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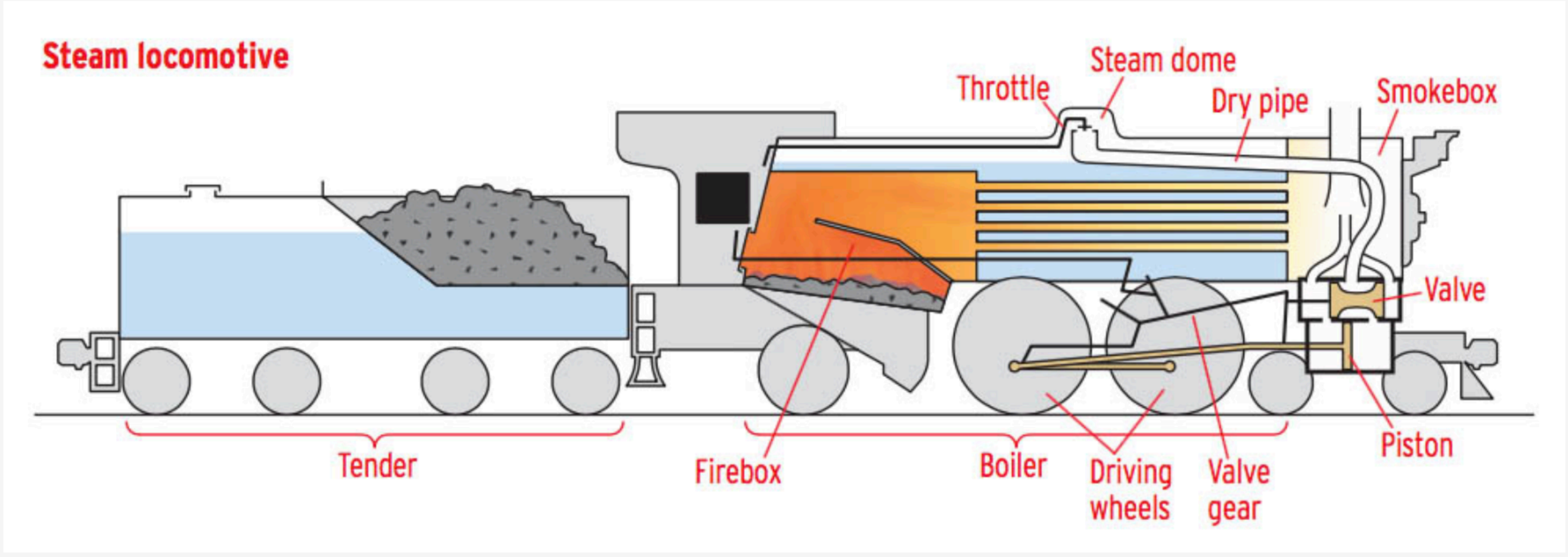
Fluctuation theorems and noise bounds for nonequilibrium nanoscale engines

Janine Splettstoesser

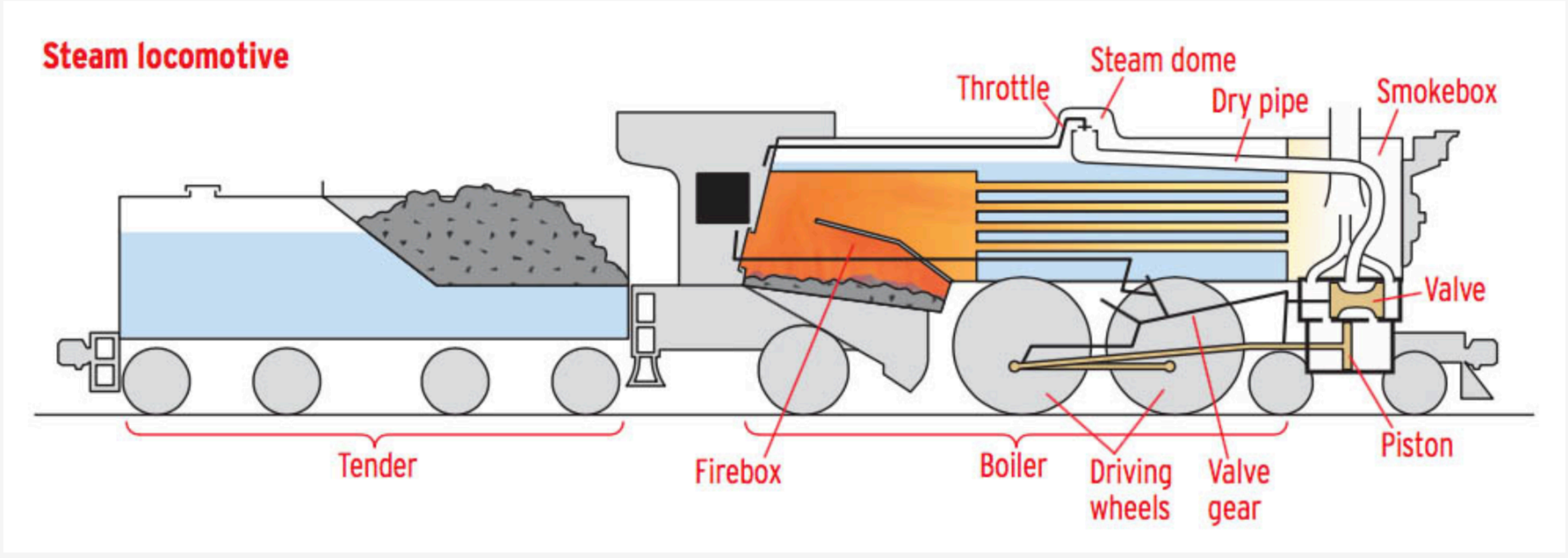
Applied Quantum Physics, Department for Microtechnology and Nanoscience, Chalmers

Quantum Carnot workshop, Dijon, November 25th - 27th 2024



R. S. McGonigal, Trains magazine, May 2006

Game-changing technology



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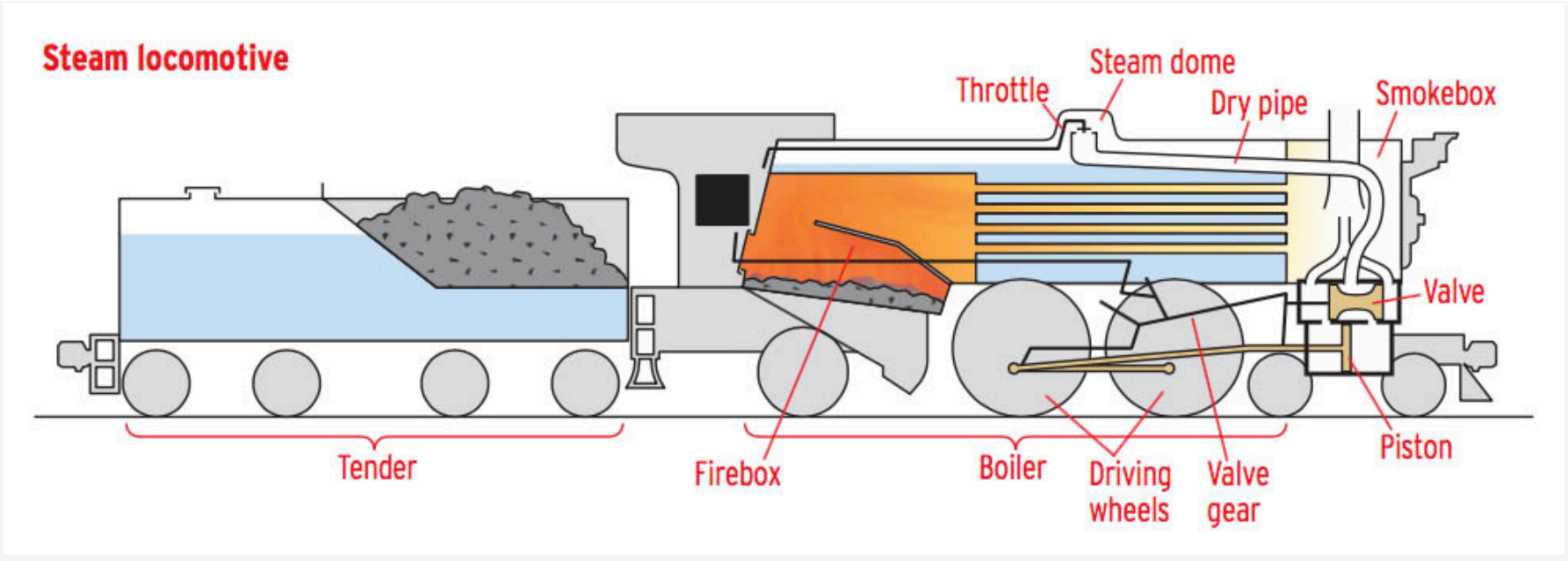

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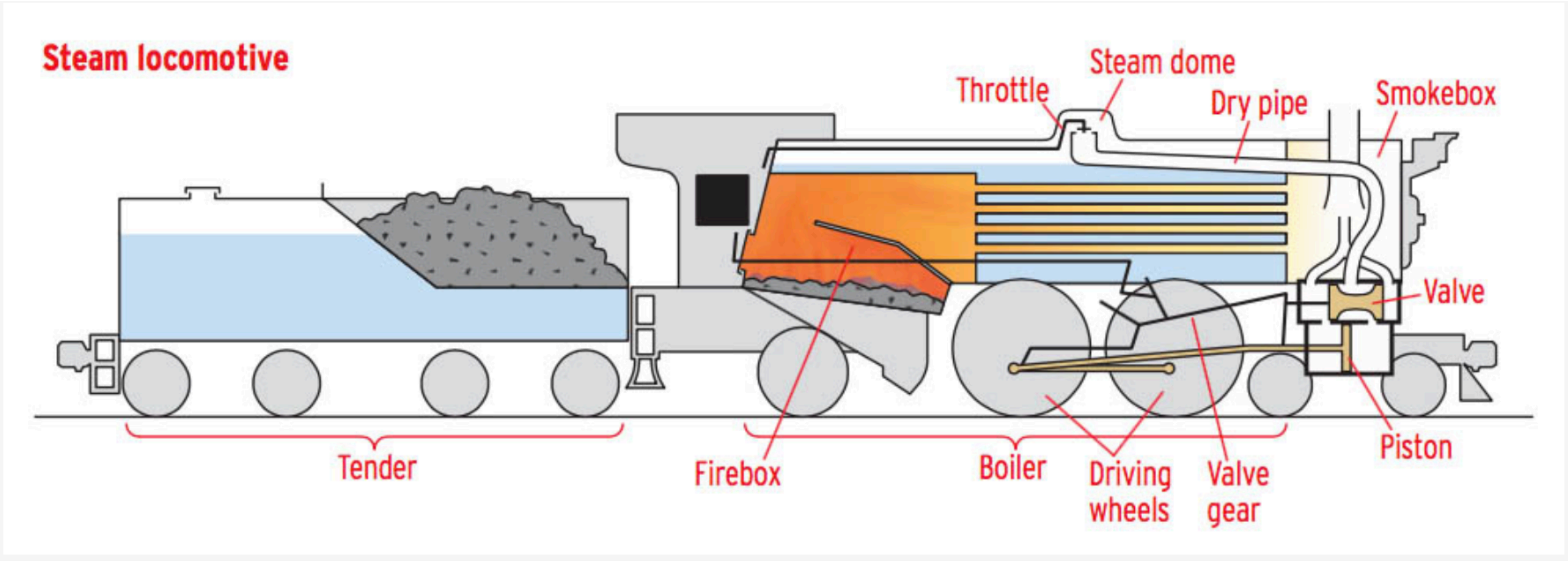
Game-changing technology

Tasks requiring energy



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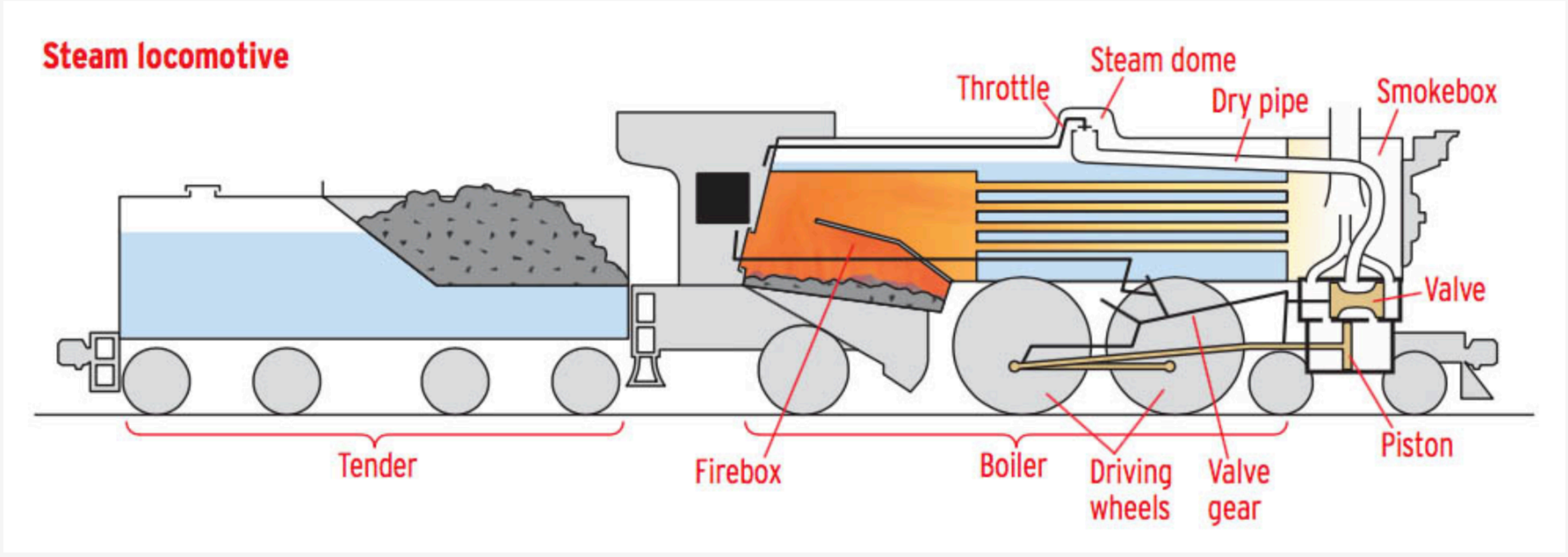
Heat as resource:
Easy to have/leftover



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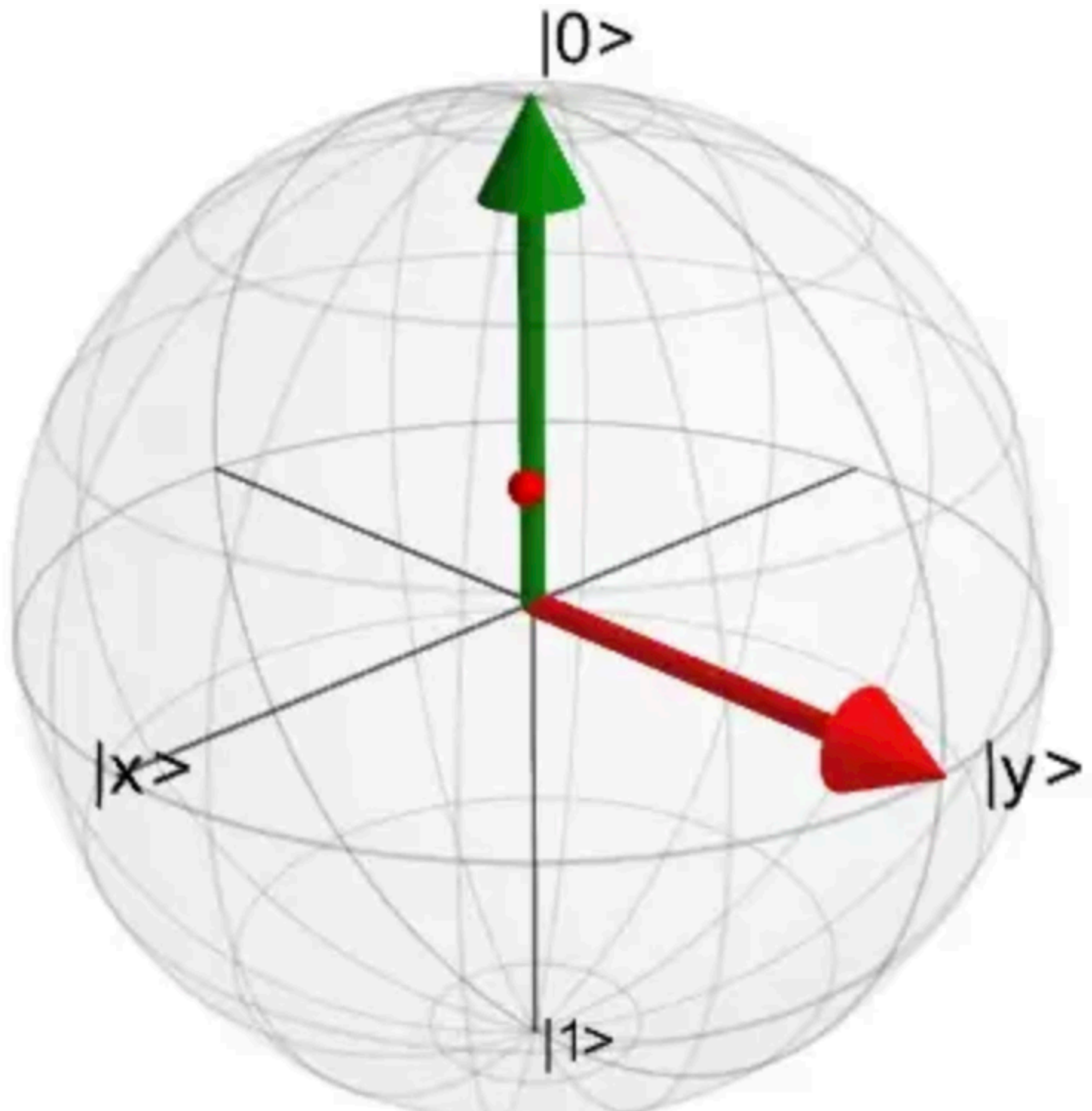

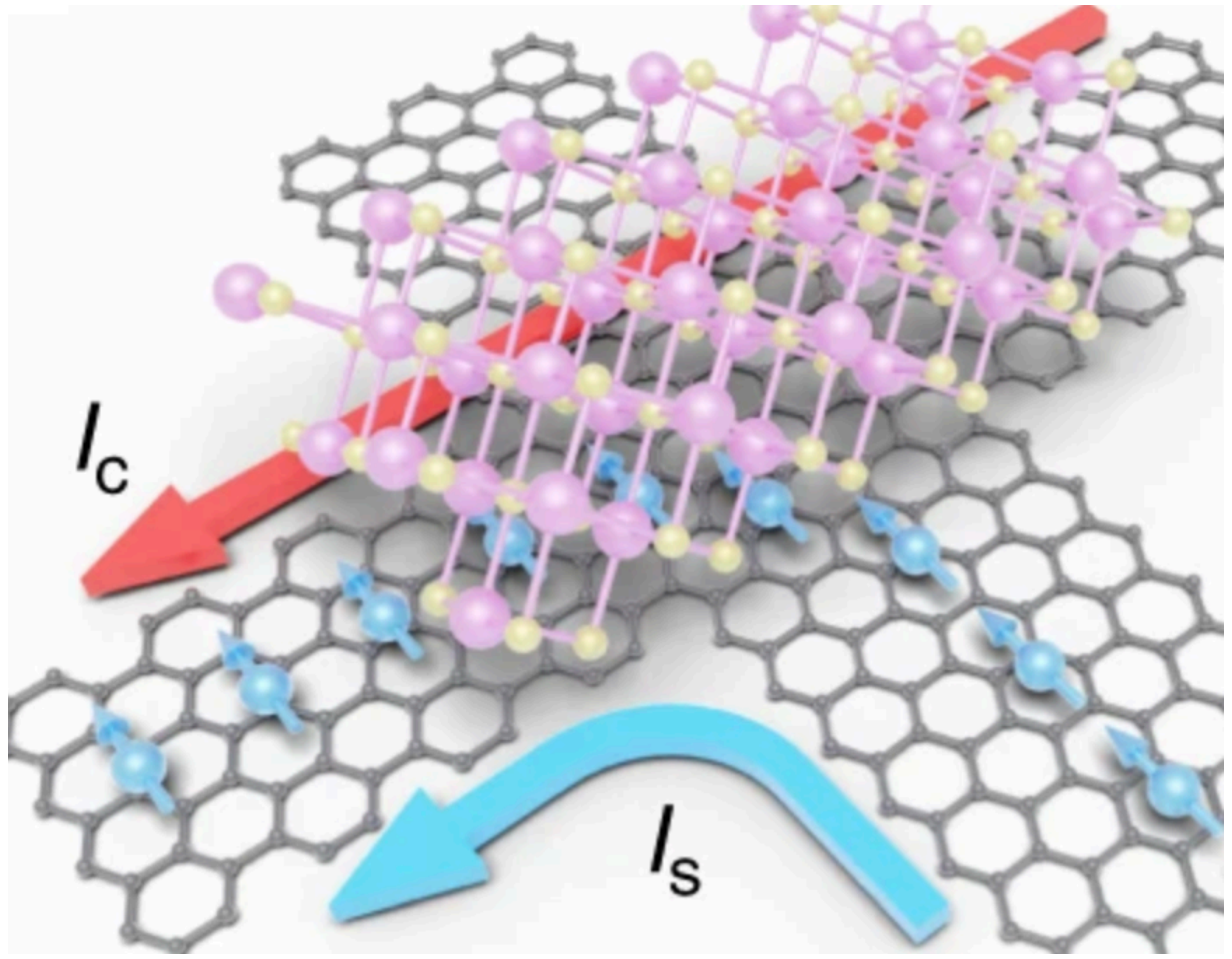

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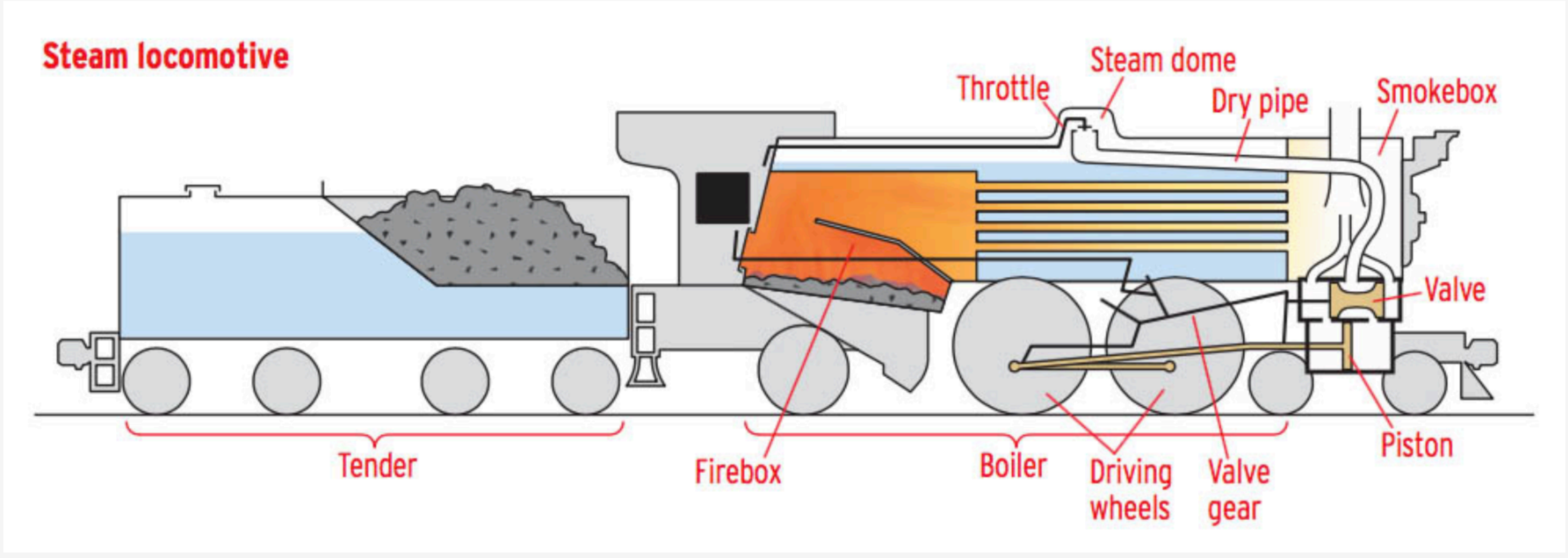

