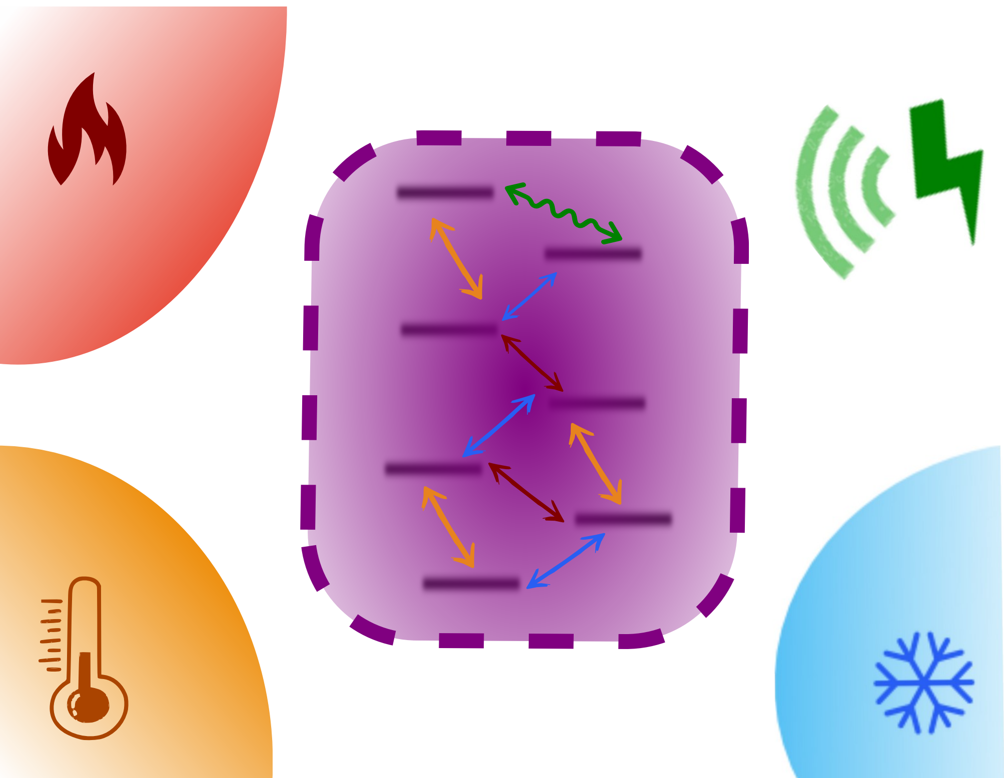
Markovian thermal machines:

Weak interaction with different reservoirs

$$\frac{d\rho(t)}{dt} = \mathcal{L}(\rho) = -i\big[H(t), \rho(t)\big] + \sum_{r=1}^{R} \sum_{k=\uparrow\downarrow} \mathcal{D}_k^{(r)}[\rho(t)],$$

Machine Hamiltonian

Dissipators in Lindblad form



Hamiltonian:

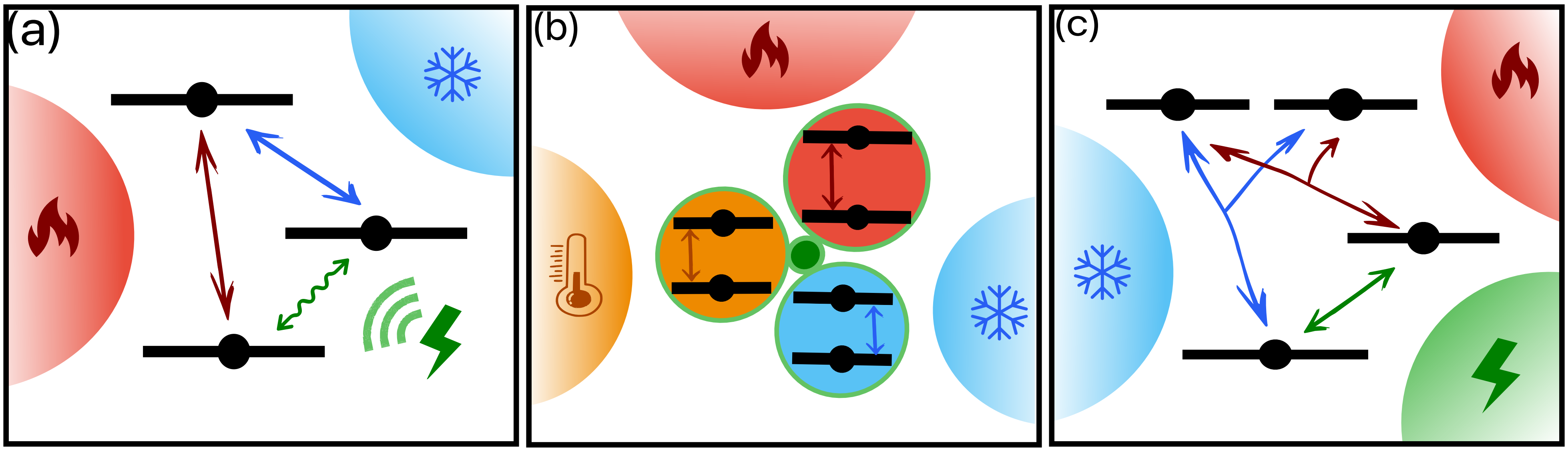
$$H(t) = H_0 + V(t)$$

N levels + weak perturbation $|V| \ll |H_0|$

Lindblad operators: jumps $\{\uparrow, \downarrow\}$

$$[H_0, L_k^{(r)}] = -\Delta\epsilon_k L_k^{(r)}$$

Examples of thermal machines considered:



a) Thee-level amplifier $g \ll \omega_{\rm d}$

$$V(t) = g(e^{-i\omega_{\rm d} t}|0\rangle\langle 1| + \mathrm{h.c})$$

b) Thee-qubits autonomous refrigerator

$$V = g\left(|101\rangle\langle 010| + \mathrm{h.c.}\right) \qquad g \ll \epsilon_i$$

Linden *et al.* PRL **105** (2010)

c) NIC machine $V(t) = 0$

$$L_{\uparrow}^{(\mathrm{h})} = \sqrt{\gamma_{1\mathrm{a}}}|2\mathrm{a}\rangle\langle 1| + \sqrt{\gamma_{1\mathrm{b}}}||2\mathrm{b}\rangle\langle 1|$$

$$L_{\downarrow}^{(\mathrm{h})} = \sqrt{\gamma_{\mathrm{a}1}}|1\rangle\langle 2\mathrm{a}| + \sqrt{\gamma_{\mathrm{b}1}}|1\rangle\langle 2\mathrm{b}|$$

collective jumps

Scully *et al.* PNAS **108** (2011)

Thermal machines performance

Average Currents: $\mathcal{L}(\pi) = 0$

$$\langle \dot{W} \rangle := -\mathrm{Tr}[\dot{H}(t)\pi] = -\mathrm{Tr}[\dot{V}(t)\pi]$$

$$\langle \dot{Q}_r \rangle := \sum_k \mathrm{Tr}[H\mathcal{D}_k^{(r)}[\pi]] \simeq \sum_k \mathrm{Tr}[H_0\mathcal{D}_k^{(r)}[\pi]],$$