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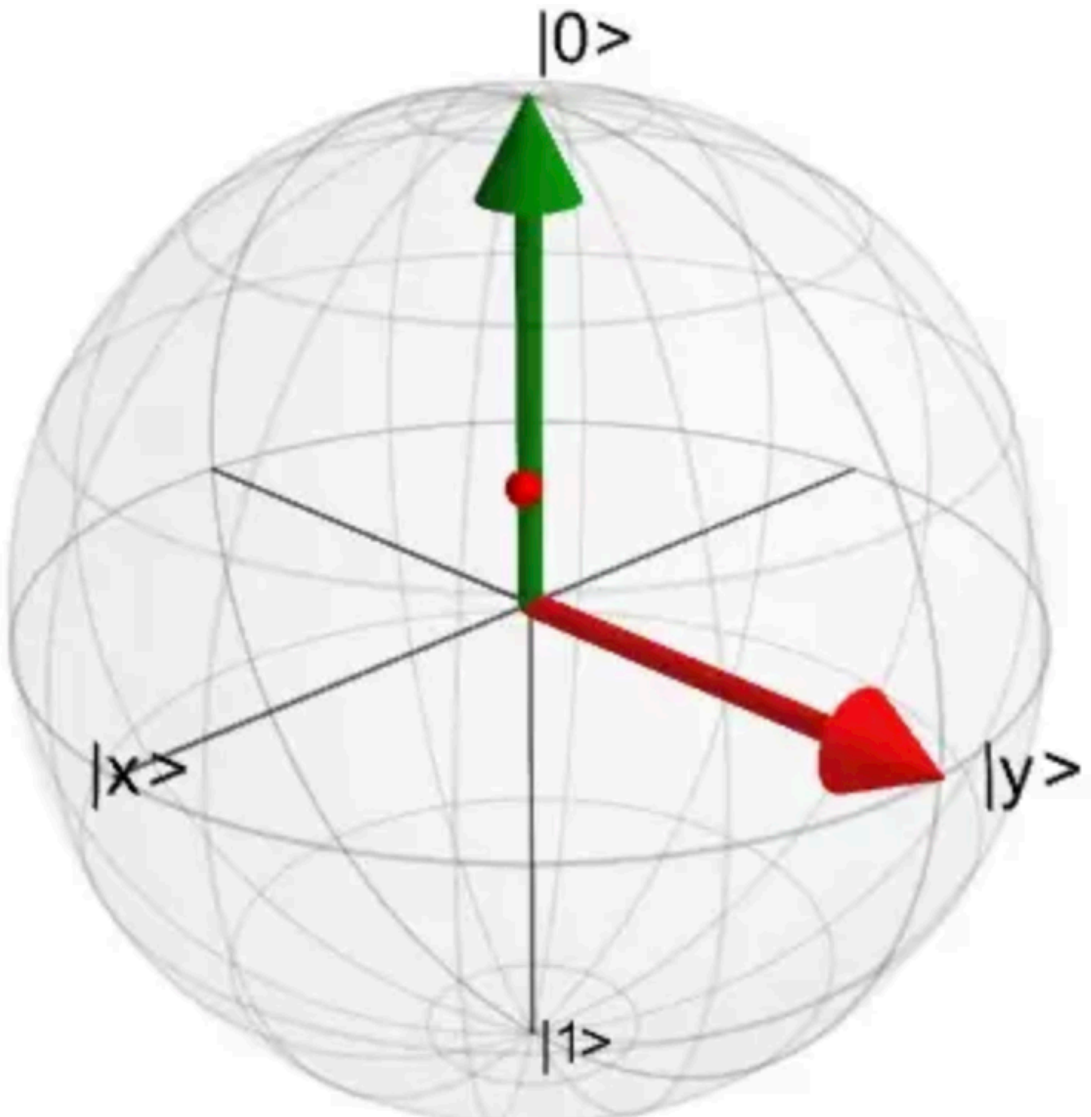

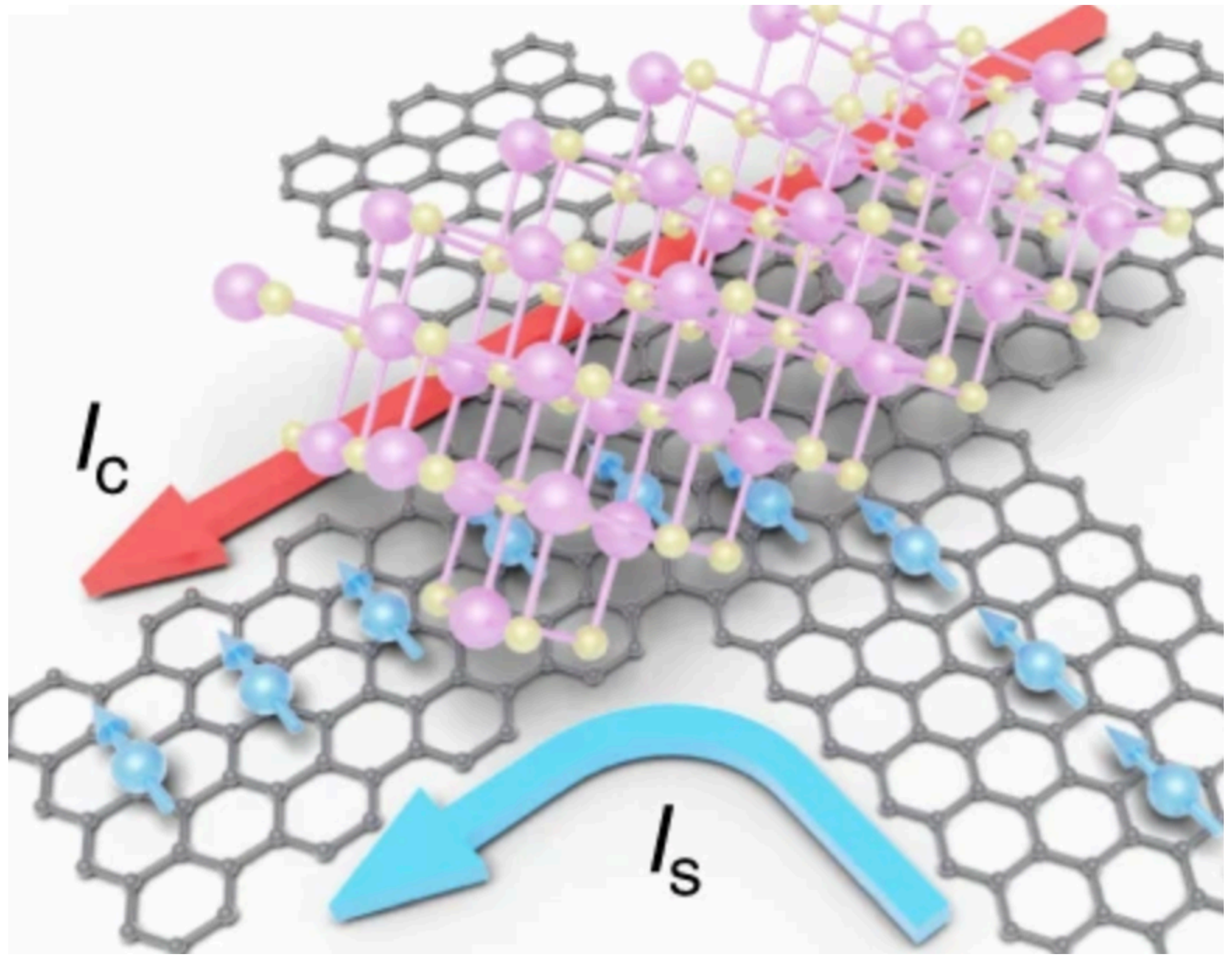

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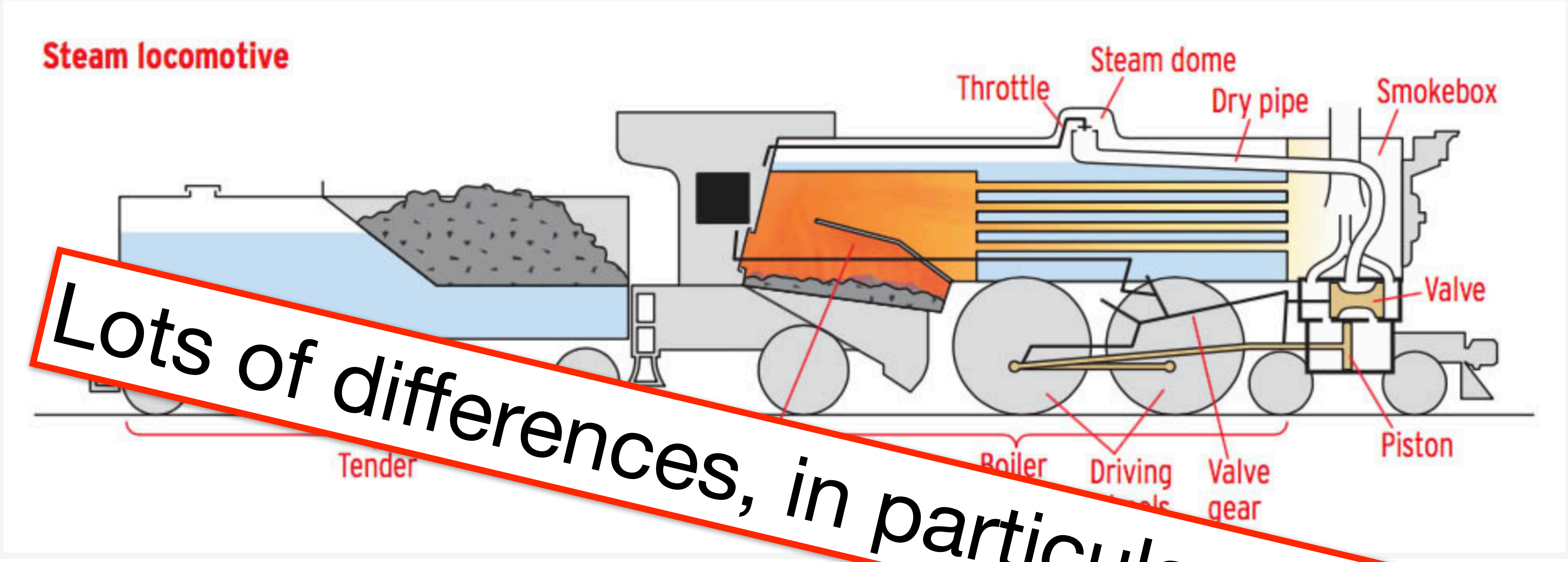

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Lots of differences, in particular, but not only due to their size!

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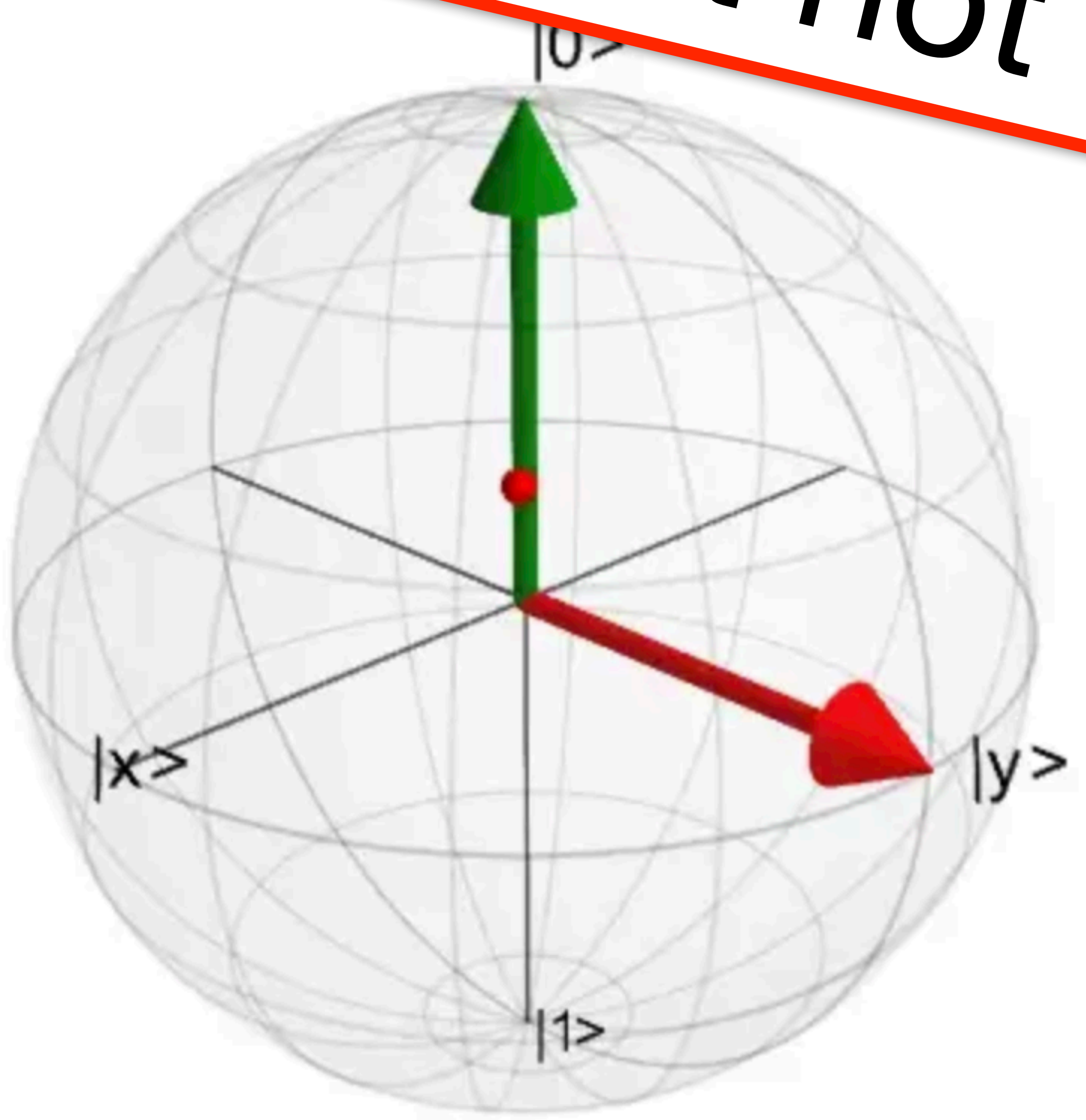
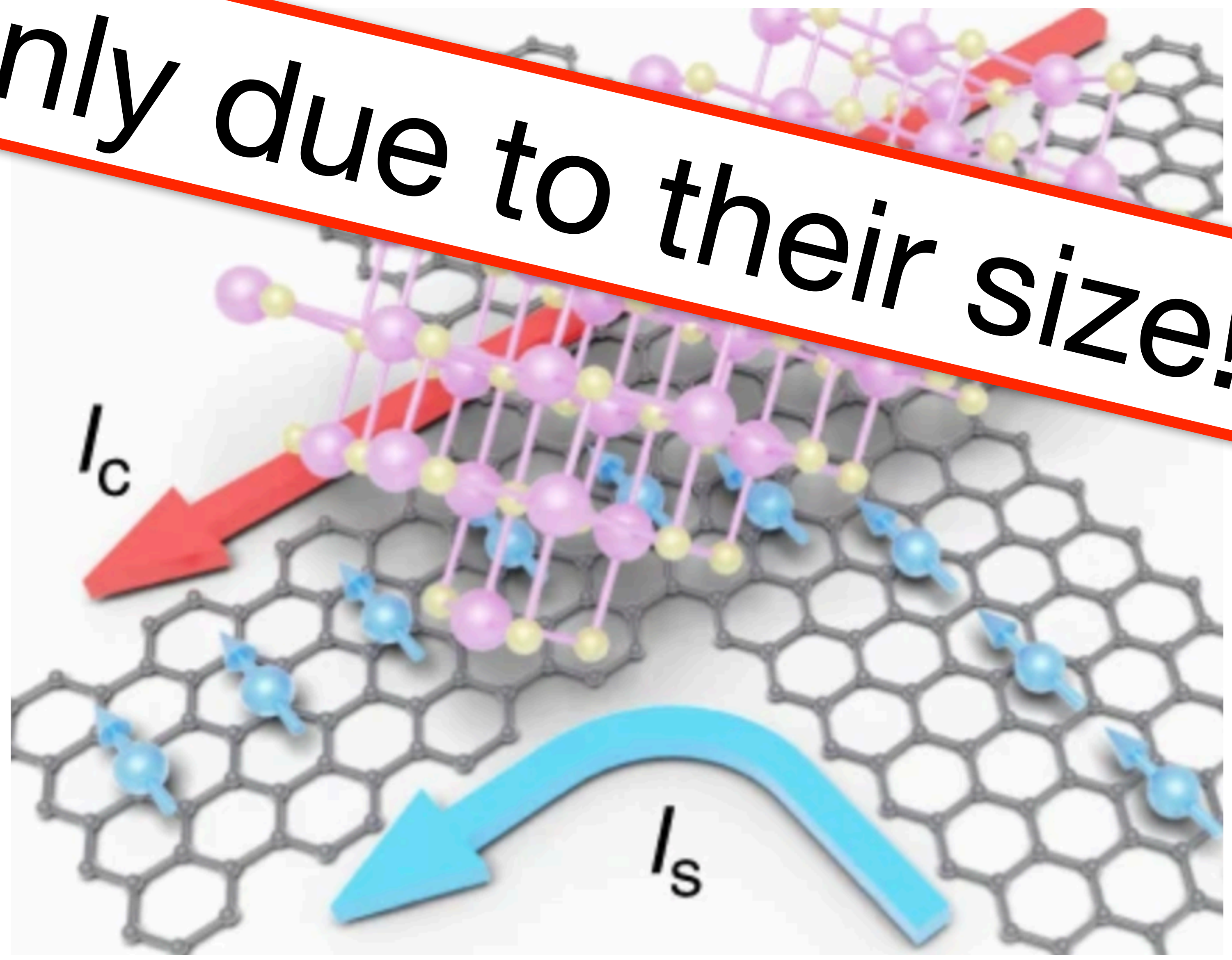
Cooling as task