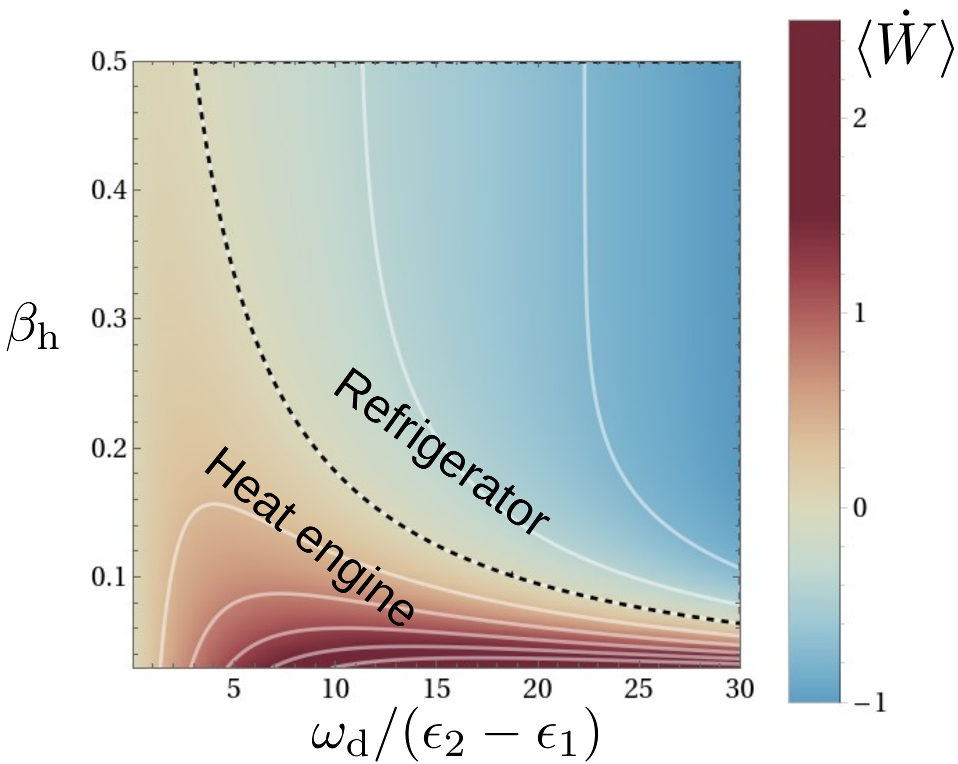
Important to neglect third-order terms to ensure consistency with local master equation approximations !

$$[V(t)\mathcal{D}_k^{(r)}[\pi]] \sim g\gamma_0$$

Trushechkin & Volovich EPL **113** (2016)

Hewgill et al. PRR **3** (2021)

Manzano et al. PRX **8** (2018)



Ensure first and second laws:

$$\langle \dot{W} \rangle = \sum_r \langle \dot{Q}_r \rangle$$

$$\langle \dot{S}_{\mathrm{tot}} \rangle = -\sum_r \beta_r \langle \dot{Q}_r \rangle \geq 0$$

Efficiency:

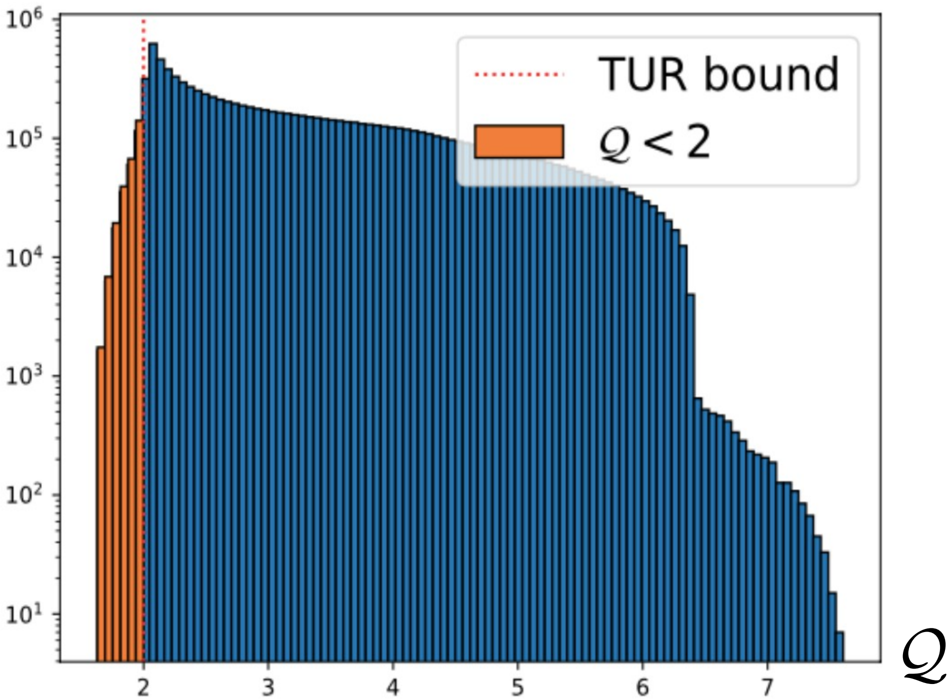
Output

$$\eta := \frac{\langle J_{\text{out}} \rangle}{\langle J_{\text{in}} \rangle}$$

Input source

$$\eta = \frac{\dot{W}}{\dot{Q}_{\text{h}}} \leq \eta_C = 1 - \frac{\beta_{\text{c}}}{\beta_{\text{h}}}$$ Heat engines

$$\eta = \frac{\dot{Q}_{\text{c}}}{\dot{Q}_{\text{h}}} \leq \eta_{\text{abs}} = \frac{\beta_{\text{m}} - \beta_{\text{h}}}{\beta_{\text{c}} - \beta_{\text{m}}}$$ Absorption refrigerator



Fluctuations of the output current:

Precise current, small $\text{Var}[J_{\text{out}}]$

These quantities are related by TUR

$$\mathcal{Q} = \langle \dot{S}_{\text{tot}} \rangle \frac{\text{Var}[J_{\text{out}}]}{\langle J_{\text{out}} \rangle^2} \geq 2$$

Classical Thermodynamic-Equivalent Machines

Define a classical machine that uses the same set of incoherent resources than the quantum machine:

- Same energy structure H_0
- All transitions between levels given by stochastic jumps
- Same temperatures in the thermal reservoirs
- Generates same average currents in the steady-state than original machine

In line with 'classical emulability':

O. González, et al. PRE **99** (2019)

In contrast with other notions:

R. Uzdin, et al. PRX **5** (2015)

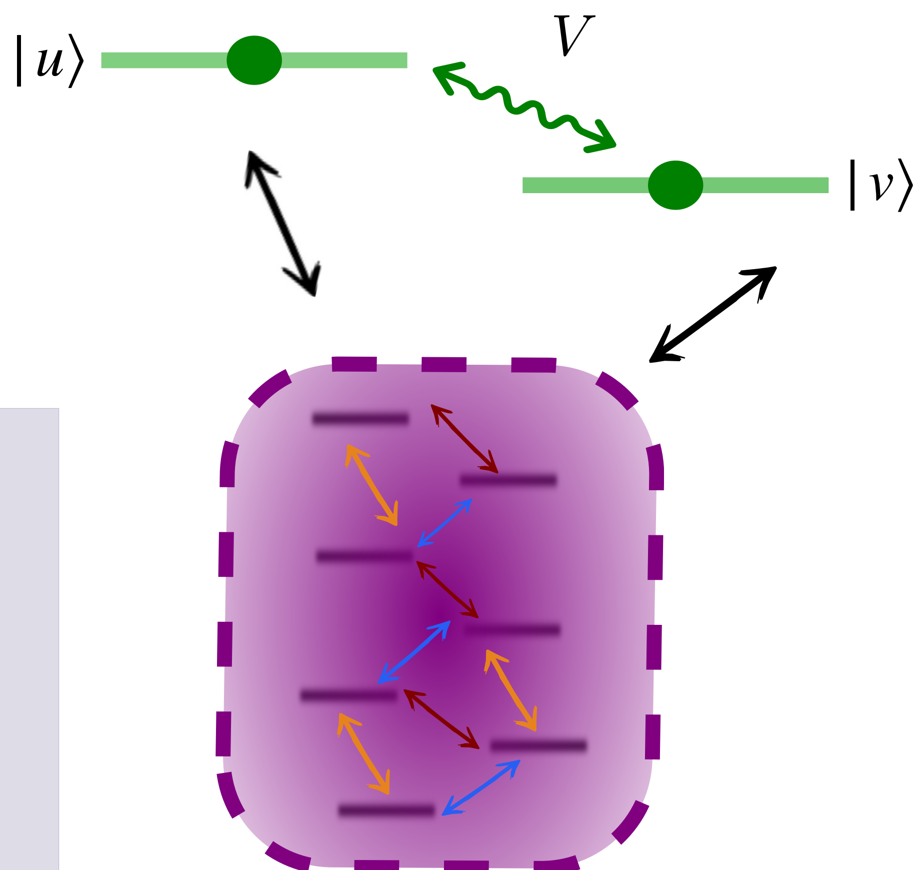
As a consequence, the classical thermodynamic equivalent machine has same power and efficiency, but can differ in the fluctuations