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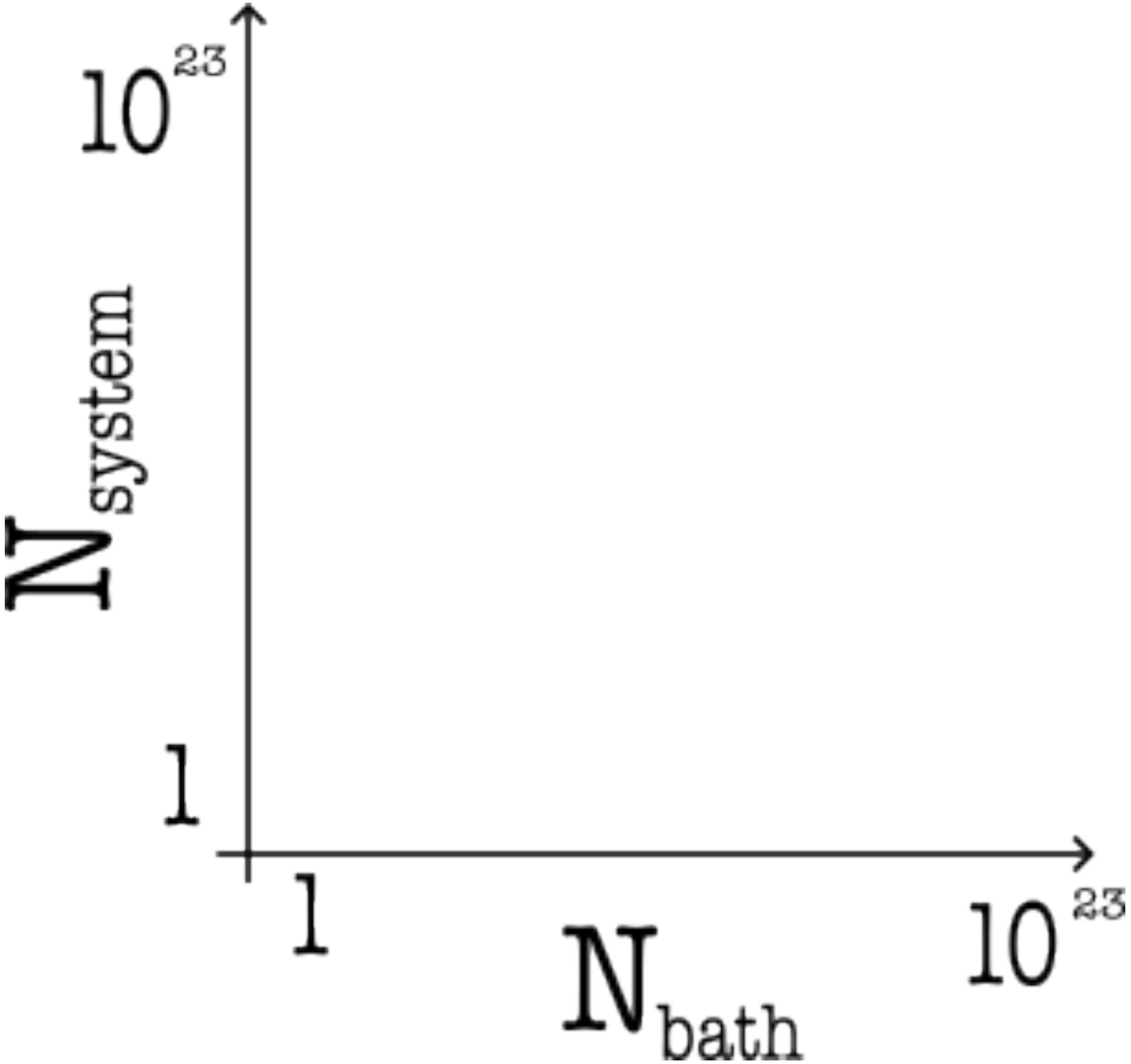
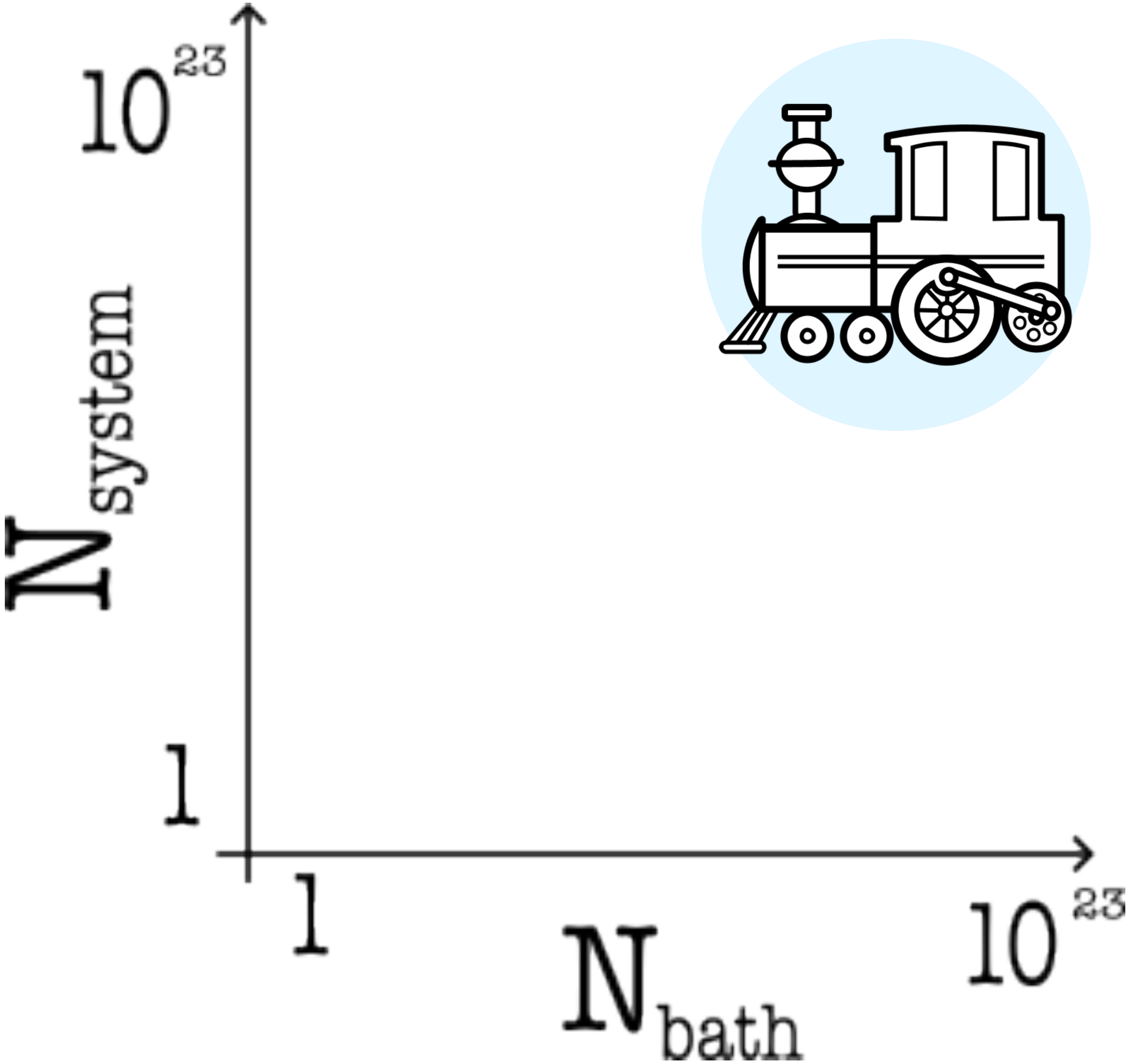


10^{23}
N_{system}
1
1
N_{bath}
10^{23}



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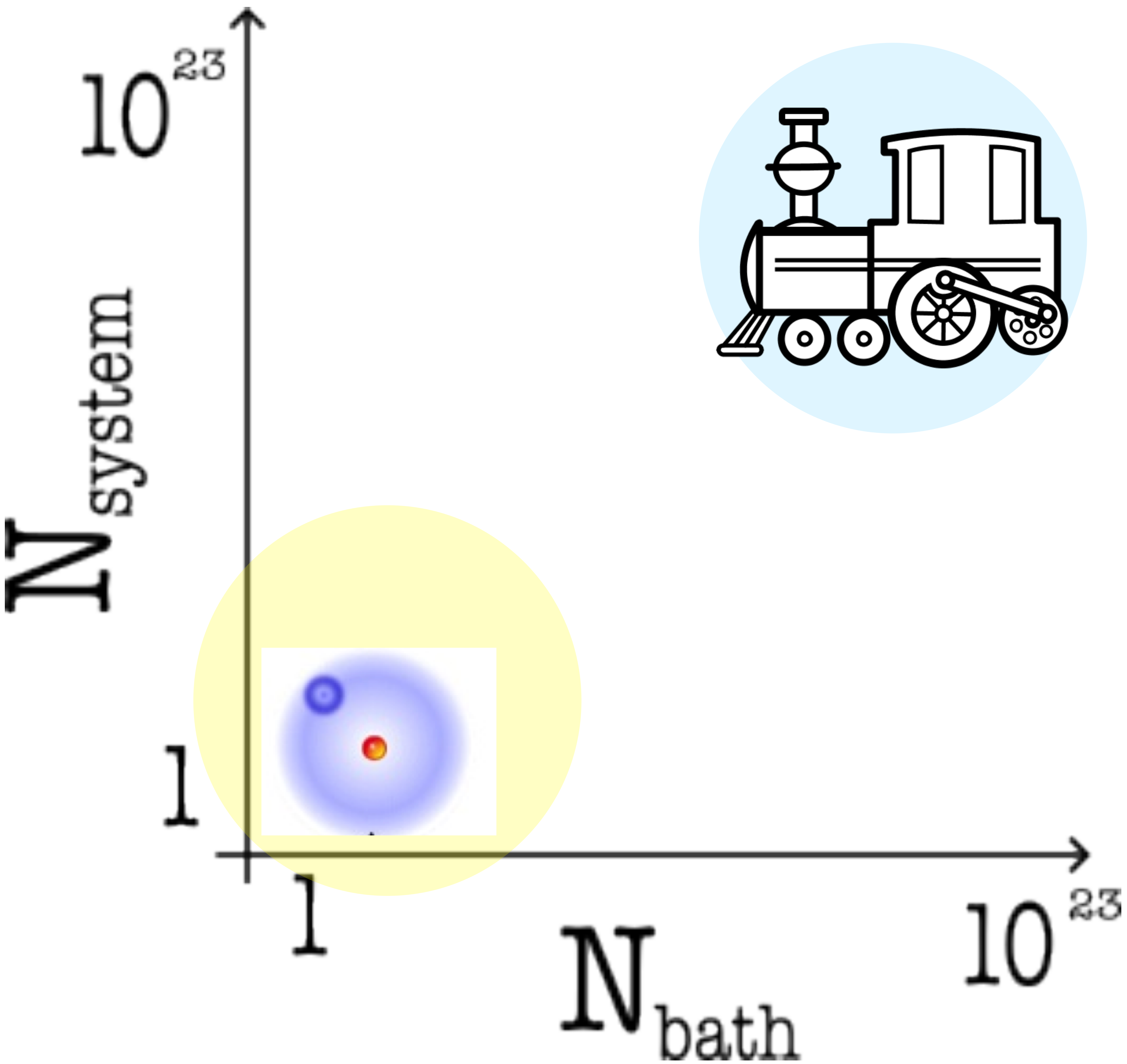

10^23
N_system
1
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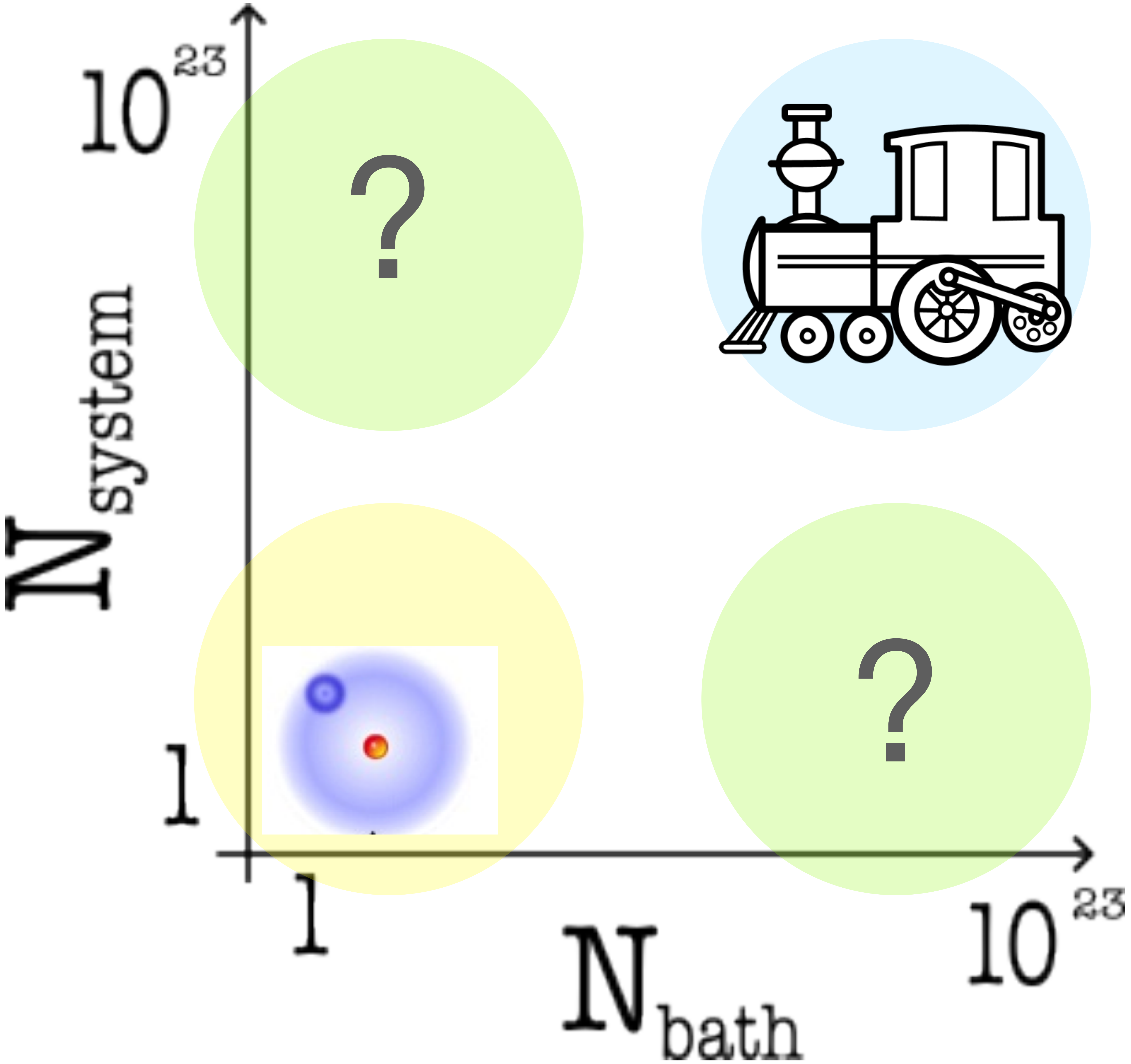
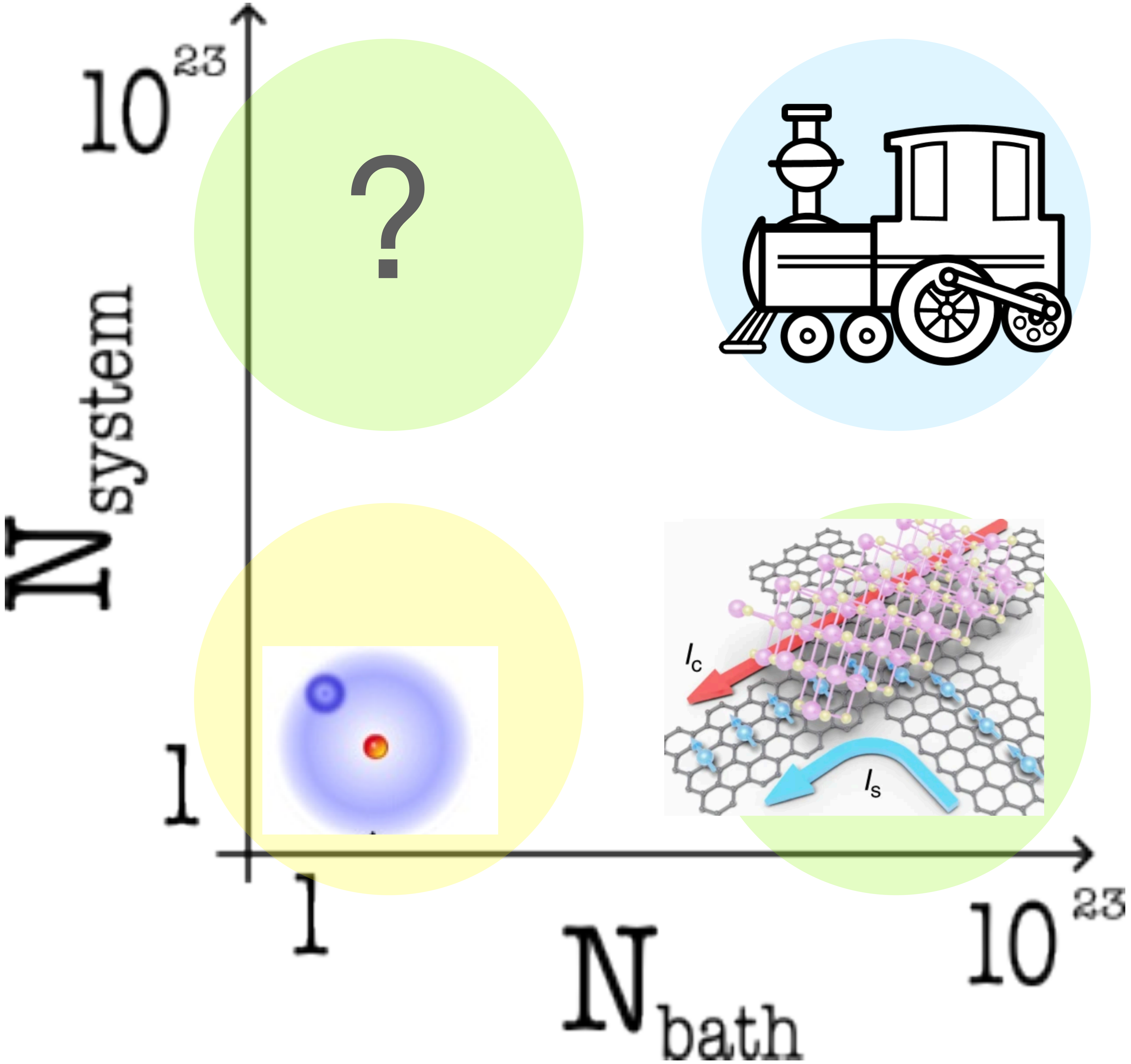
10^23



10^{23}
N_{system}
?
1
1
?
N_{bath}
10^{23}



10^{23}
N_{system}
1
?
I_c
I_s
1
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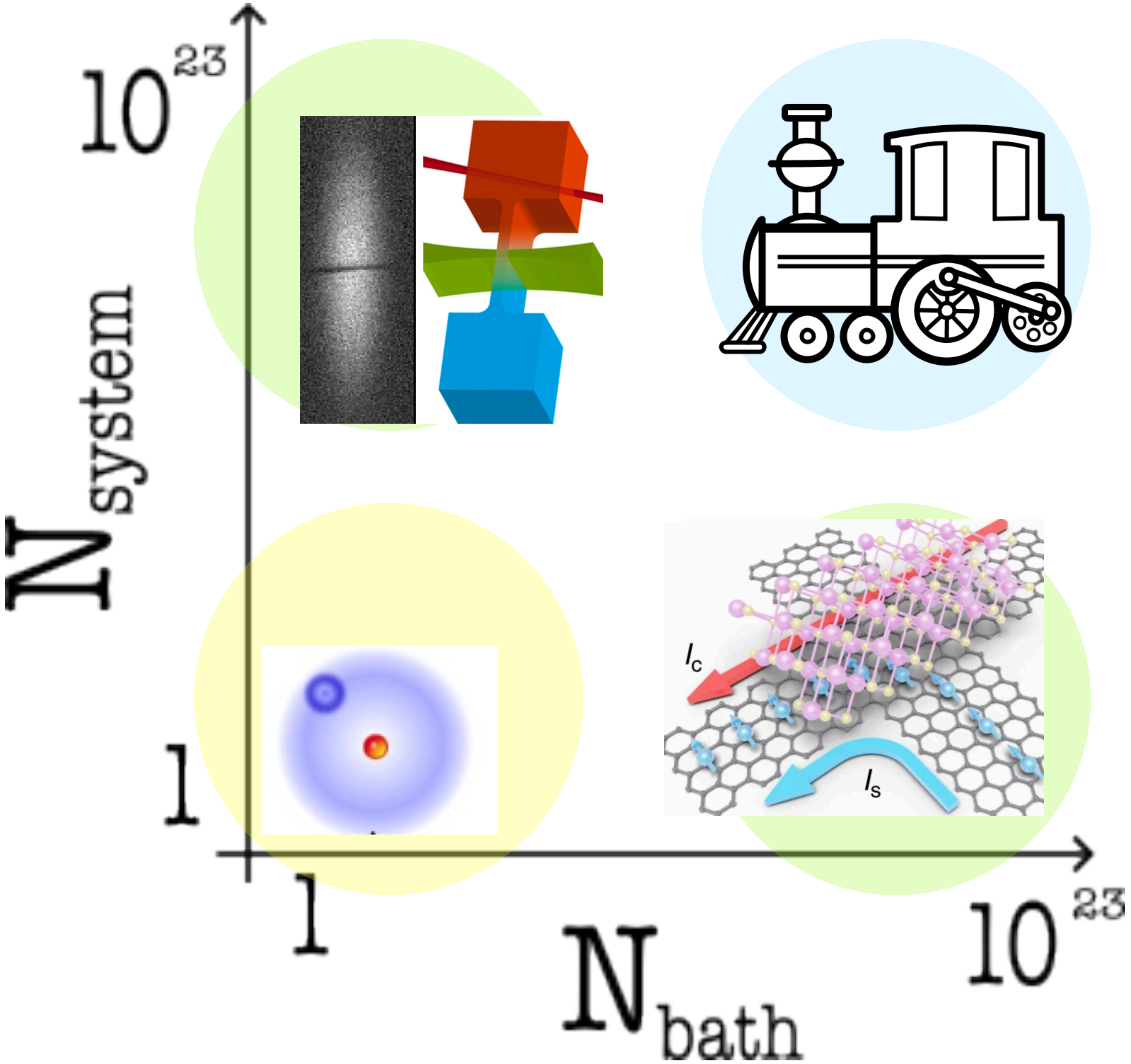

10^{23}
N_{system}
1
1
N_{bath}
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I_c
I_s

Energy conversion in small-scale devices?

- “waste” from operations: heat or unused energy excitations
- …can harm the operation and need to be evacuated (equivalent to cooling)
- Use/recycle for other tasks?

Needs better fundamental understanding….

Outline

- Introduction
  - Thermodynamics of steady-state quantum transport

Resources and output of nanoscale thermodynamic systems are diverse!