Classical Thermodynamic-Equivalent Machines

Hamiltonian-induced coherence

$$\langle \dot{Q}_r \rangle = \sum_{i<j}^{\in B_r} (\epsilon_j - \epsilon_i)\left(\gamma_{ij}\pi_{ii} - \gamma_{ji}\pi_{jj}\right)$$

One needs same steady-state diagonal elements



$$\dot{\rho}_{\mathrm{uu}} = \sum_{j\neq \mathrm{u}} \gamma_{j\mathrm{u}}\rho_{jj} - \rho_{\mathrm{uu}} \sum_i \gamma_{\mathrm{u}i} - 2g\,\mathrm{Im}\left(\rho_{\mathrm{uv}}\right)$$

$$\dot{\rho}_{\mathrm{vv}} = \sum_{j\neq \mathrm{v}} \gamma_{j\mathrm{v}}\rho_{jj} - \rho_{\mathrm{vv}} \sum_i \gamma_{\mathrm{v}i} + 2g\,\mathrm{Im}\left(\rho_{\mathrm{uv}}\right)$$

$$\dot{\rho}_{\mathrm{uv}} = -\frac{1}{2}\sum_i \left(\gamma_{\mathrm{u}i} + \gamma_{\mathrm{v}i}\right)\rho_{\mathrm{uv}} - ig\left(\rho_{\mathrm{vv}} - \rho_{\mathrm{uu}}\right)$$

$$\dot{\rho} \to 0 \qquad \Rightarrow \qquad \pi_{\mathrm{uv}} = \frac{-2ig\left(\pi_{\mathrm{vv}} - \pi_{\mathrm{uu}}\right)}{\sum_i \left(\gamma_{\mathrm{u}i} + \gamma_{\mathrm{v}i}\right)}.$$

Classical Thermodynamic-Equivalent Machines

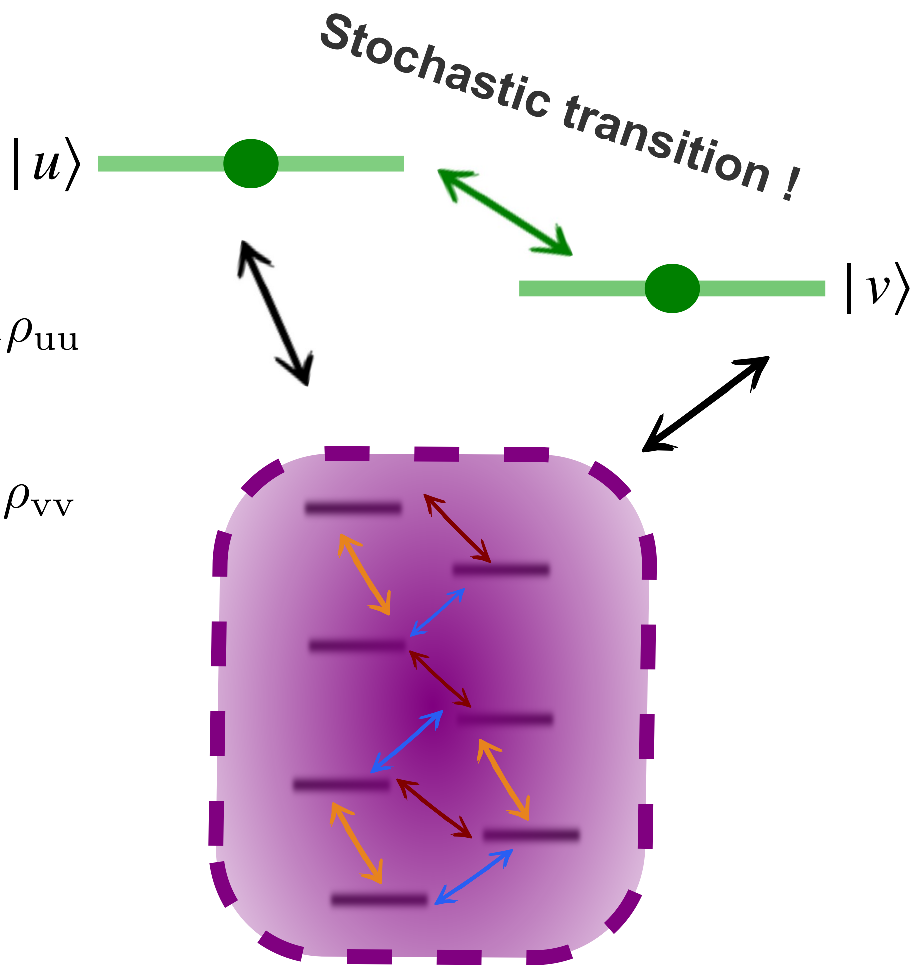
Replacing back into the equations:

$$\frac{d}{dt}\rho_{\mathrm{uu}} = \sum_{j\neq \mathrm{u}} \gamma_{j\mathrm{u}}\rho_{jj} - \rho_{\mathrm{uu}} \sum_{i} \gamma_{\mathrm{u}i} + \gamma^{\mathrm{cl}}_{\mathrm{vu}}\rho_{\mathrm{vv}} - \gamma^{\mathrm{cl}}_{\mathrm{uv}}\rho_{\mathrm{uu}}$$

$$\frac{d}{dt}\rho_{\mathrm{vv}} = \sum_{j\neq \mathrm{v}} \gamma_{j\mathrm{v}}\rho_{jj} - \rho_{\mathrm{vv}} \sum_{i} \gamma_{\mathrm{v}i} + \gamma^{\mathrm{cl}}_{\mathrm{uv}}\rho_{\mathrm{uu}} - \gamma^{\mathrm{cl}}_{\mathrm{vu}}\rho_{\mathrm{vv}}$$

New rate (symmetric):

$$\gamma^{\mathrm{cl}}_{\mathrm{uv}} = \gamma^{\mathrm{cl}}_{\mathrm{vu}} = \frac{4g^2}{\sum_i \left(\gamma_{\mathrm{u}i} + \gamma_{\mathrm{v}i}\right)}$$



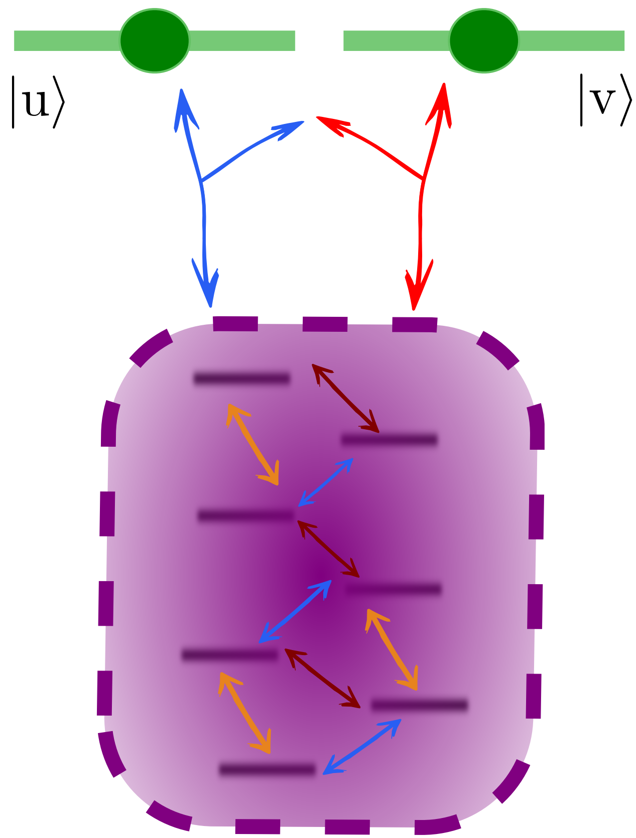
We have substituted the coherent work source by an equivalent stochastic work source, leading to same steady-state populations and thence same currents !

Classical Thermodynamic-Equivalent Machines

Noise-induced coherence

$$\langle \dot{Q}_r \rangle = \sum_{i<j} (\epsilon_j - \epsilon_i)(\gamma_{ij}\pi_{ii} - \gamma_{ji}\pi_{jj}) + 2\sum_j (\epsilon_j - \epsilon_{\mathrm{v}})(\gamma_{\mathrm{u}j} + \gamma_{\mathrm{v}j})\mathrm{Re}(\pi_{\mathrm{uv}})$$

Same steady-state diagonal elements are sufficient!