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- Fluctuations in mesoscopic conductors operating as engines
    - Fluctuation dissipation theorem and beyond
    - Implications for the precision of steady-state nanoscale engines

L. Tesser, J. Splettstoesser: Phys. Rev. Lett. **132**, 186304 (2024).

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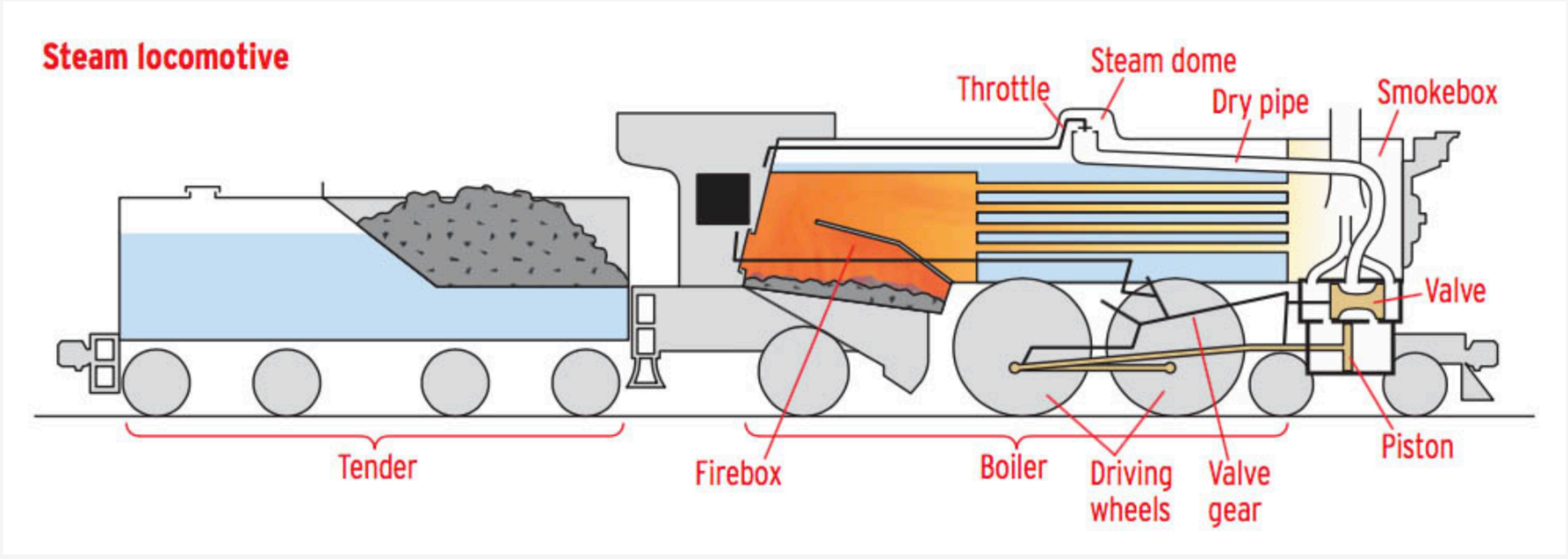
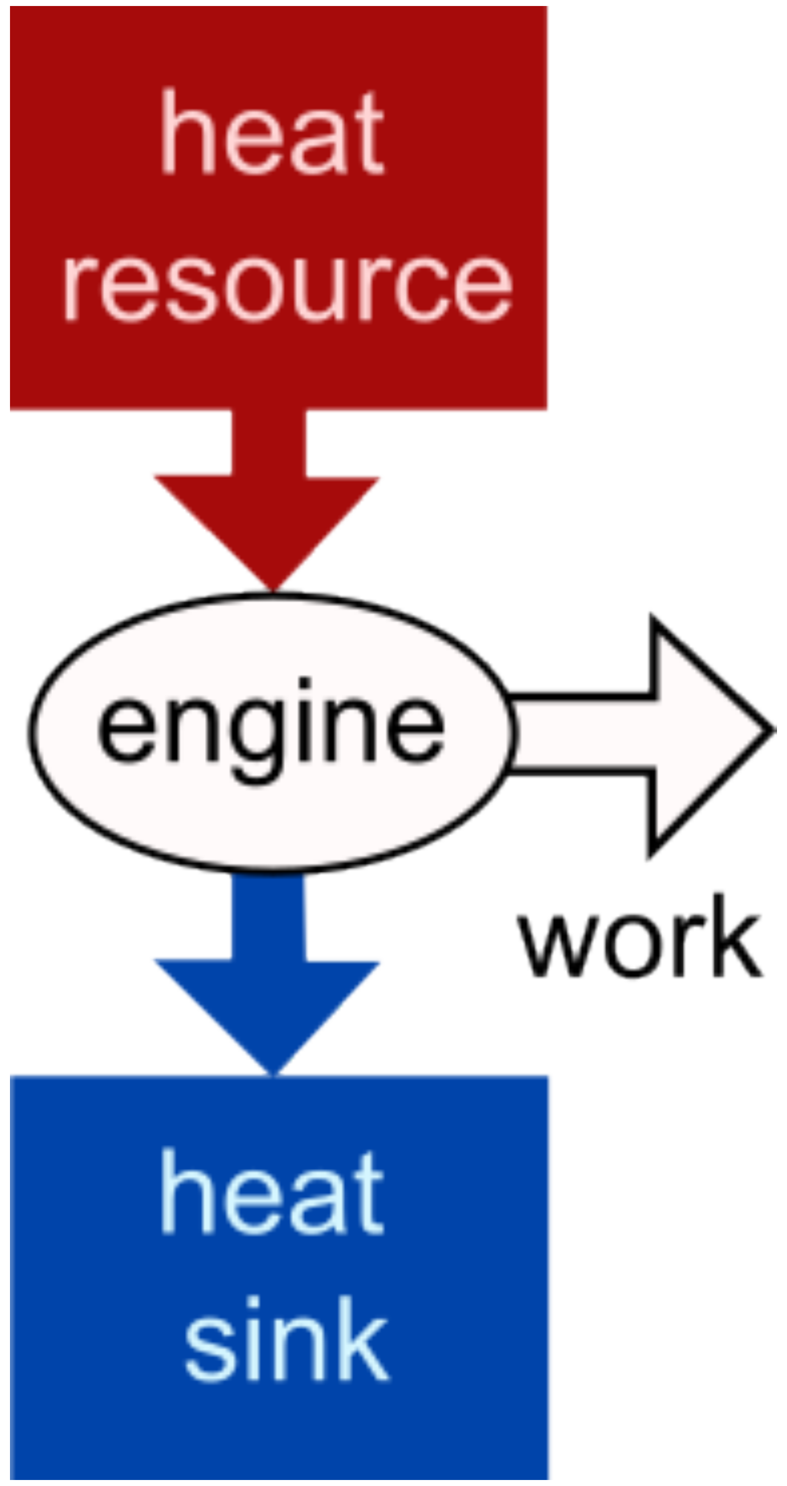
- Precision bounds for quantum transport of generic transport observables
  - Kinetic uncertainty relation-like precision bounds

D. Palmqvist, L. Tesser, J. Splettstoesser: arXiv:2410.10793 (2024).

M. Acciai*, L. Tesser*, J. Eriksson, R. Sánchez, R. S. Whitney, J. Splettstoesser, Phys. Rev. B **109**, 075405 (2024).

Good old heat engines…





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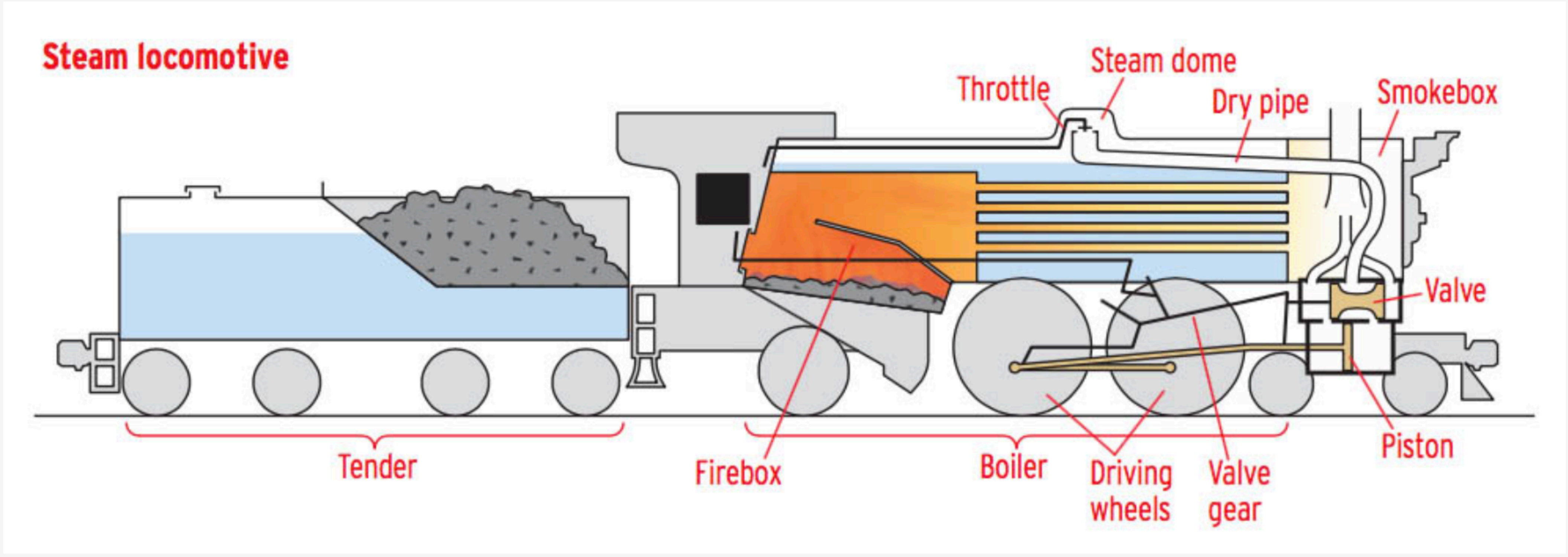

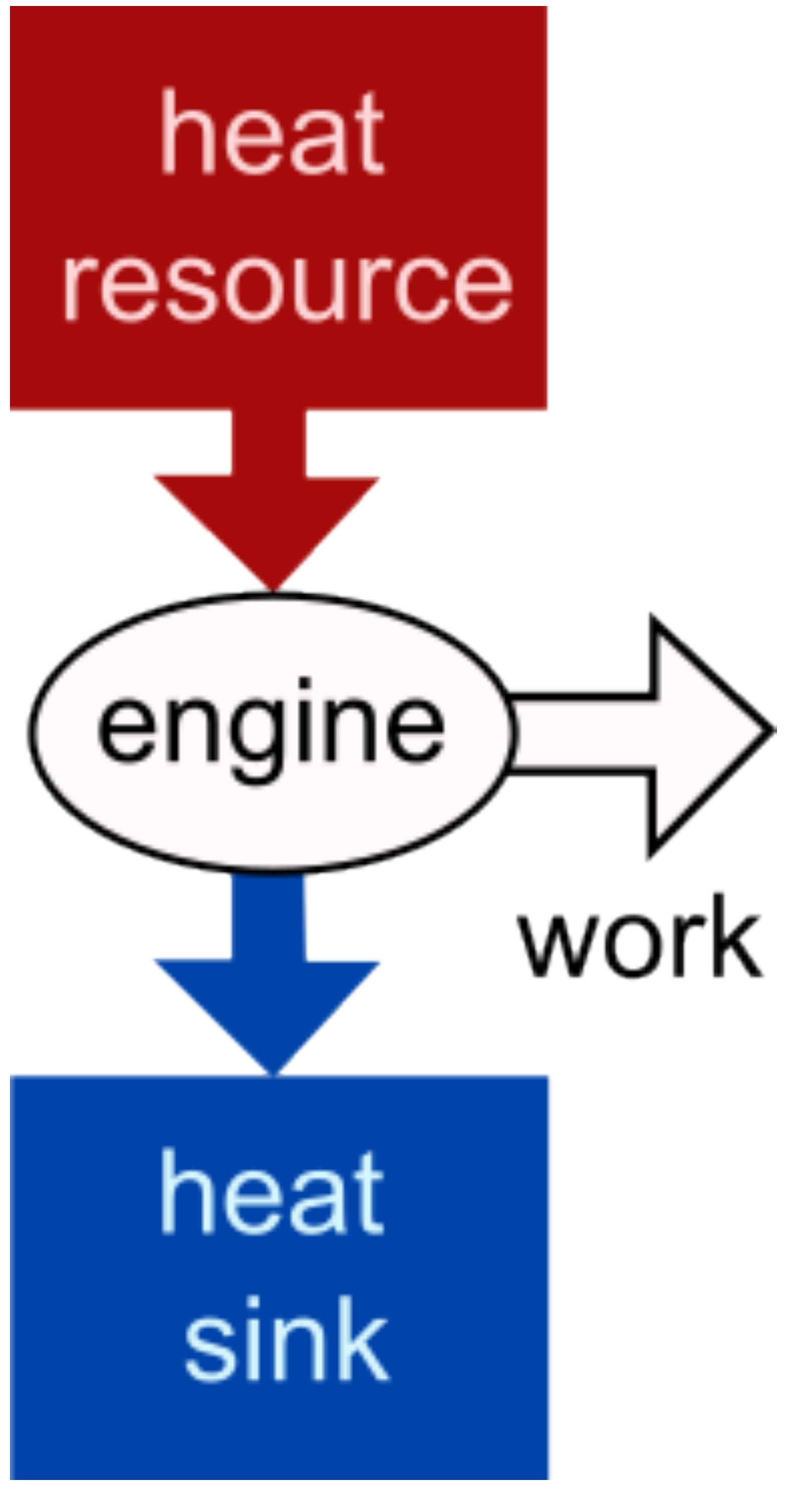

Goal: produce work with a resource that is easy to have.

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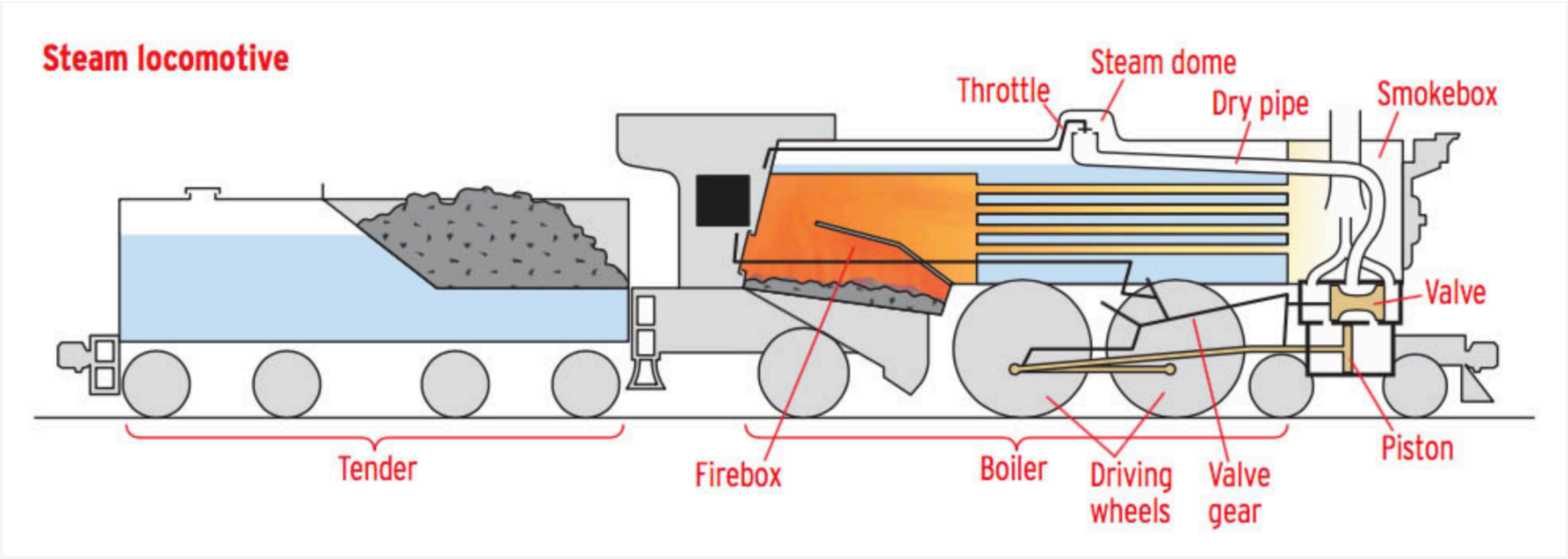

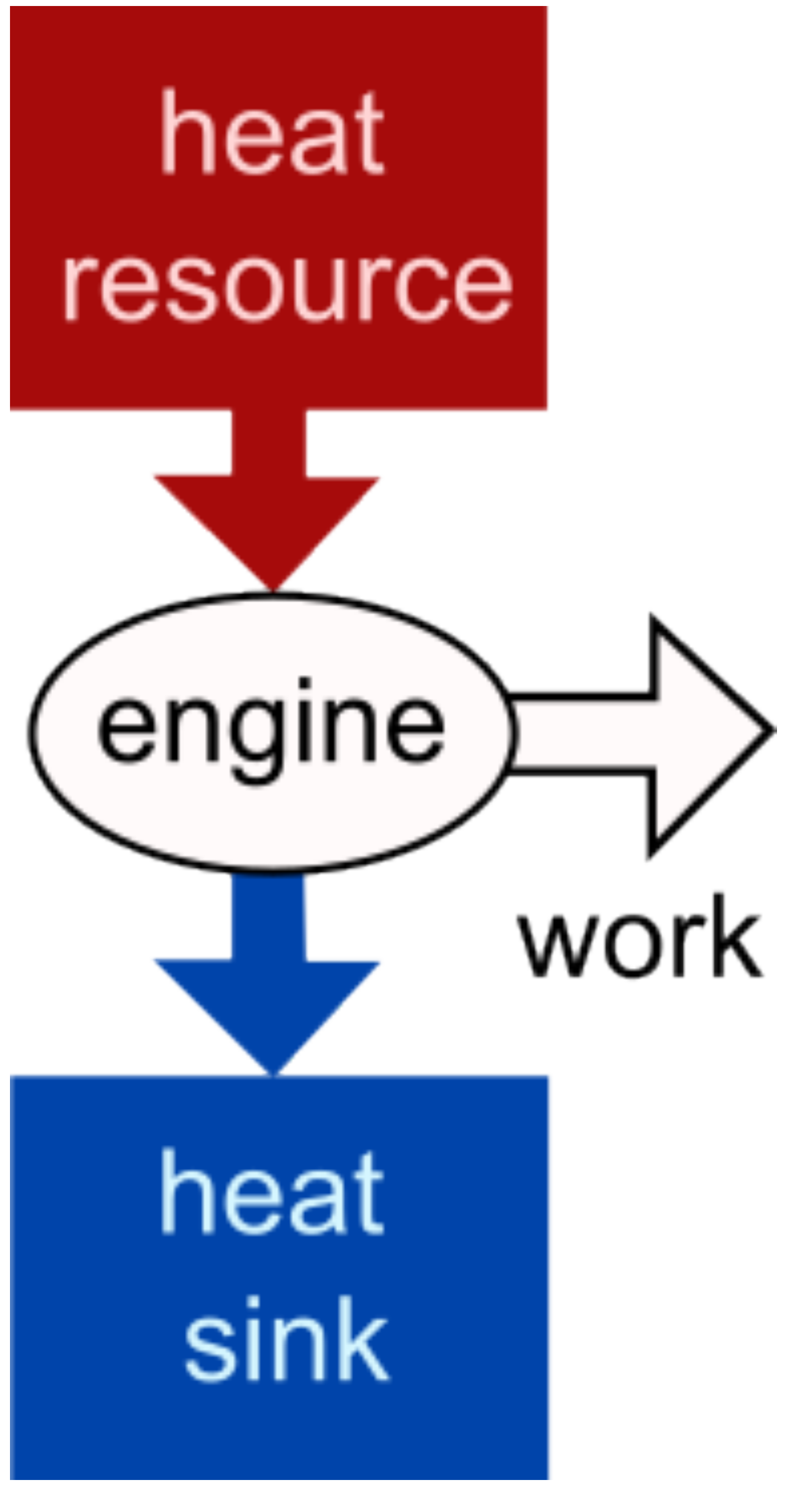

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Typically: cyclic operation

(but steady state is in many applications more practical)

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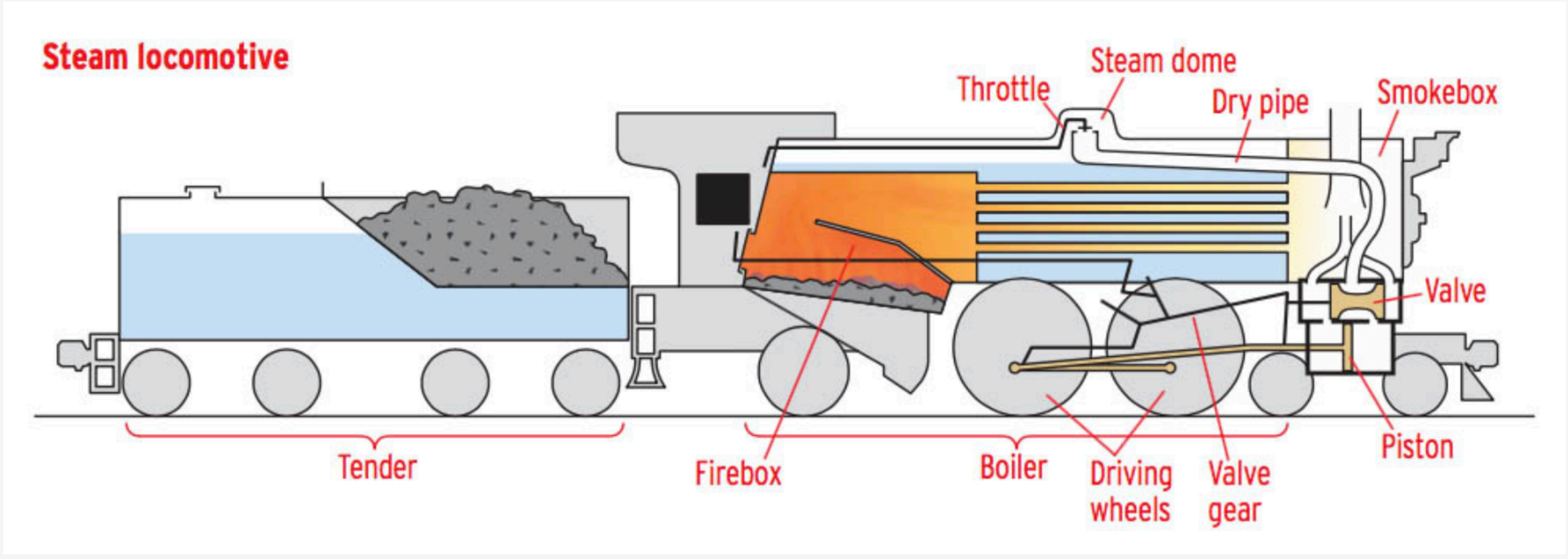

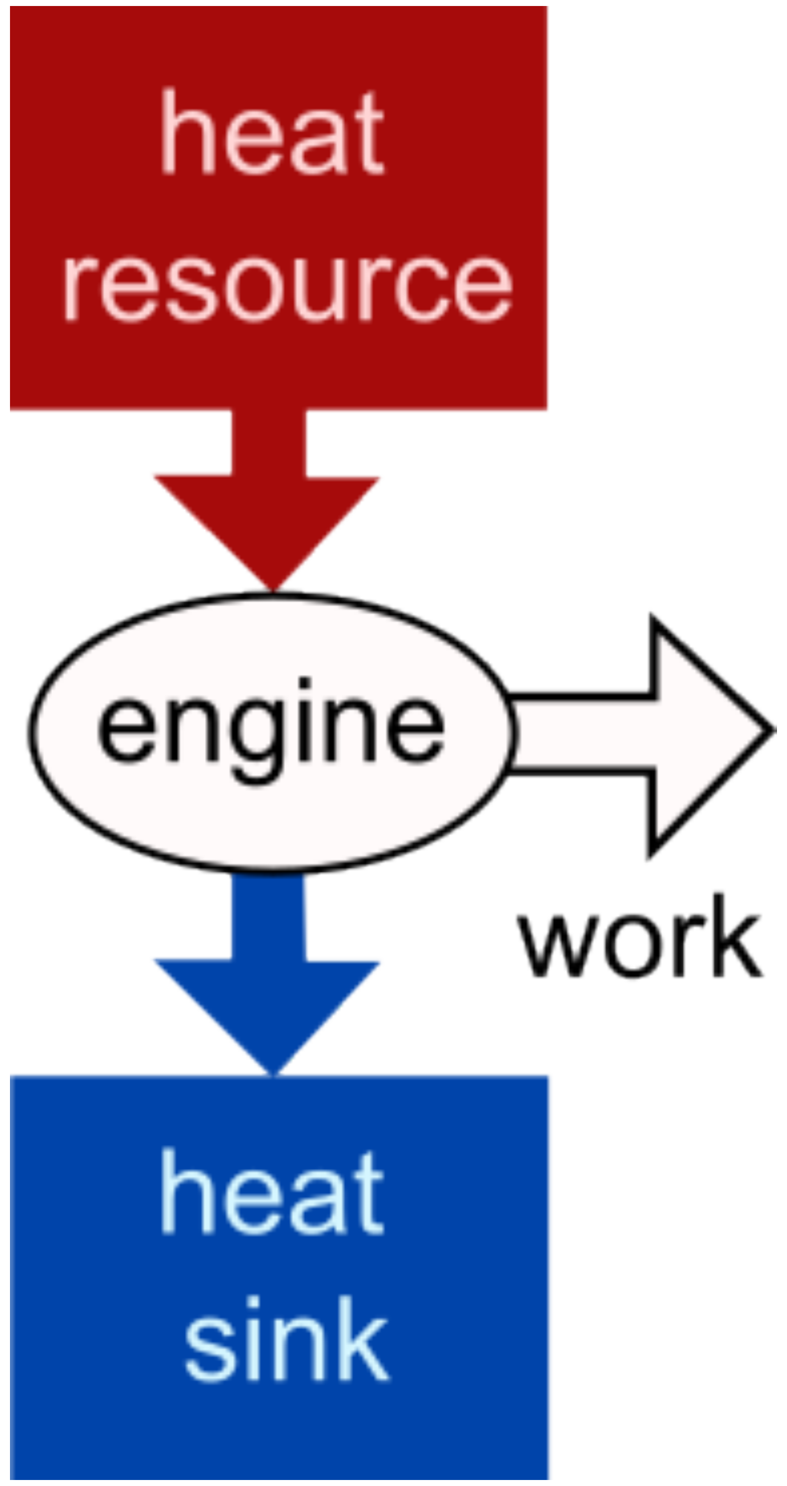

$$P = \dot{W}$$

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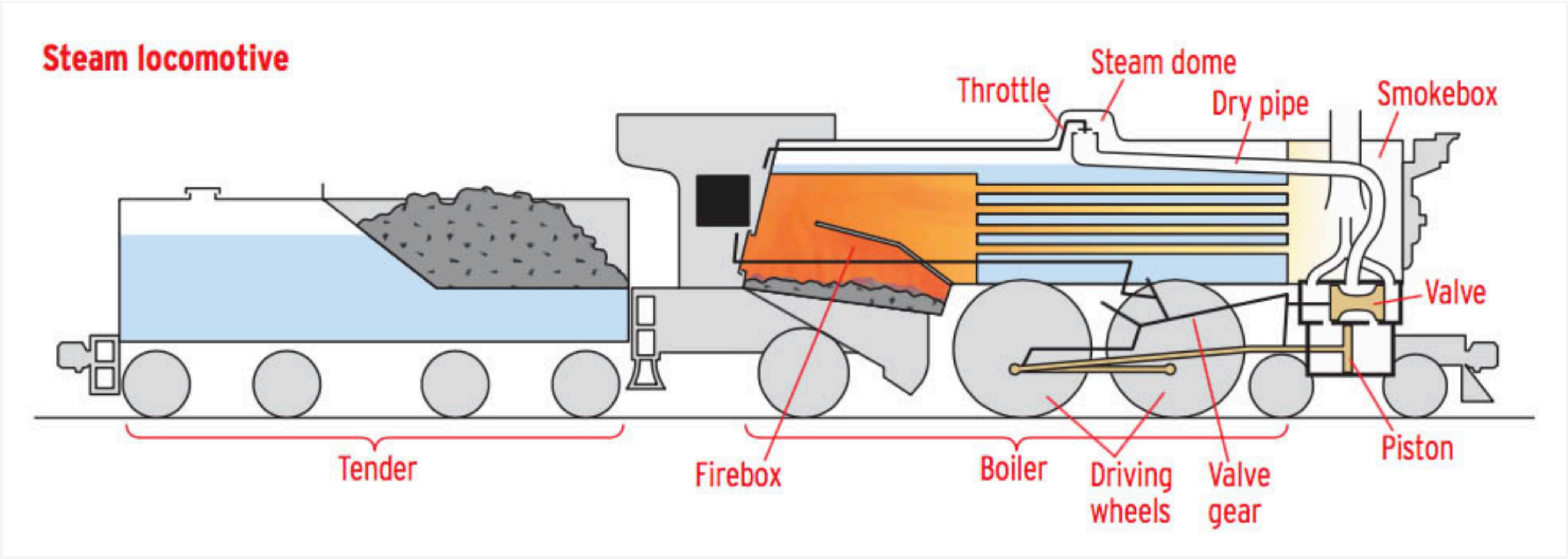

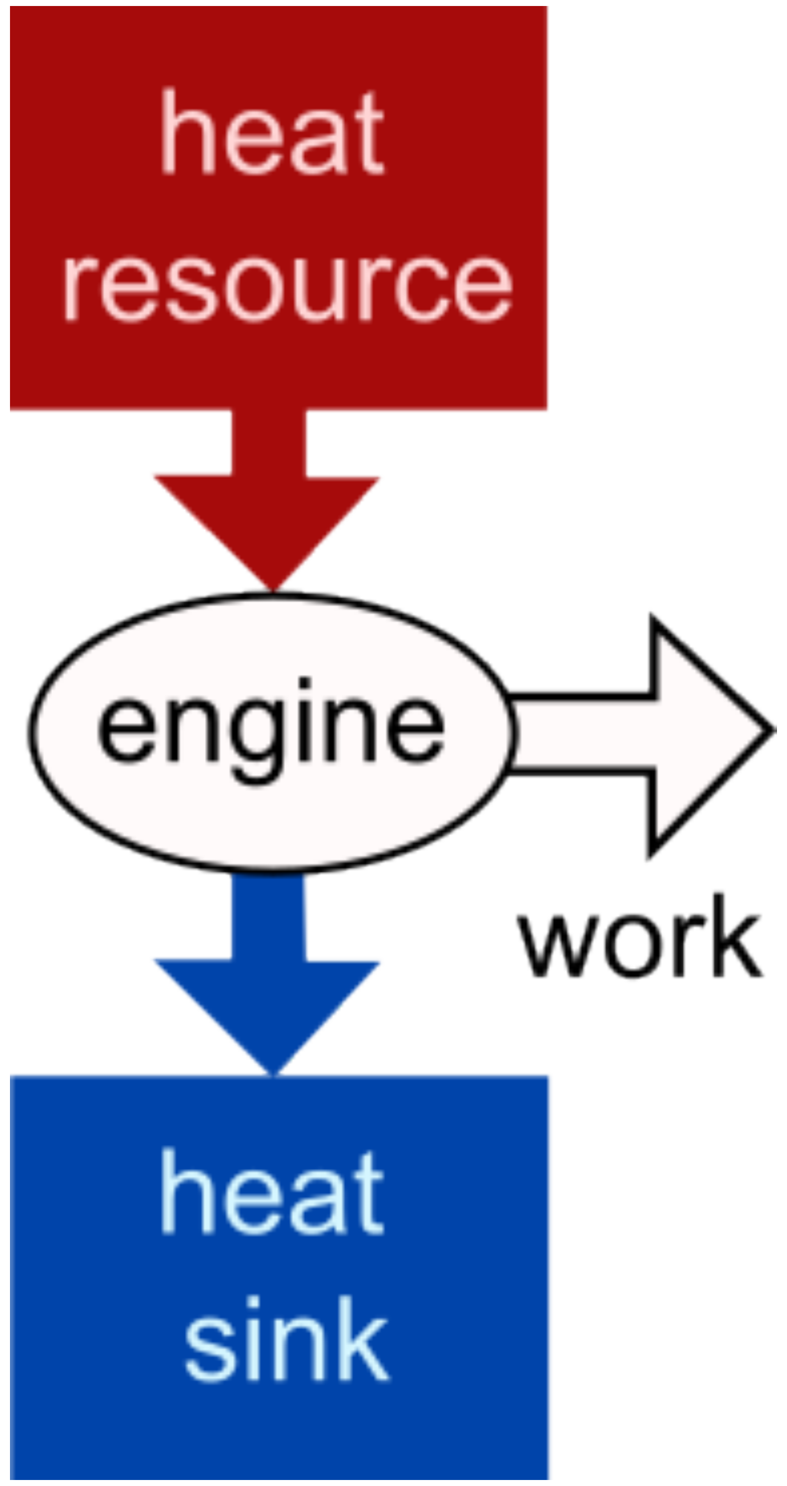

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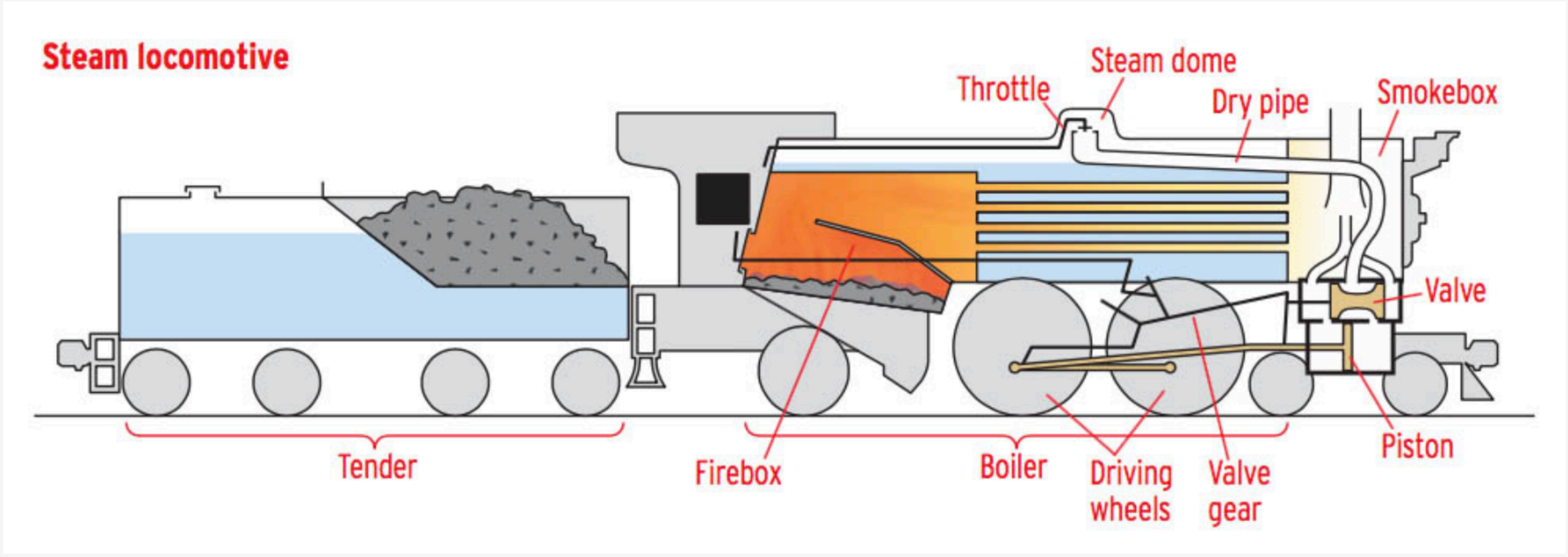

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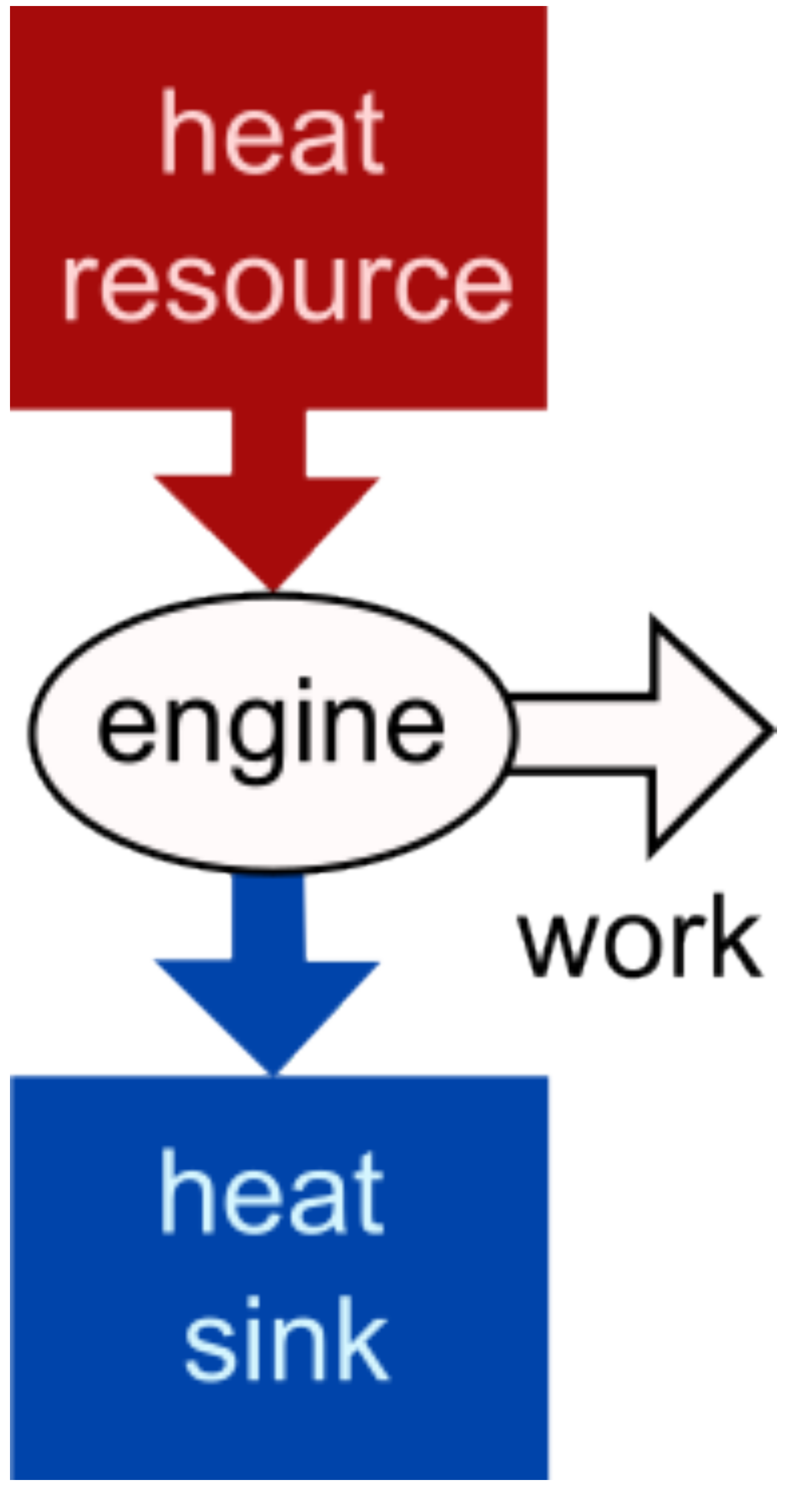


$$J_{\text{heat}}$$

$$P = \dot{W}$$

Performance goals: large power

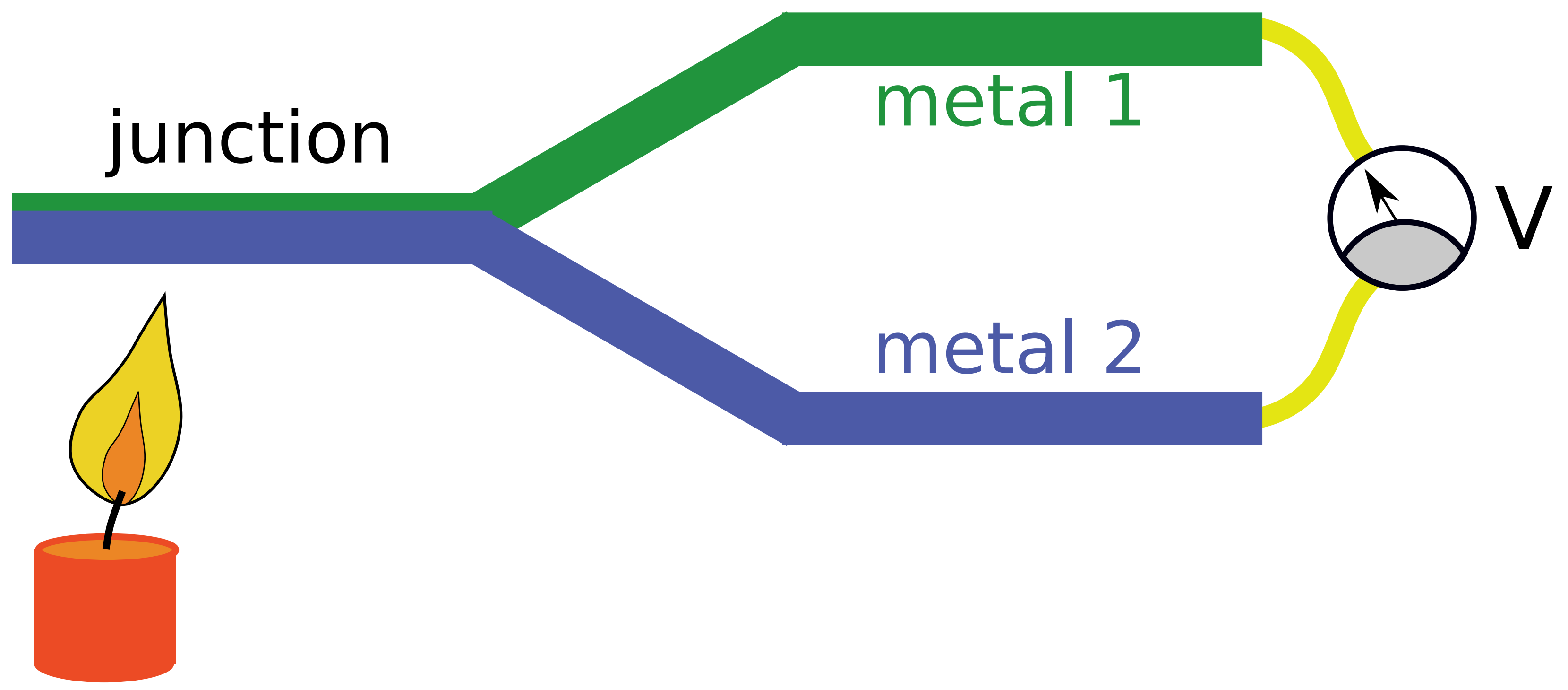
$$\eta = \frac{P}{J_{\text{heat}}}$$

Using available heat flow

$$\leq 1 - T_c/T_h$$

Limited by **Carnot efficiency** (only reached at zero power)

Steady-state (thermoelectric) heat engine



Simple and reliable!

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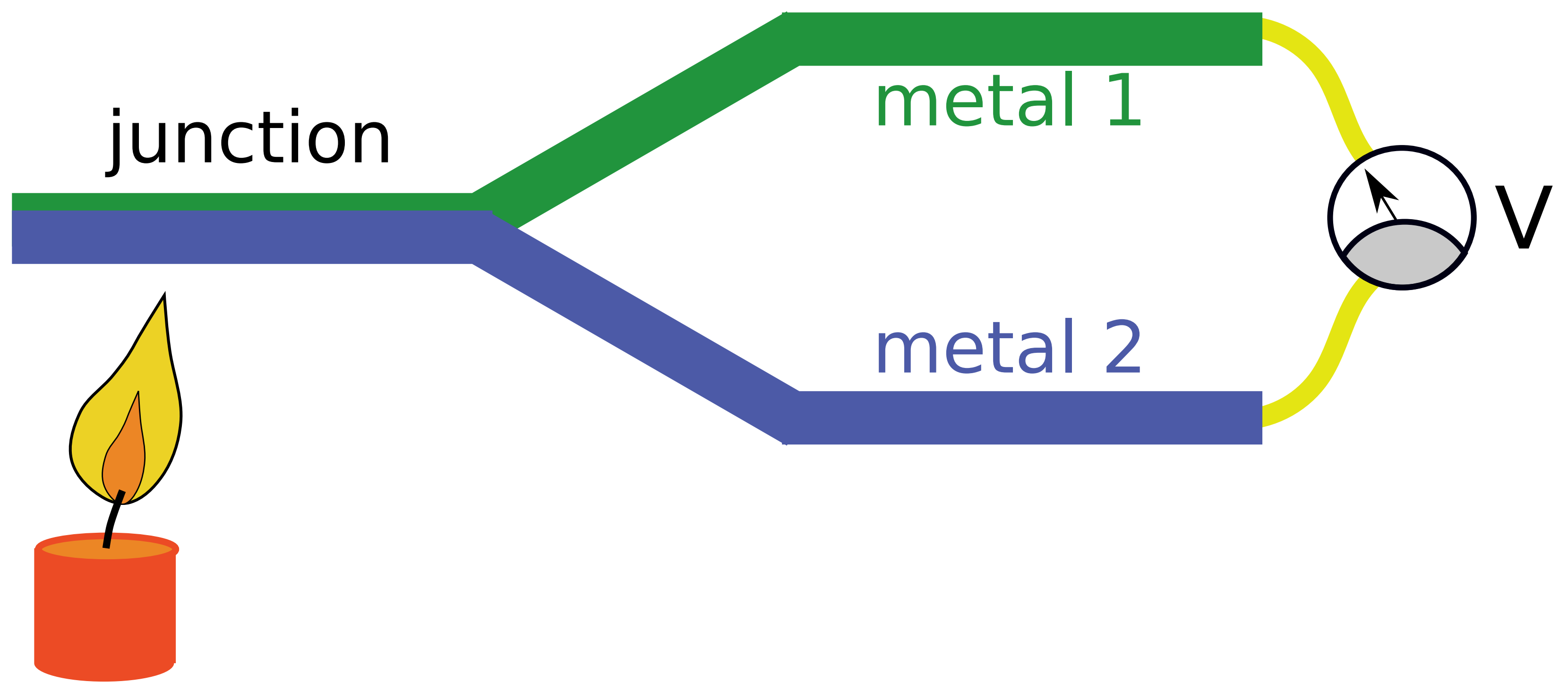