Following a similar calculation: $\dot{\rho} \to 0$

$$\pi_{\mathrm{uv}} = \frac{\sum_i \left[2\sqrt{\gamma_{\mathrm{u}i}\gamma_{\mathrm{v}i}}\pi_{ii} - \sqrt{\gamma_{i\mathrm{u}}\gamma_{i\mathrm{v}}}(\pi_{\mathrm{uu}} + \pi_{\mathrm{vv}})\right]}{\sum_i (\gamma_{i\mathrm{u}} + \gamma_{i\mathrm{v}})}$$

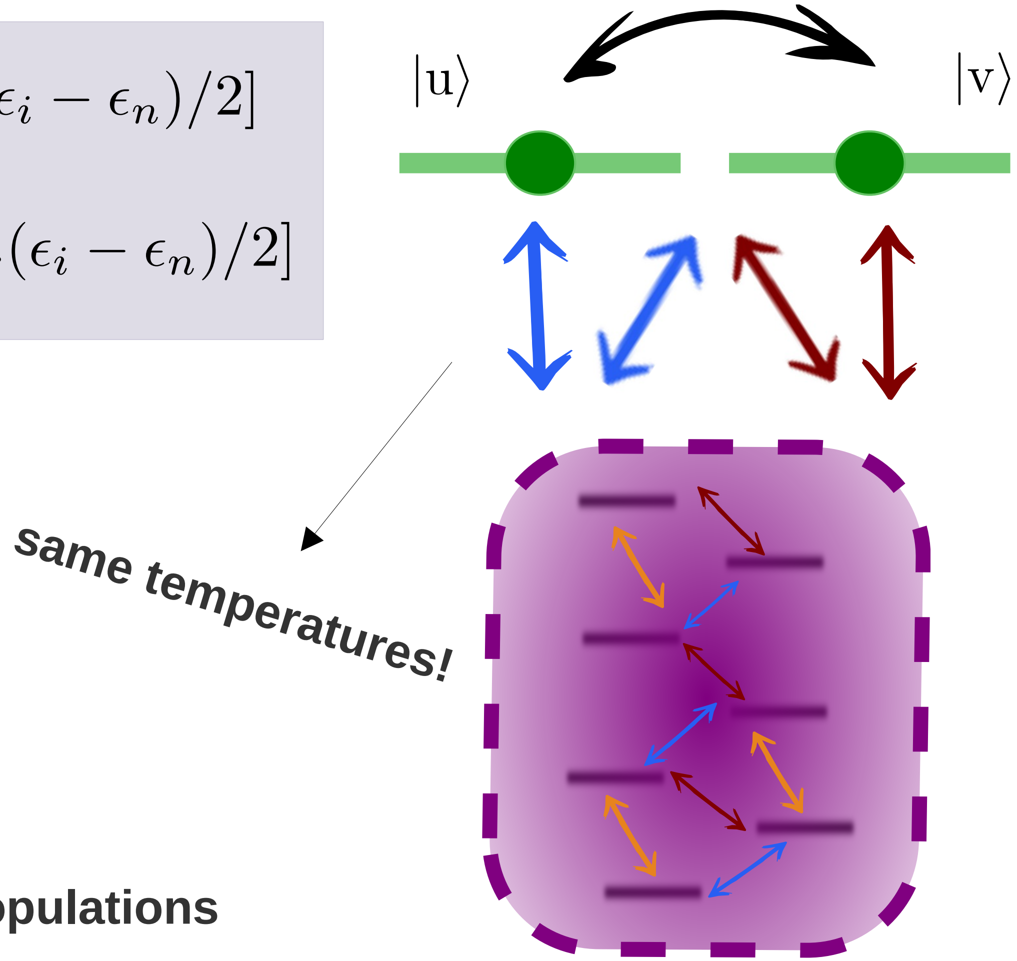
and replace back into the equations of motion

Classical Thermodynamic-Equivalent Machines

Replaces the collective jumps by simple jumps to $i = \mathrm{u}, \mathrm{v}$

$$\gamma_{in}^{\mathrm{cl}} = \left(\gamma_r^i - 2\gamma_{\mathrm{uv}}^* \sqrt{\gamma_r^{\mathrm{u}} \gamma_r^{\mathrm{v}}}\right) \exp[\beta_r(\epsilon_i - \epsilon_n)/2]$$

$$\gamma_{ni}^{\mathrm{cl}} = \left(\gamma_r^i - 2\gamma_{\mathrm{uv}}^* \sqrt{\gamma_r^{\mathrm{u}} \gamma_r^{\mathrm{v}}}\right) \exp[-\beta_r(\epsilon_i - \epsilon_n)/2]$$



Adds a new stochastic transition:

$$\gamma_{\mathrm{uv}}^{\mathrm{cl}} = \gamma_{\mathrm{vu}}^{\mathrm{cl}} = \frac{\left(\sum_j \sqrt{\gamma_{\mathrm{u}j}\gamma_{\mathrm{v}j}}\right)^2}{\sum_j (\gamma_{\mathrm{u}j} + \gamma_{\mathrm{v}j})}$$

We obtain again same steady-state populations and same currents !

Benchmarking quantum enhacements

Classical equivalents have the same average currents and efficiency

The Fluctuation Ratio:

$$\mathcal{R} := \frac{\mathrm{Var}[J_{\mathrm{out}}^{\mathrm{cl}}] - \mathrm{Var}[J_{\mathrm{out}}]}{\mathrm{Var}[J_{\mathrm{out}}]}$$

$\mathcal{R} > 0$ Quantum is better

$\mathcal{R} = 0$ Classical and Quantum are equal

$\mathcal{R} < 0$ Classical is better

portion of fluctuations in the quantum machine that are due to quantum effects

This can be applied to any quantum machine for which the Classical Thermodynamic Equivalent is defined !

Full Counting Statistics

Esposito et al. Rev. Mod. Phys. **81** (2009)

Inverse counting

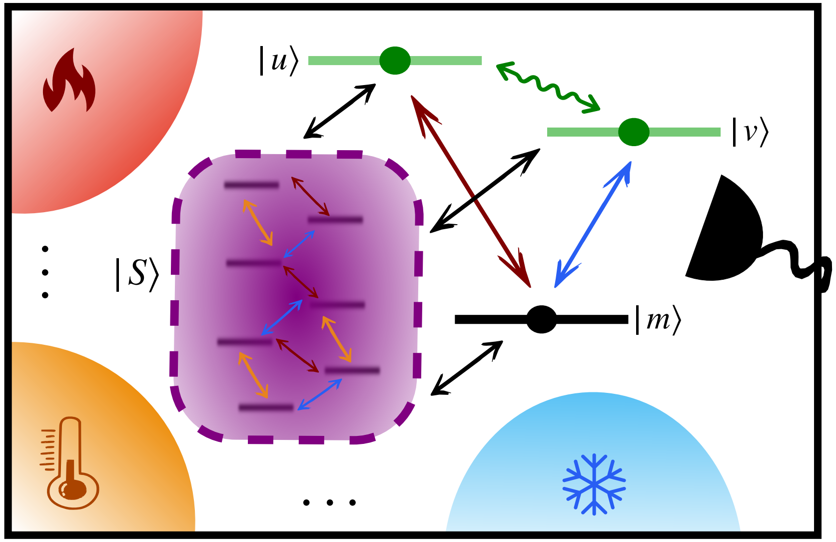
Bruderer et al. New J. Phys. **16** (2014)

Benchmarking quantum enhacements

Universal results for Hamiltonian-induced coherence:

Define a mesostate S: $\rho_{SS} := \sum_{j\in S} \rho_{jj}$

$$\Gamma_{Si} := \frac{1}{p_S} \sum_{j\in S} \gamma_{ji}\rho_{jj} \qquad \Gamma_{iS} := \sum_{j\in S} \gamma_{ij}$$