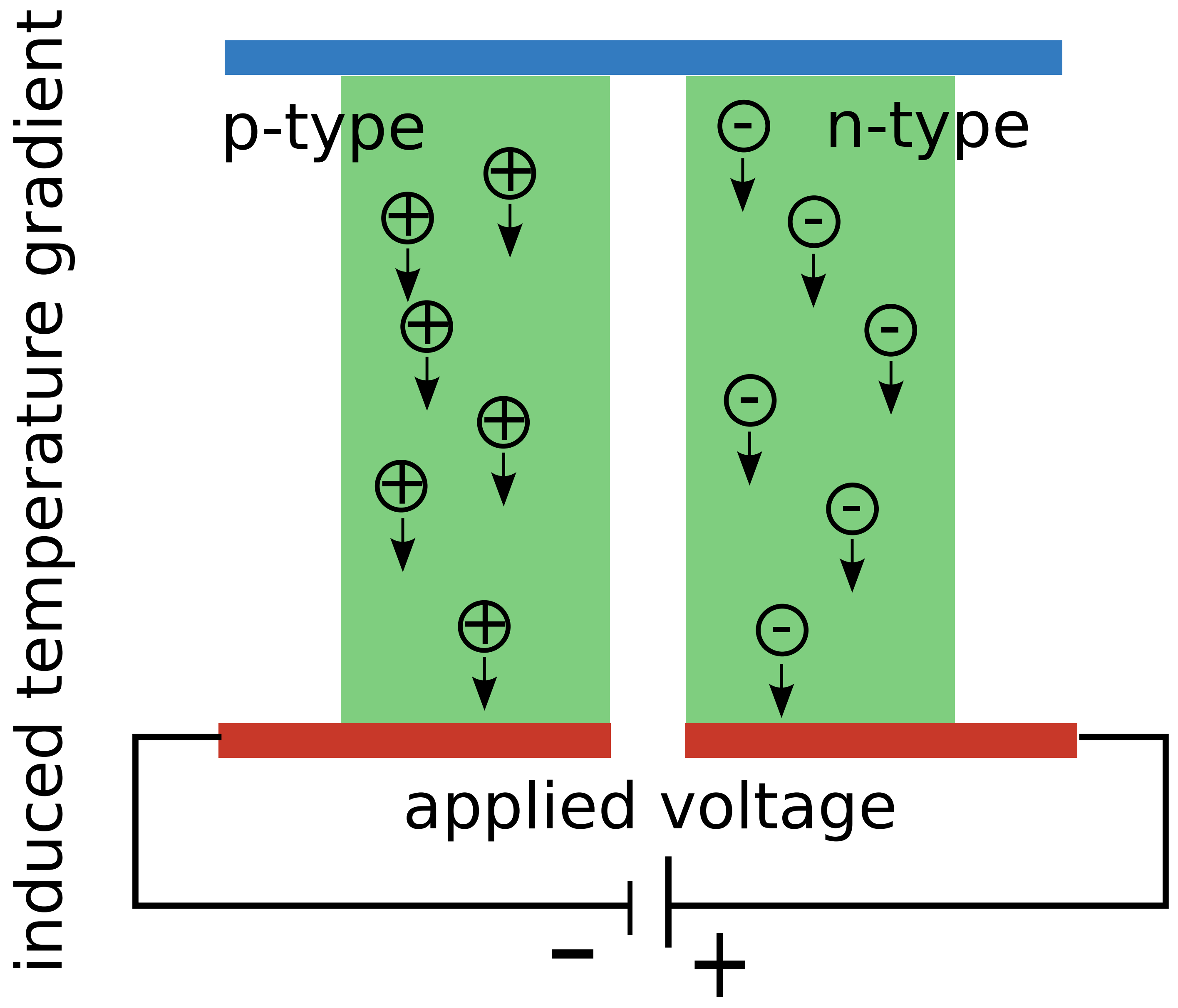

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Exploits energy-filtering (e.g. different transport for electrons and holes)

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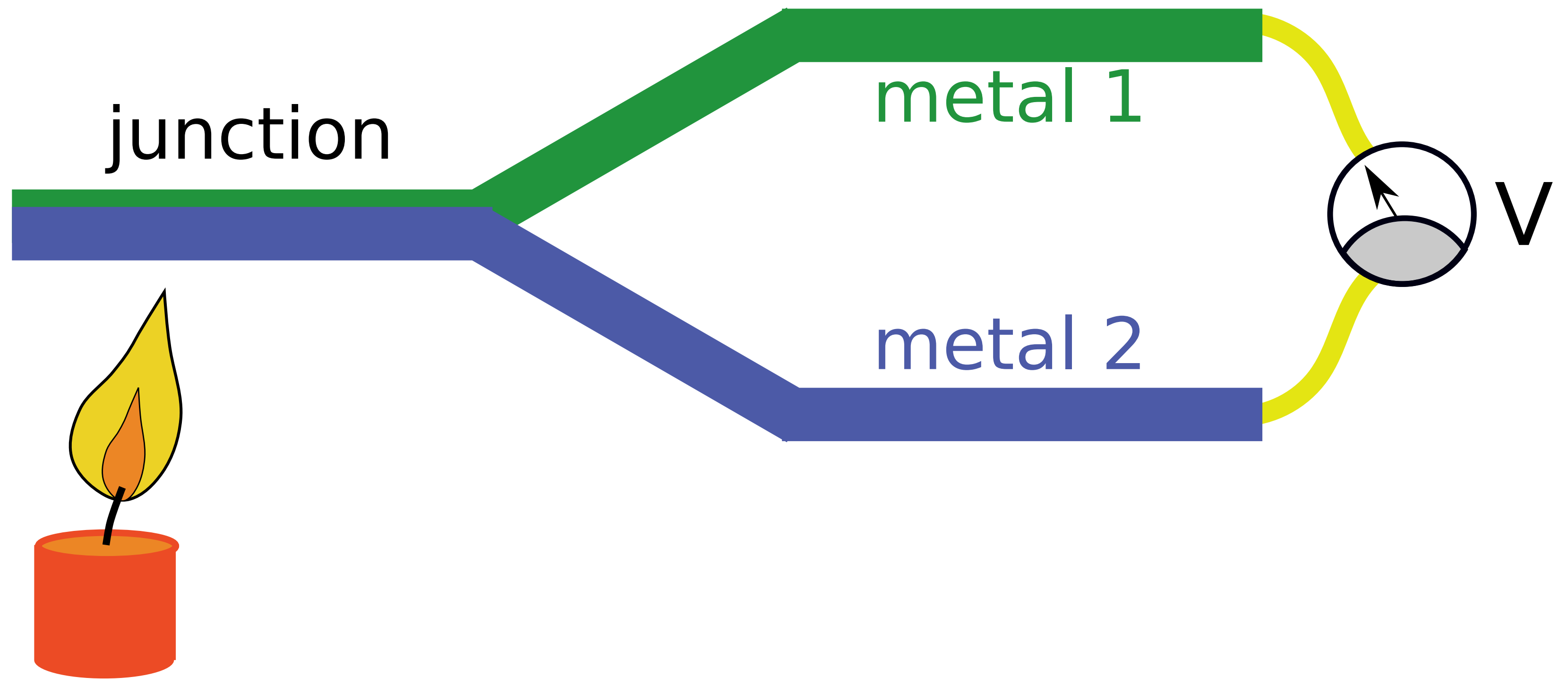

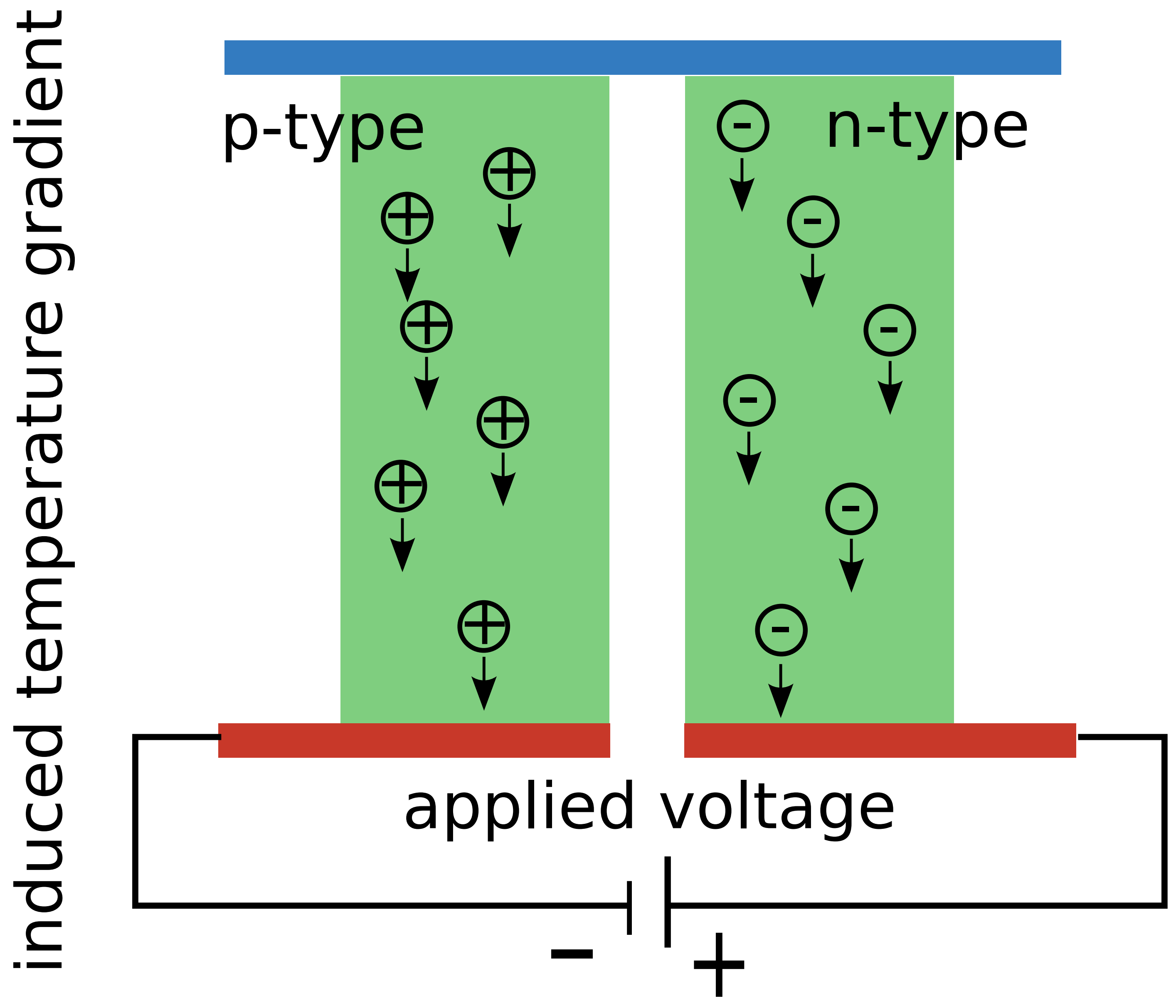

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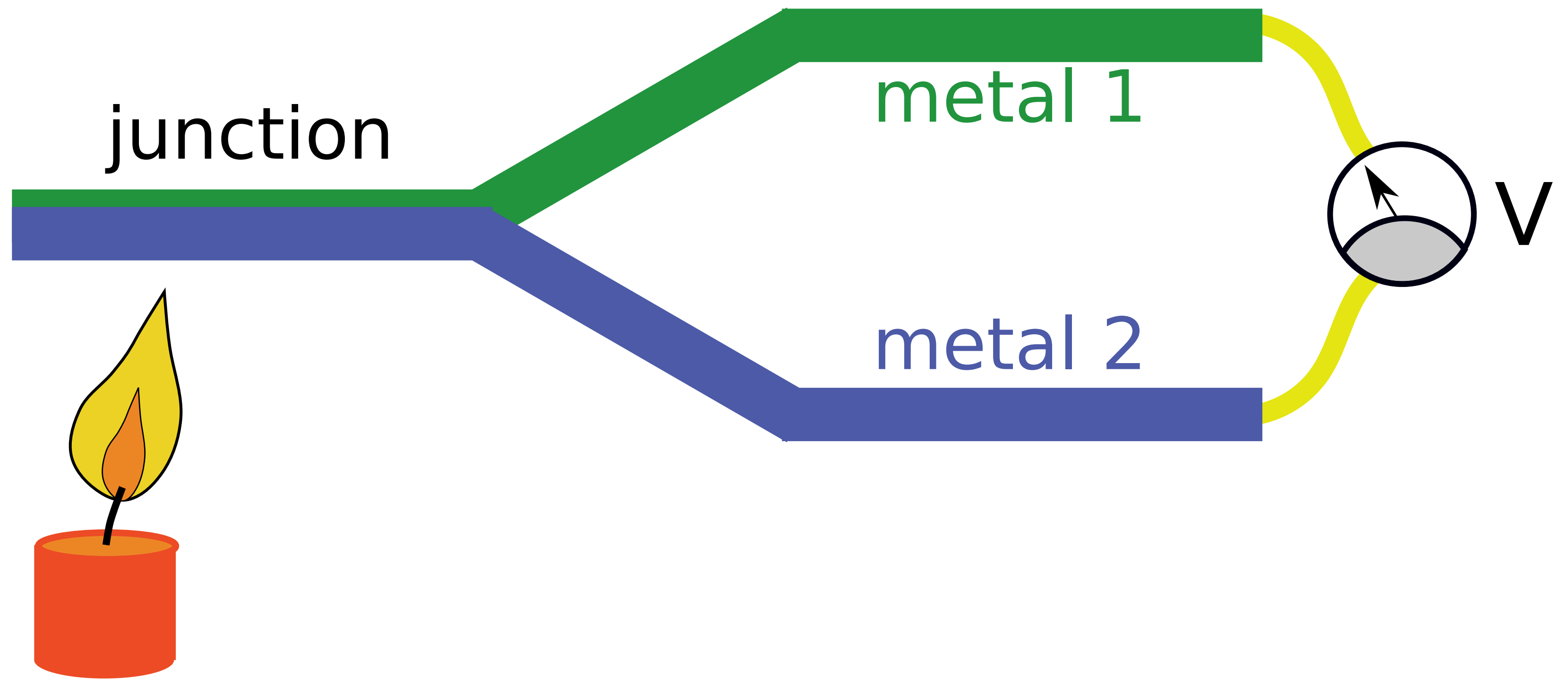

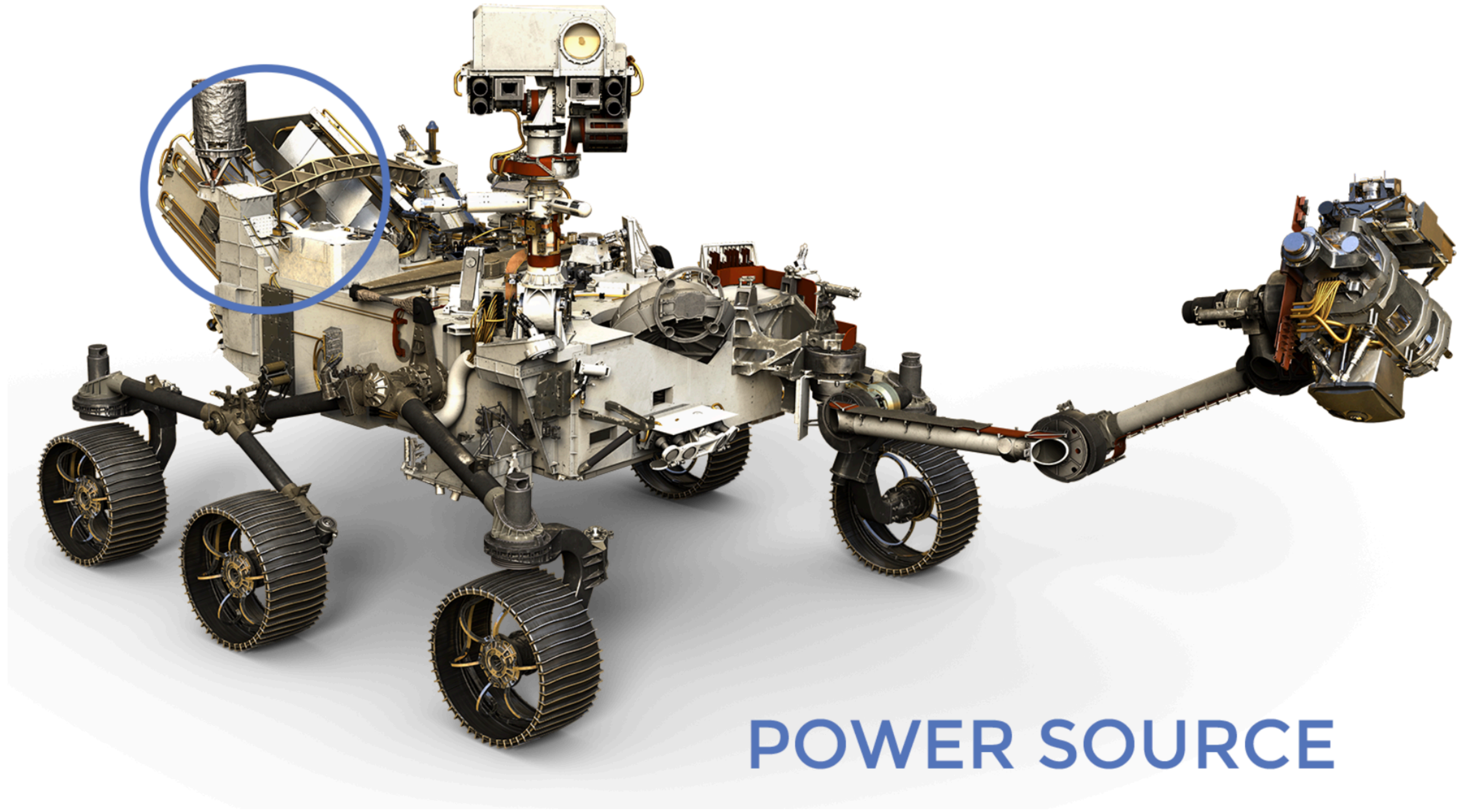

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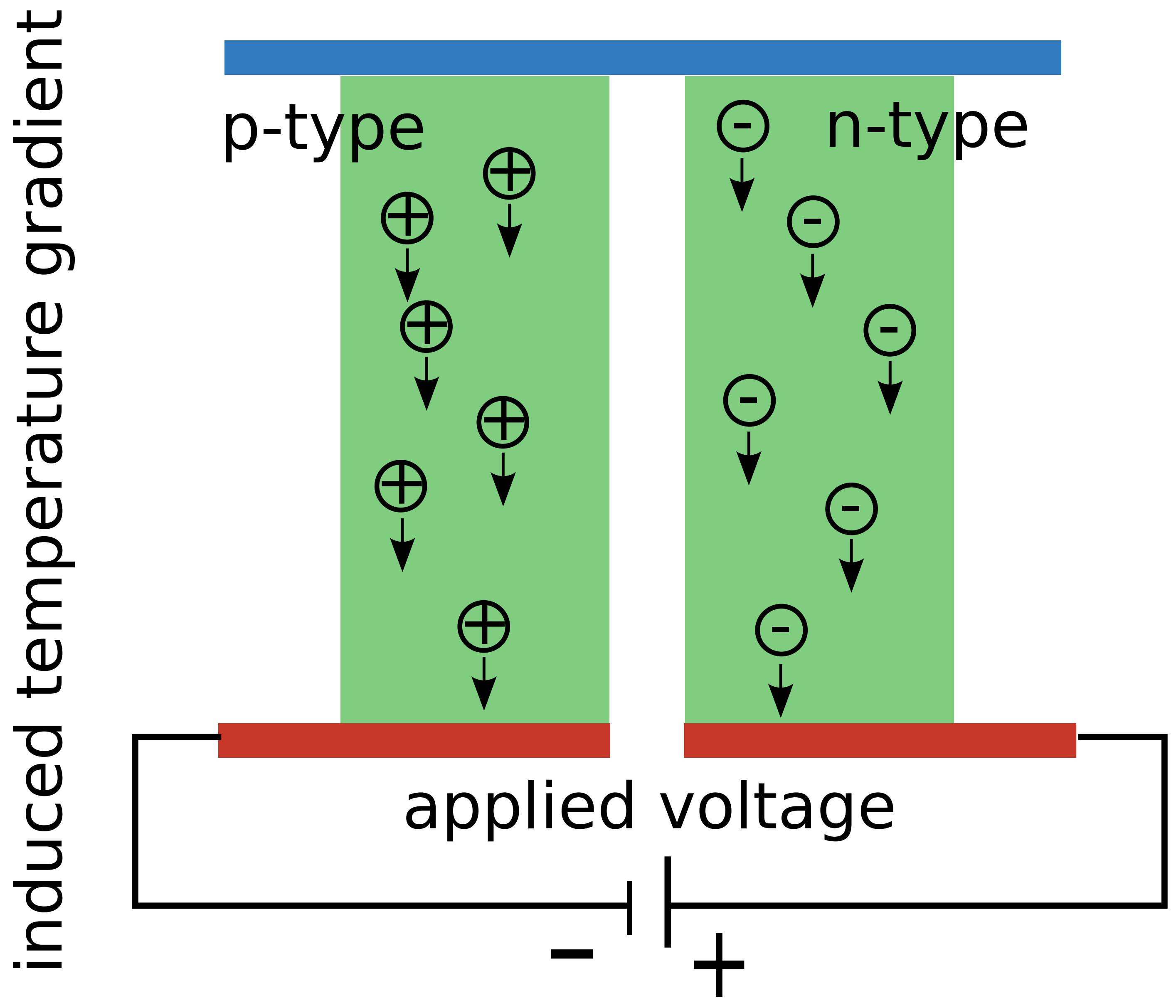

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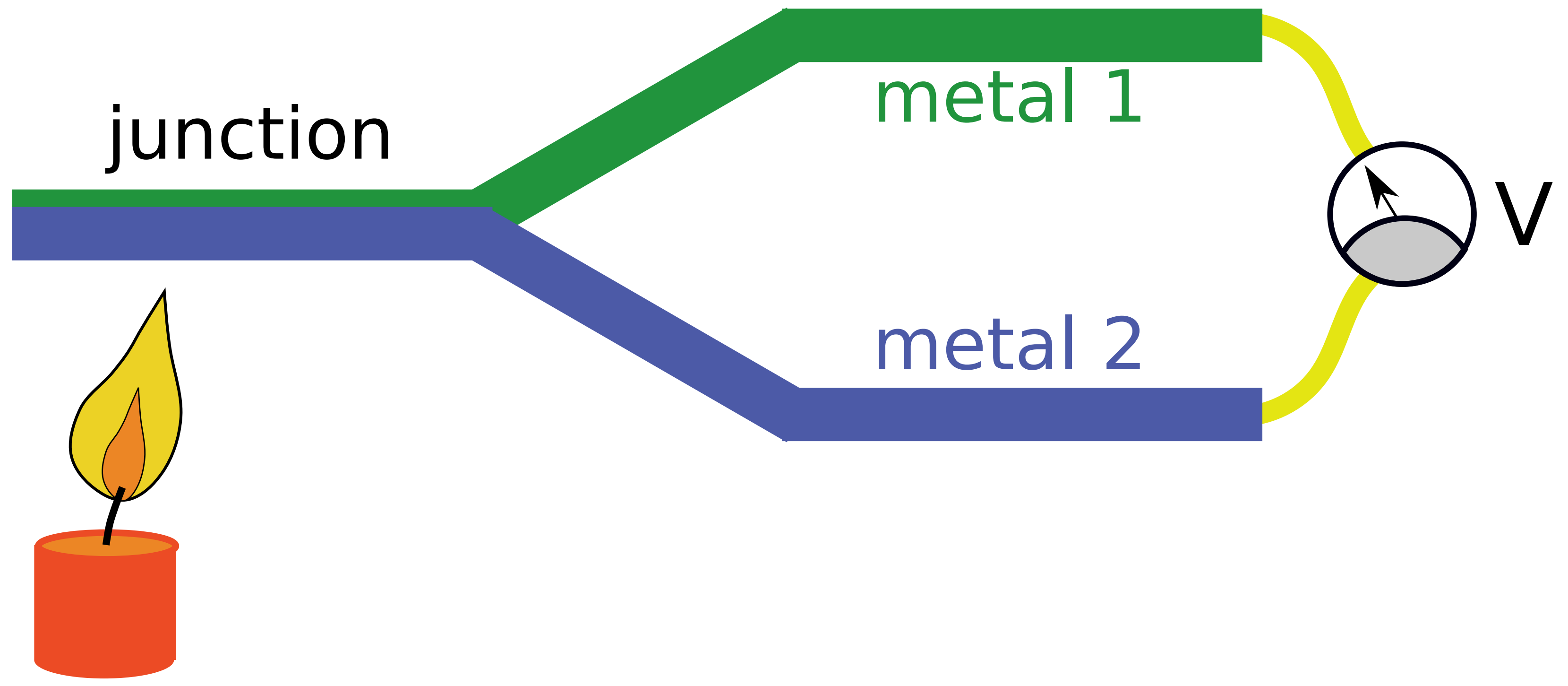

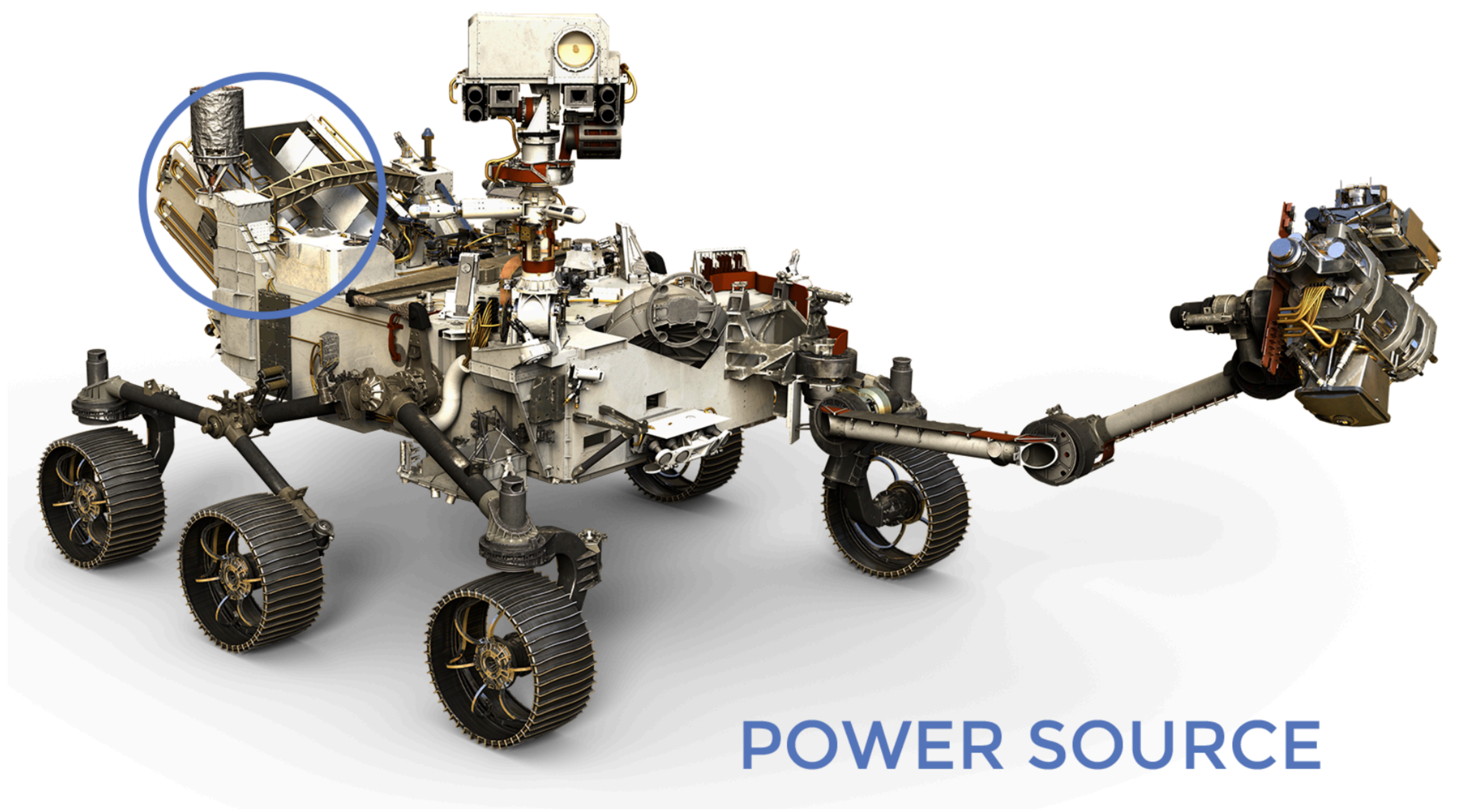

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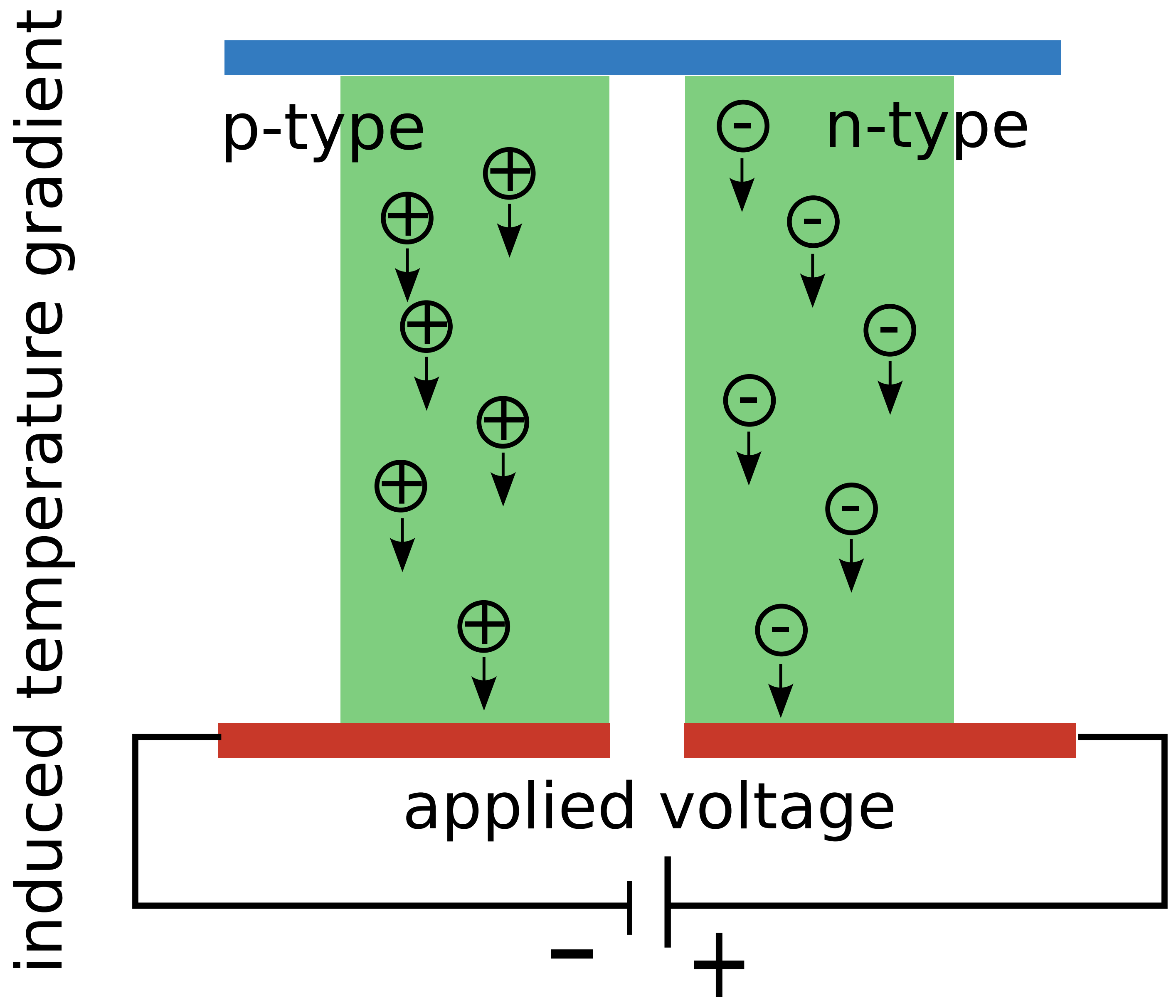

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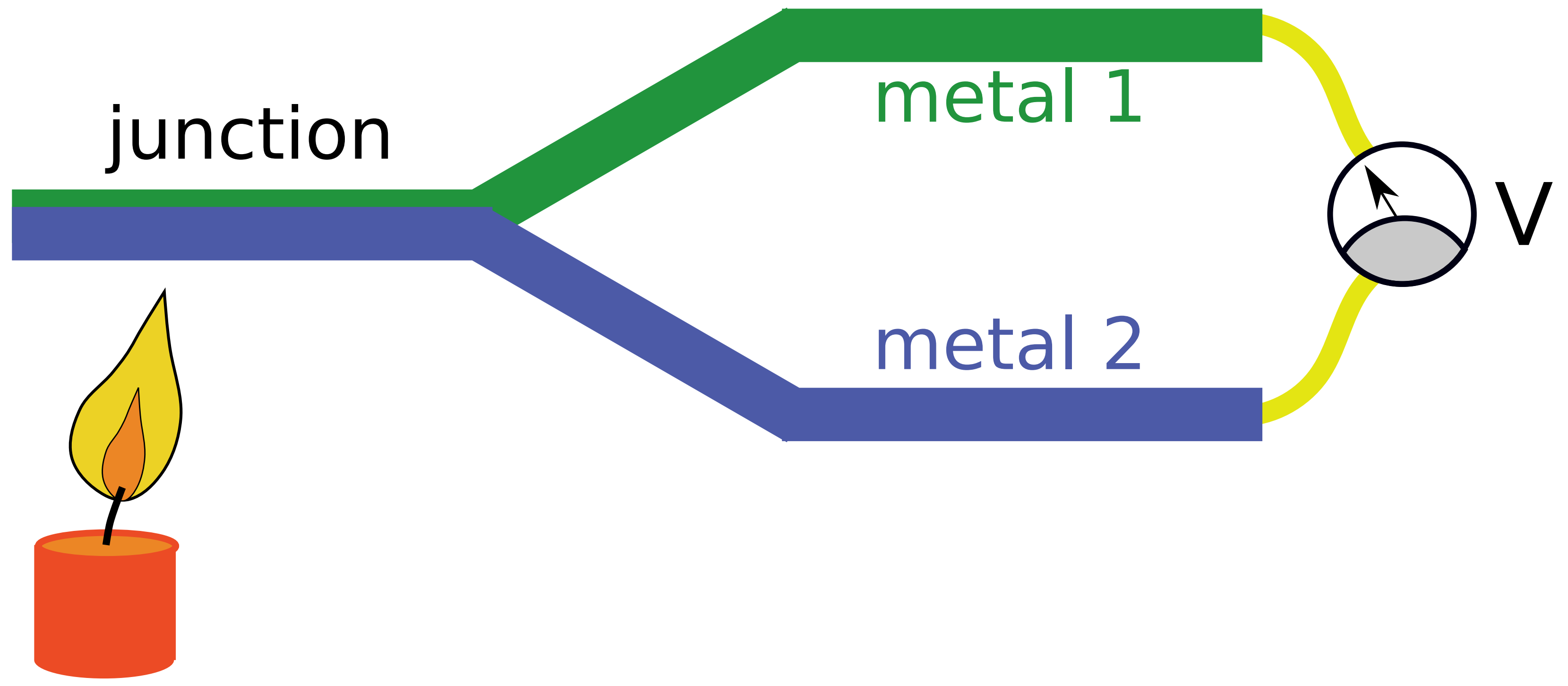

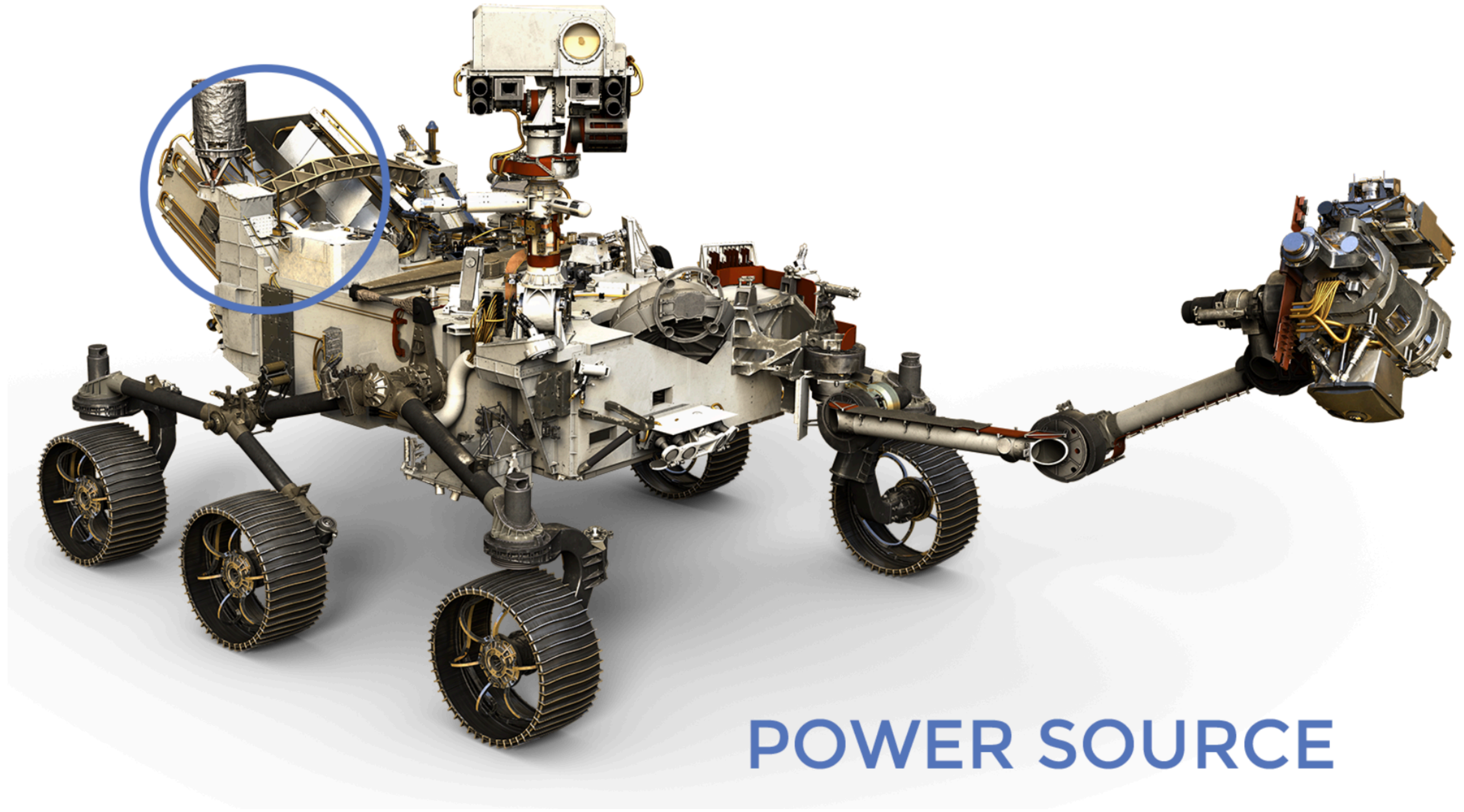

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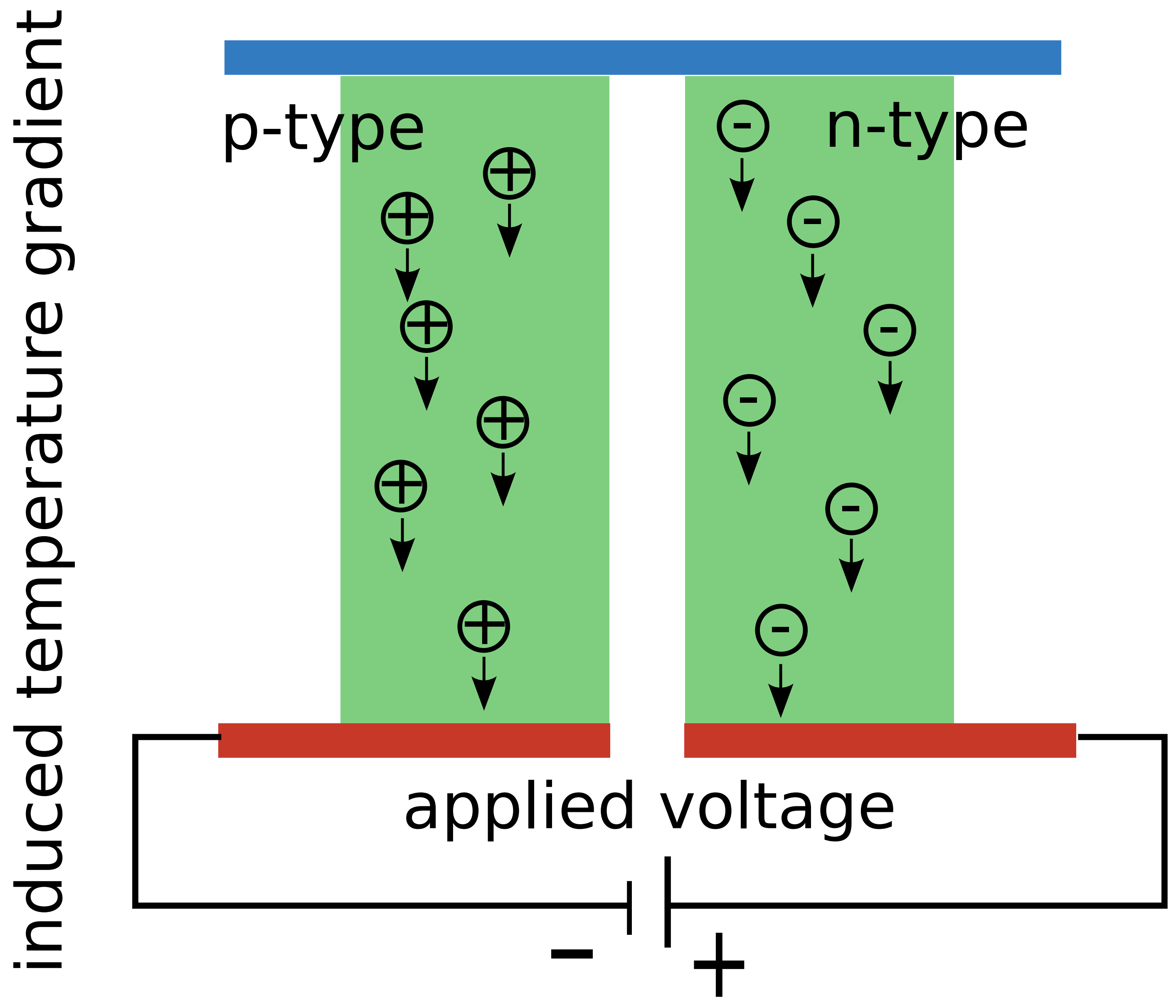