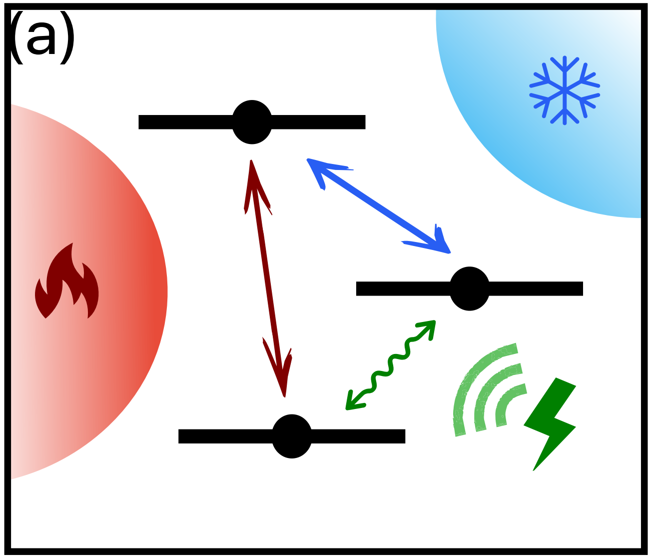
N levels $\longrightarrow$ 4 levels

Theorem 1: $\mathcal{R} \geq 0$ **reaching zero at equilibrium only**

For all quantum thermal machines with Hamiltonian-induced coherence (levels can be degenerate or not) which are either *unicyclic or multicyclic with a multipartite structure*.

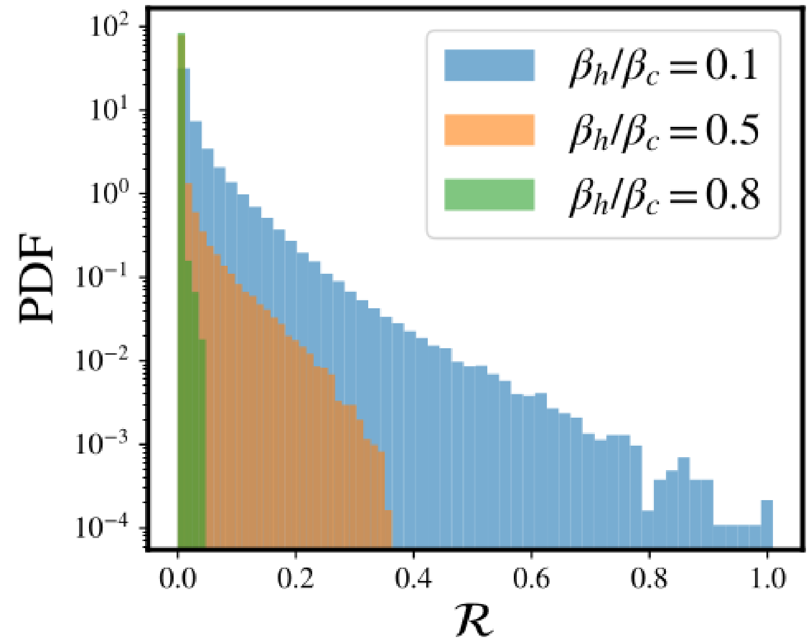
Same net number of quanta from each reservoir in every cycle

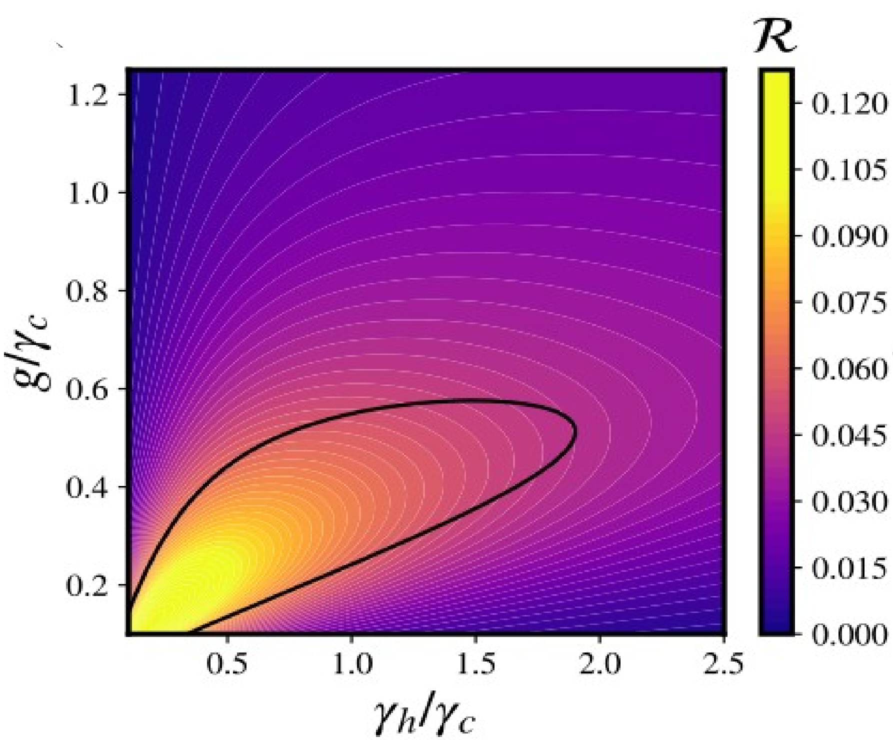
$$\mathrm{Var}[\dot{W}] = \mathrm{Var}[\dot{Q}_r] = \Delta\epsilon_r^2 \mathrm{Var}[\dot{N}]$$

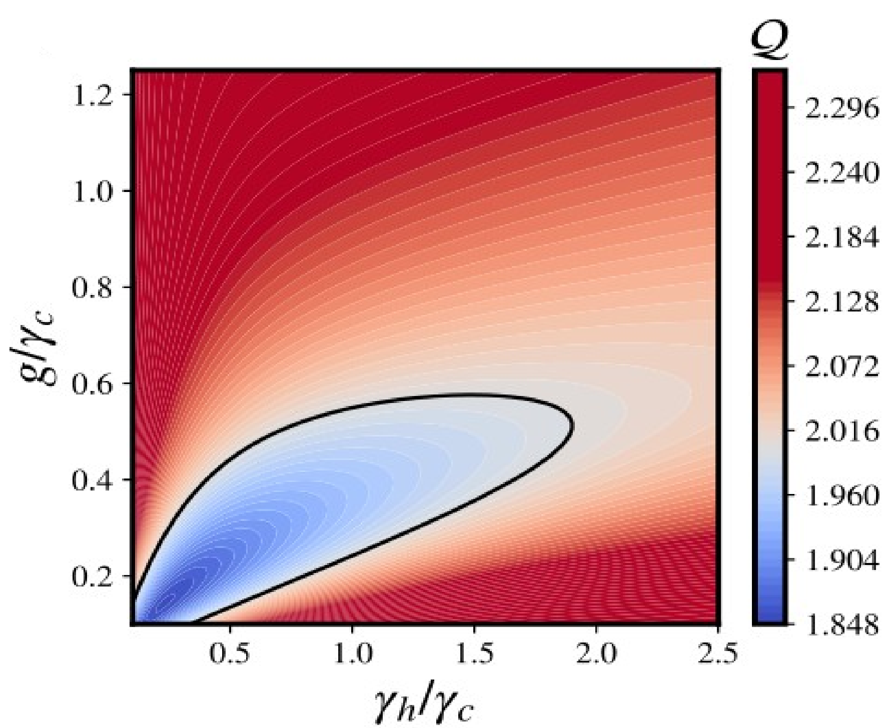


Classical equivalent:

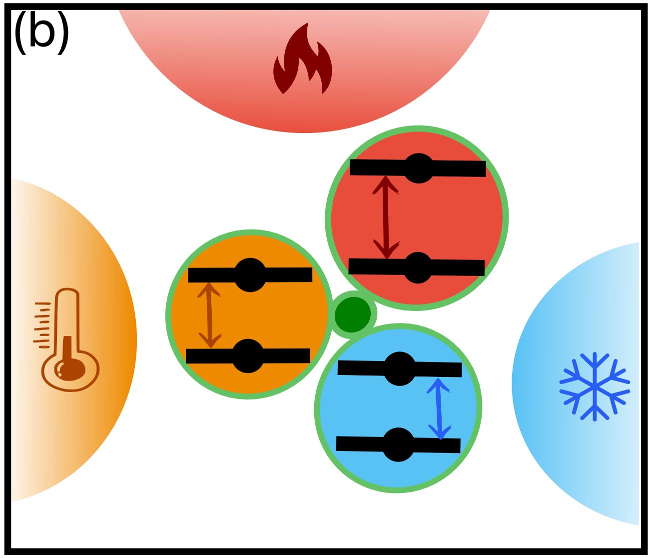
$$\gamma_{01} = \frac{4g^2}{\gamma_{\mathrm{h}}\bar{n}_{\mathrm{h}} + \gamma_{\mathrm{c}} n_{\mathrm{c}}}$$

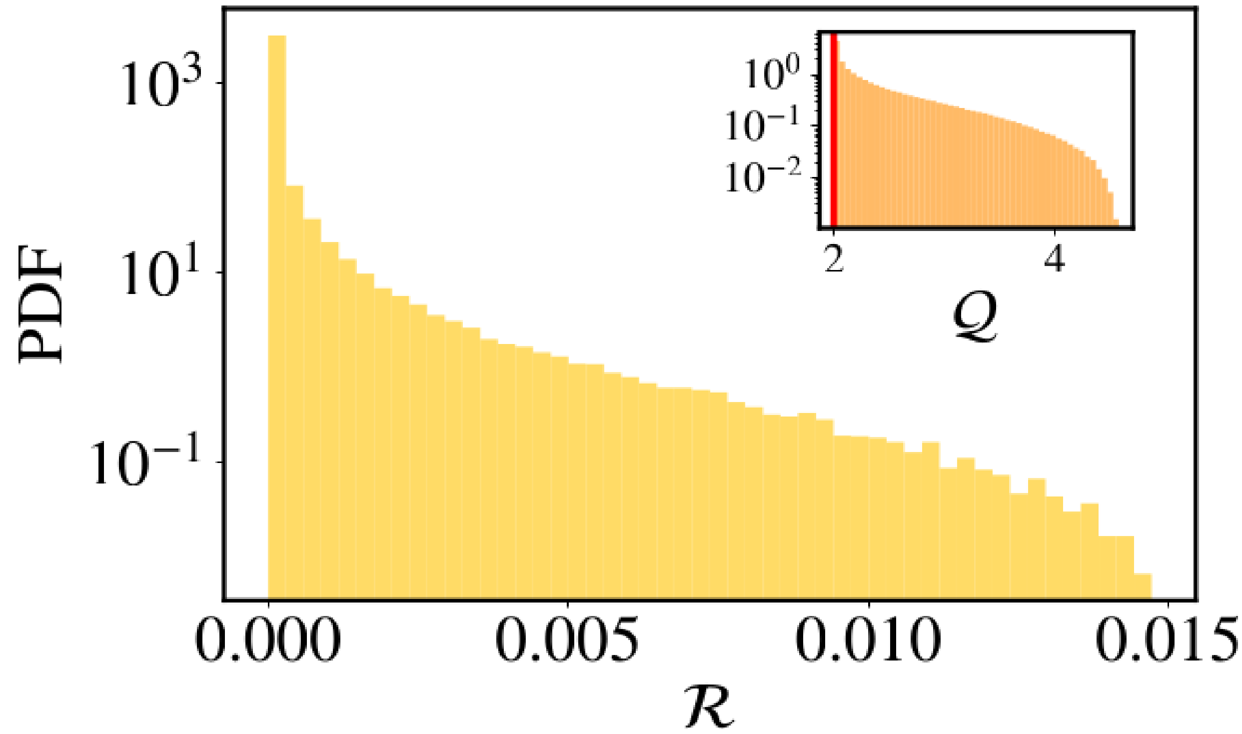


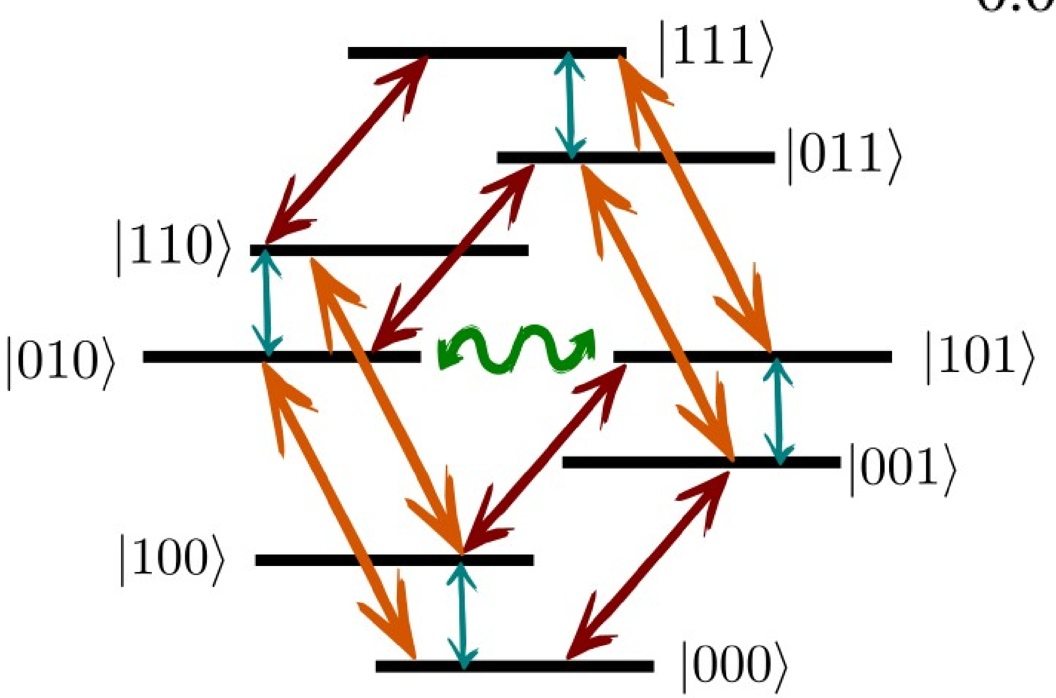




Example II : Three-qubit refrigerator





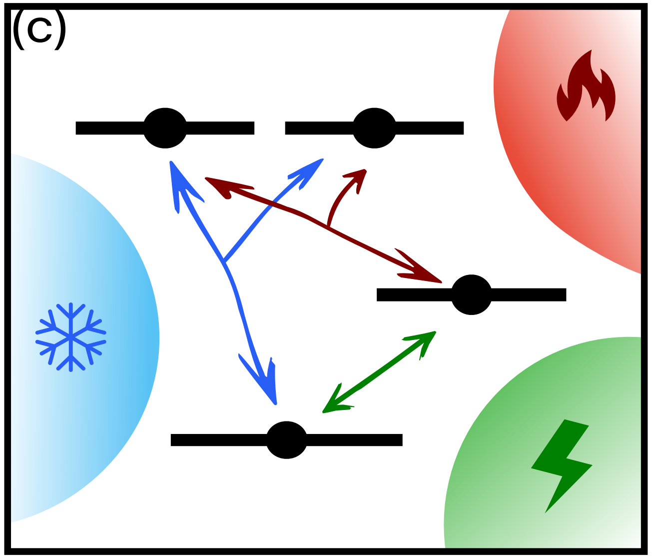


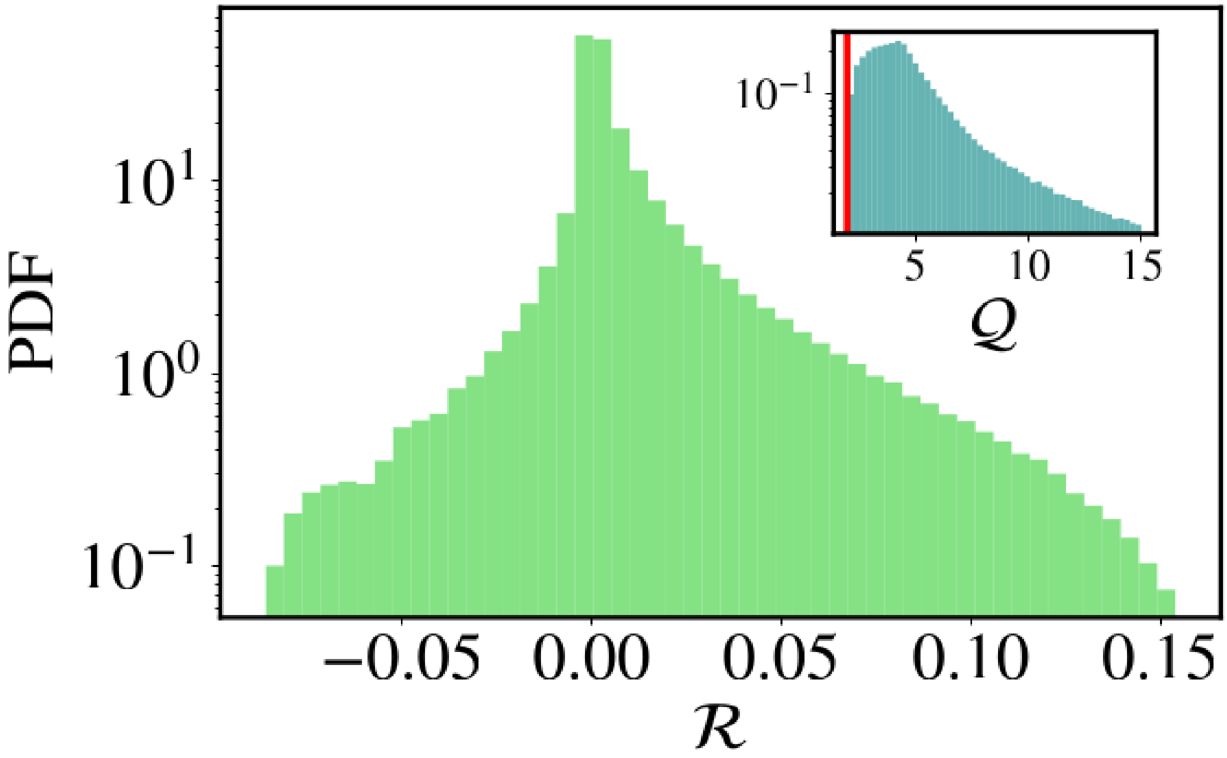
Classical equivalent:

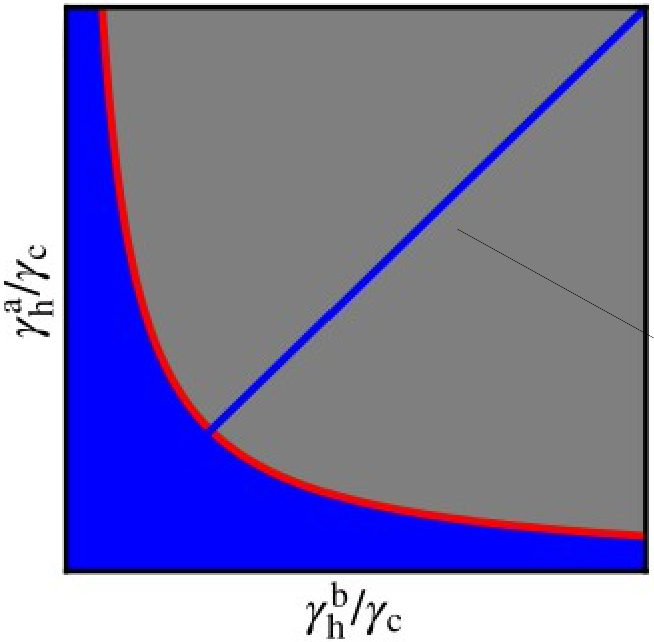
$$\gamma_{\rm uv} = \frac{4g^2}{\sum_r (2\bar{n}_r + 1)\gamma_r}$$

$$|\mathrm{u}\rangle = |101\rangle \qquad |\mathrm{v}\rangle = |010\rangle$$

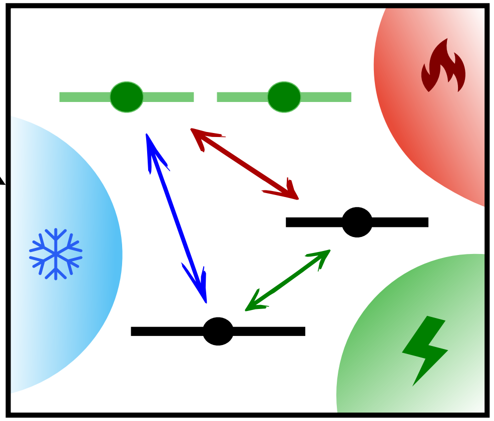
Example III : NIC machine