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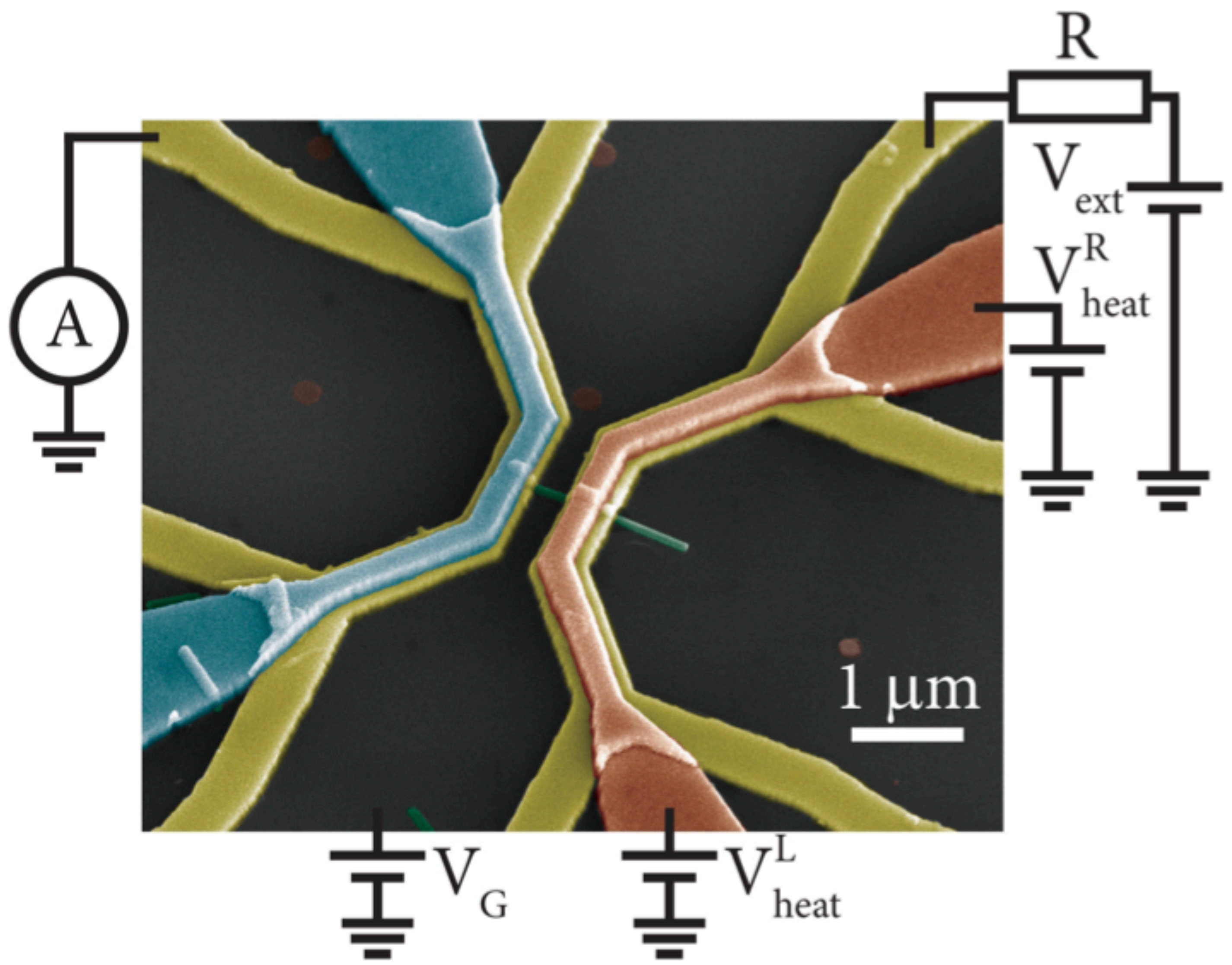
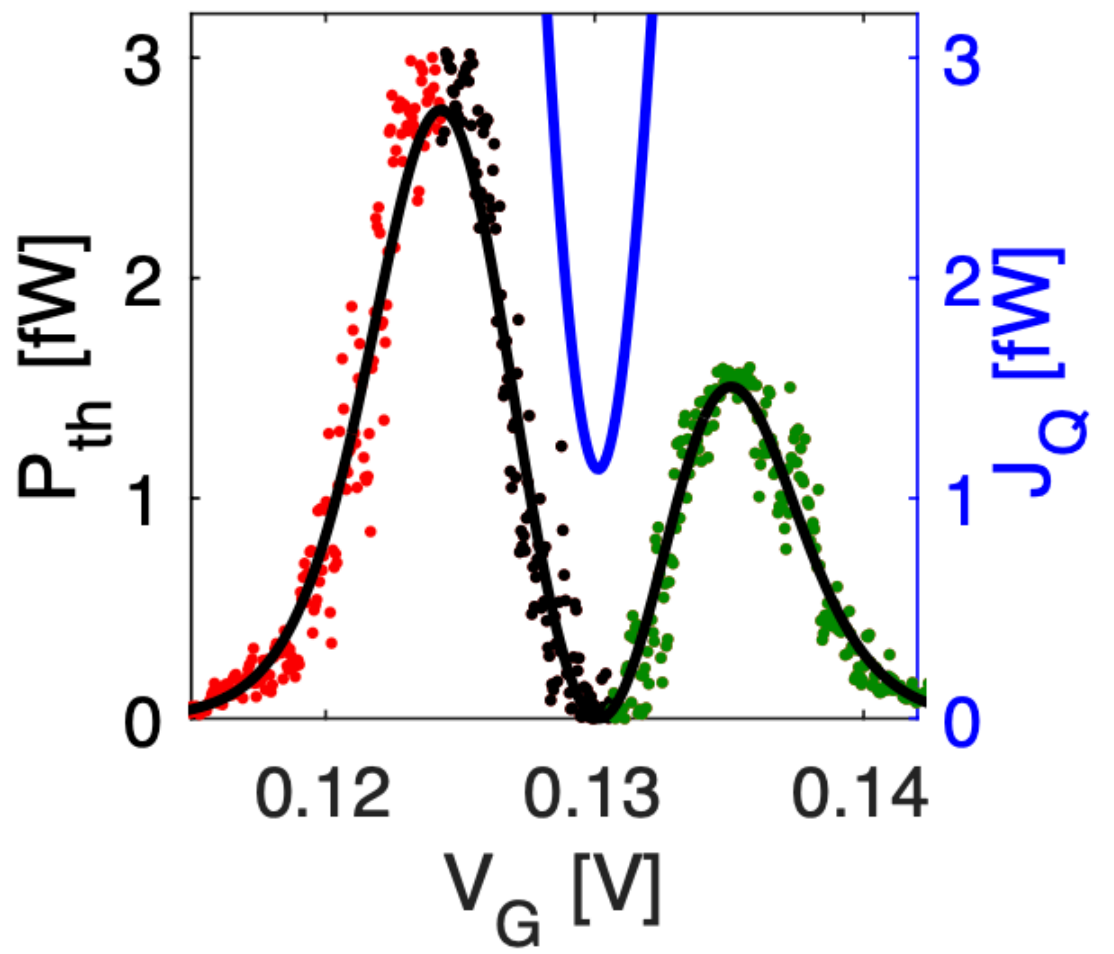
Thermoelectric effect in nanoscale conductors





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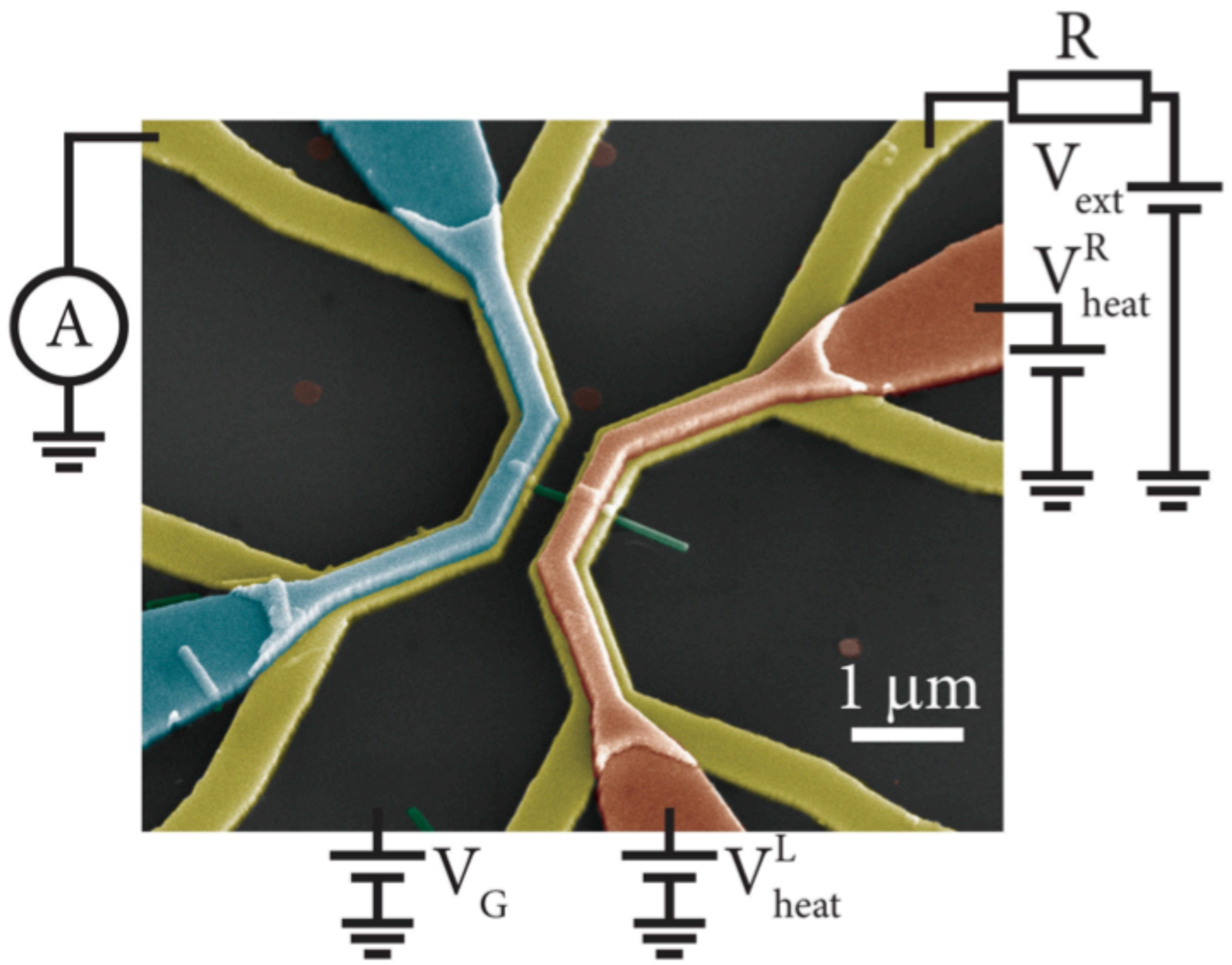

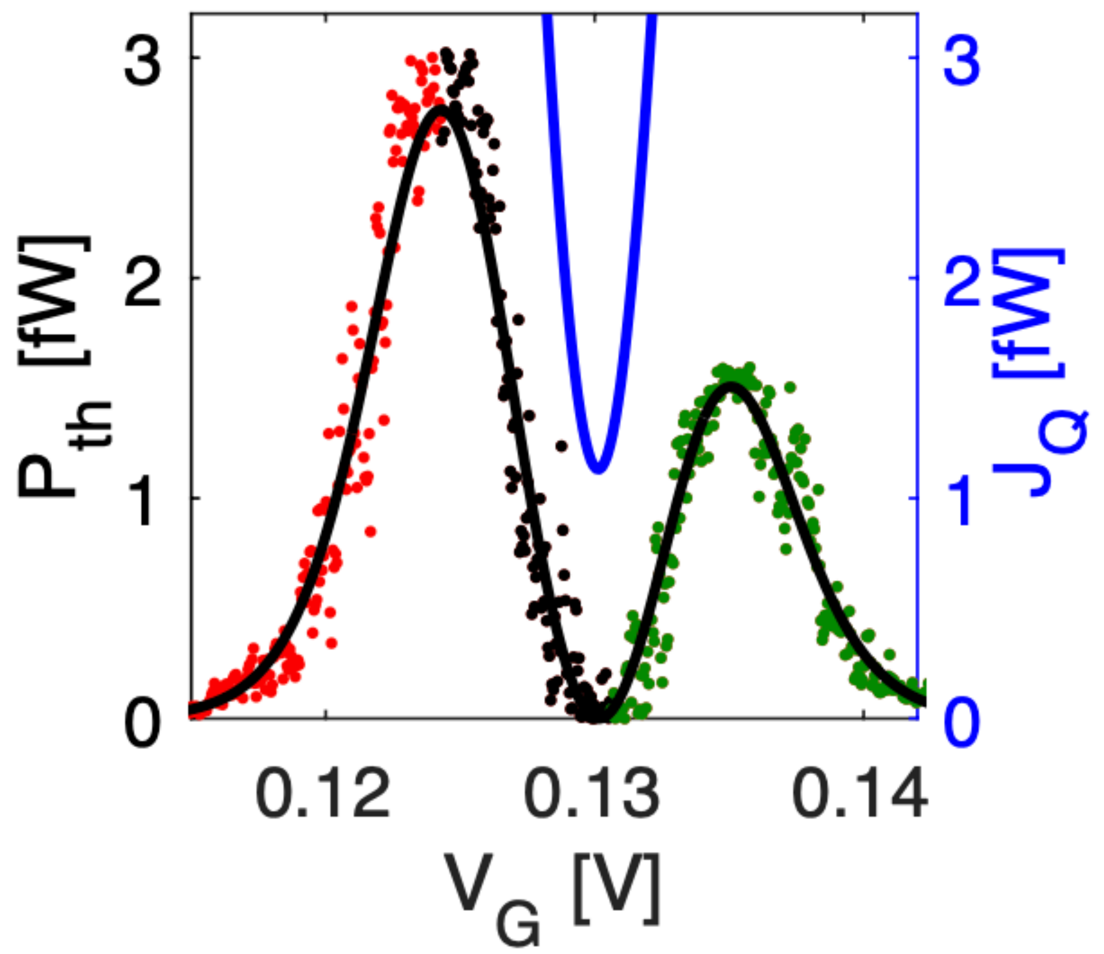

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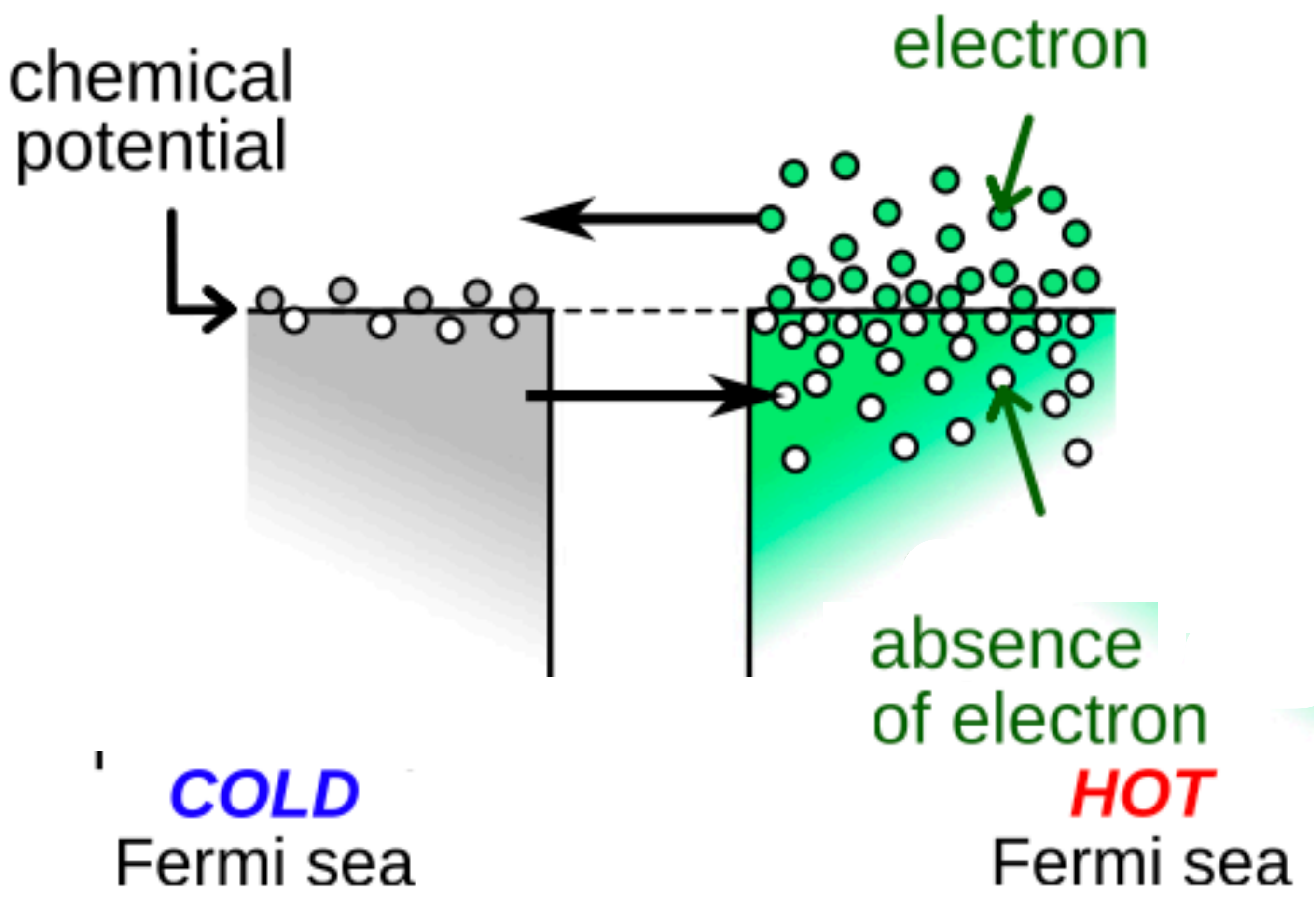
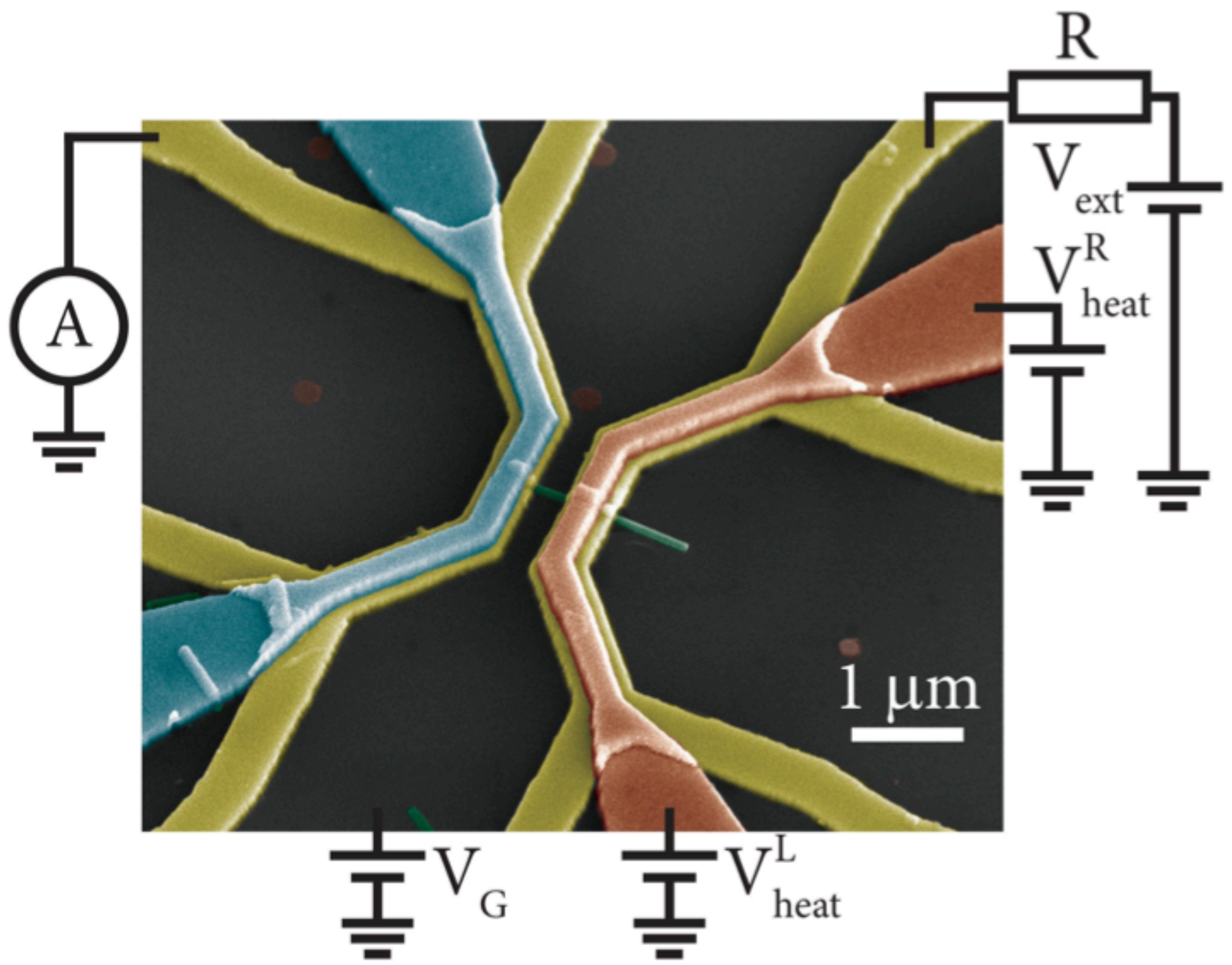
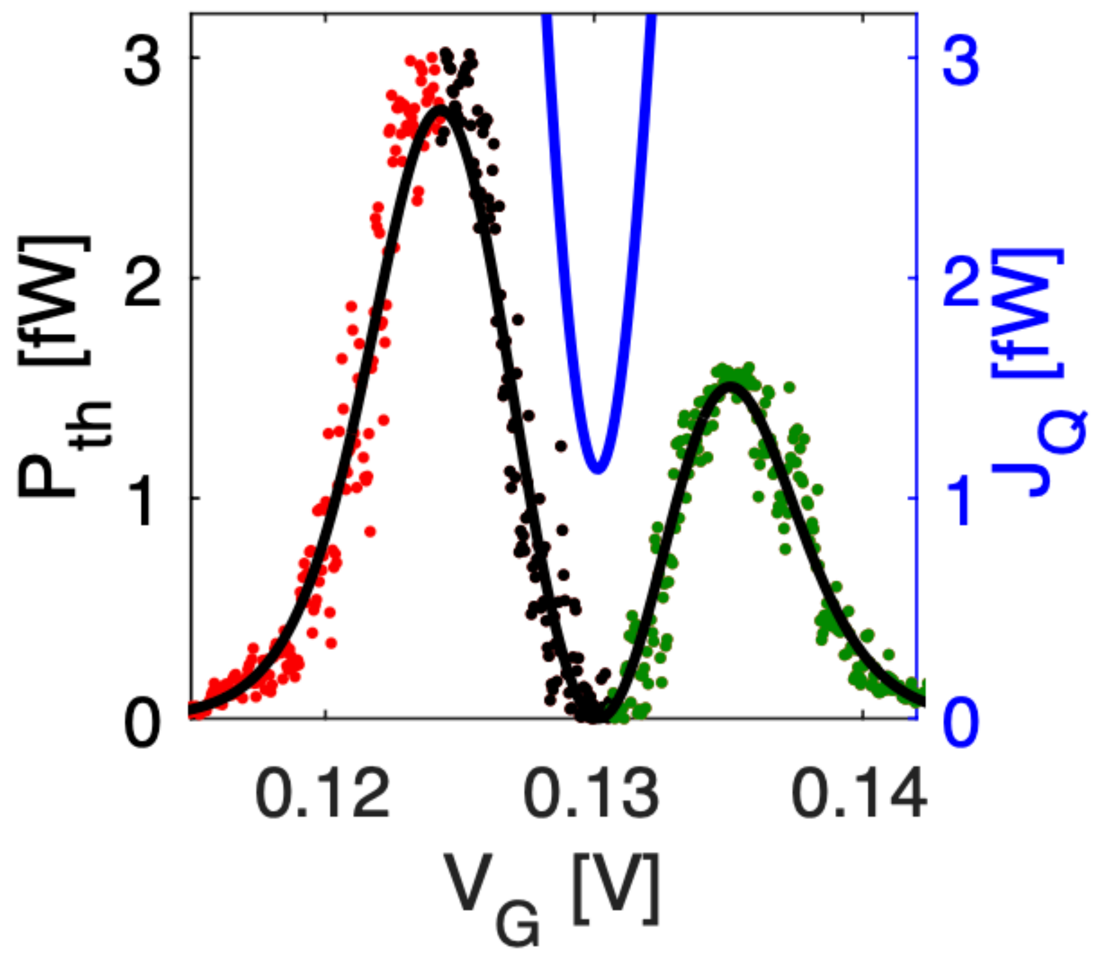


(a) Direct contact – no energy filter.





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