In the symmetric case:



Theorem 2:

Quantum thermal machines with Noise-induced coherence can either Outperform, be equal, or be outperfromed by classical equivalent machines

+ Can we have quantum-thermodynamic advantage even if TUR is not broken?

YES!

+ Can some types of quantum engines outperform any other classical engine using the same resources?

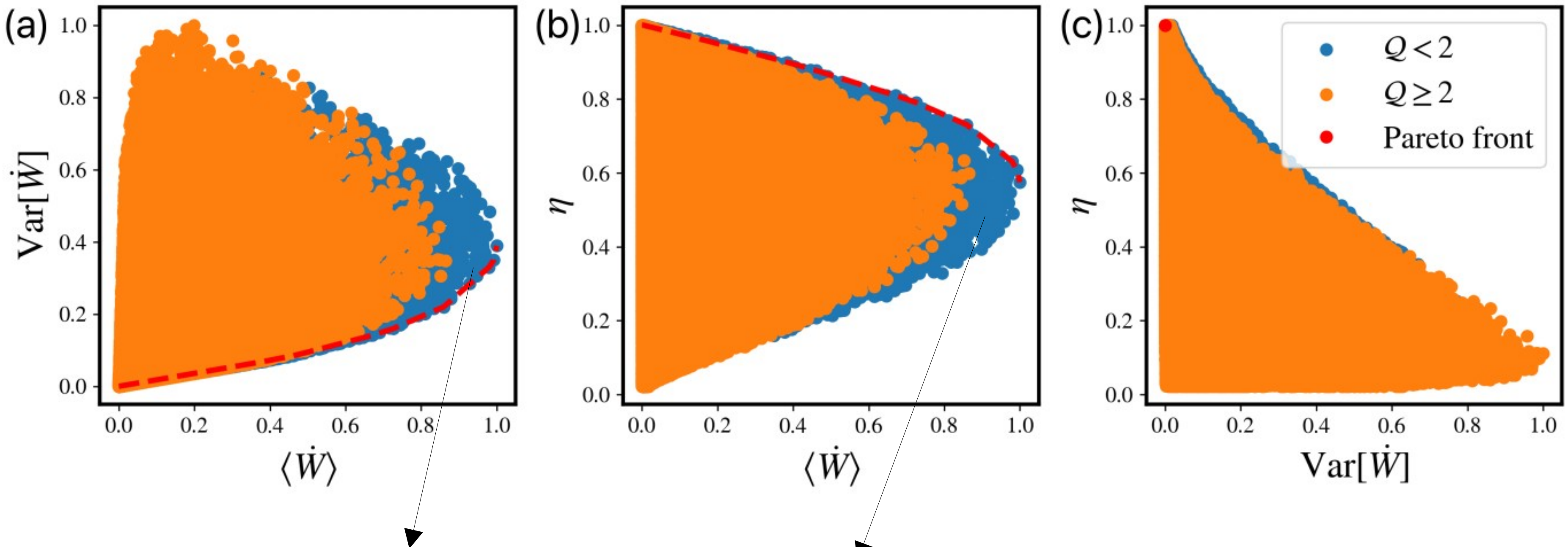
YES!

+ Moreover, when these advantages may occur in relevant regimes of operation?

Multi-objective (Pareto) optimization:

We consider three optimization problems and analyze in which regimes a quantum advantage is obtained:

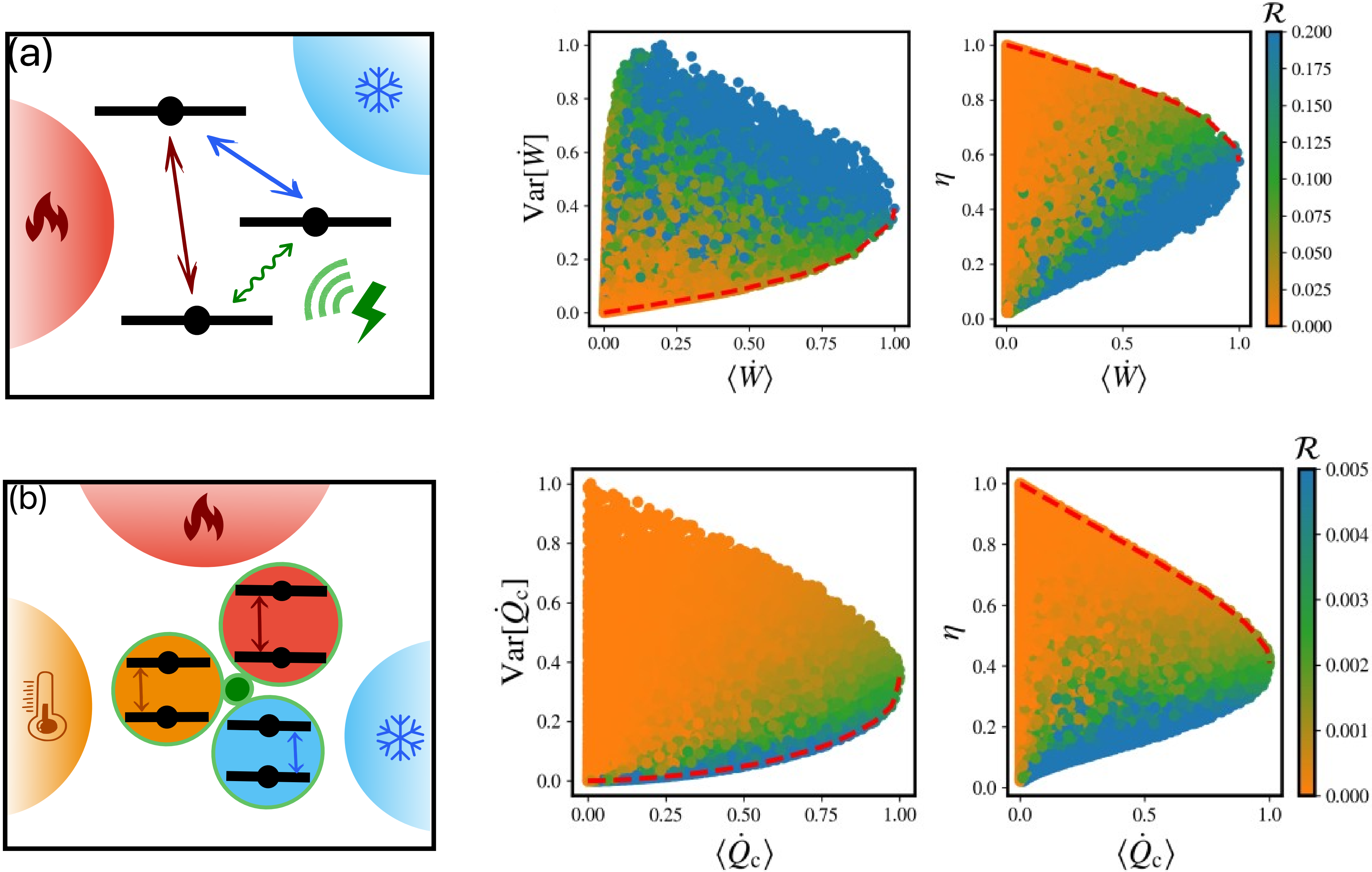
(a) power vs. fluctuations (b) power vs. efficiency (c) efficiency vs. fluctuations



Violations of TUR occur at maximum power

Quantum enhancements in relevant (optimal regimes)

Multi-objective (Pareto) optimization:



Conclusions

- Assessing genuine quantum-thermodynamic enhancements is more involved than just comparing to a dephased-like model and surprises and require an inspection of the fluctuations in the machine

- General recipe to construct classical thermodynamic-equivalent thermal machines.

- Quantum thermal machines showing Hamiltonian-induced coherence outperform their classical counterparts showing reduced fluctuations. This is always the case for weak coupling conditions and for unicyclic and multiclycic symmetric machines.

- Noise-induced coherence instead can lead to either beneficial or detrimental effects, which suggest to reassess these models in a case-by-case basis.

Outlook

Possible extensions to e.g. strong driving using Floquet theory?

Finding a model using degenerate coherence with big quantum advantages?

THANK YOU

for your attention

More information: arXiv: 2403.19280 [quant-ph]

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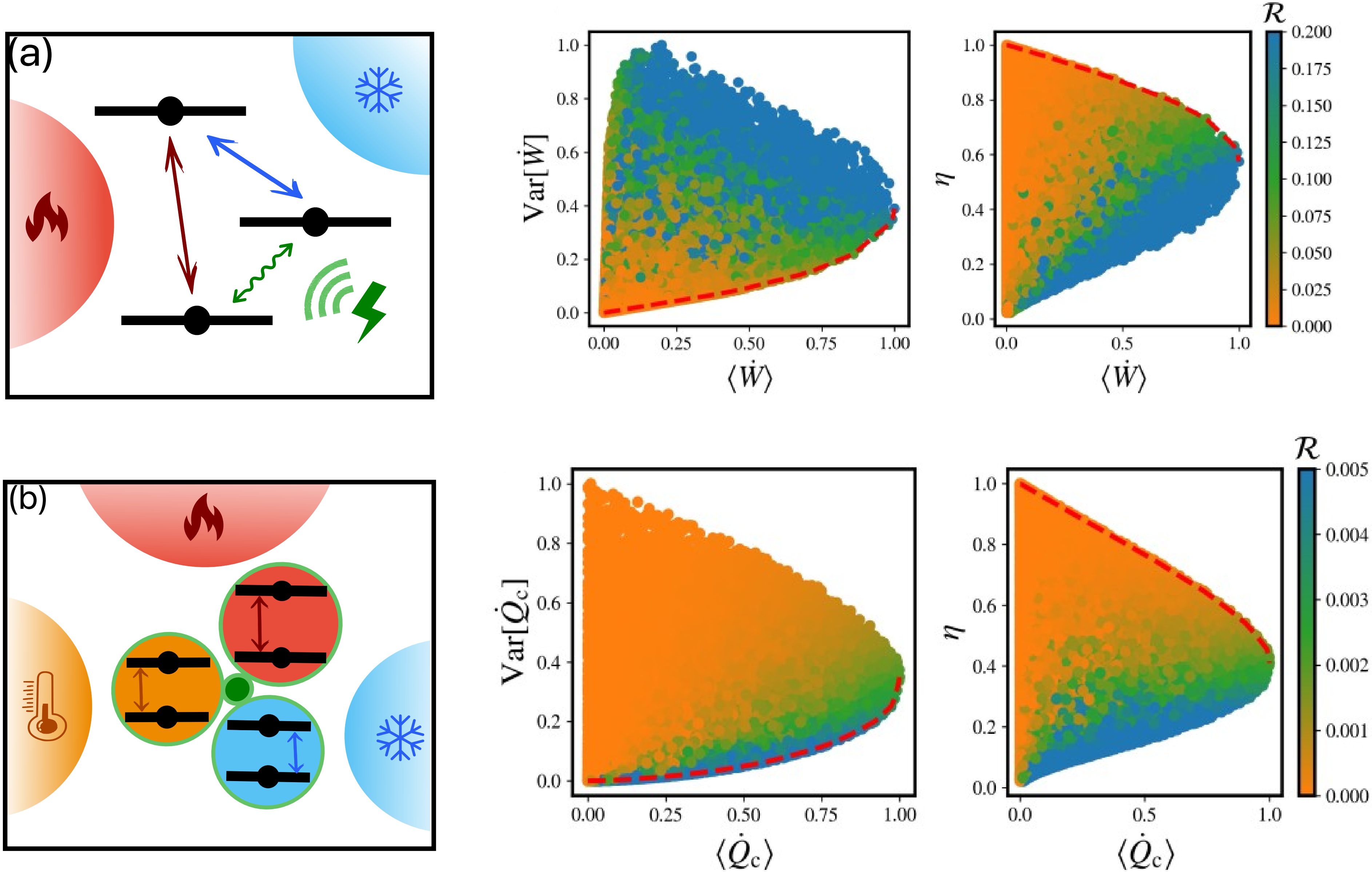

Multi-objective (Pareto) optimization: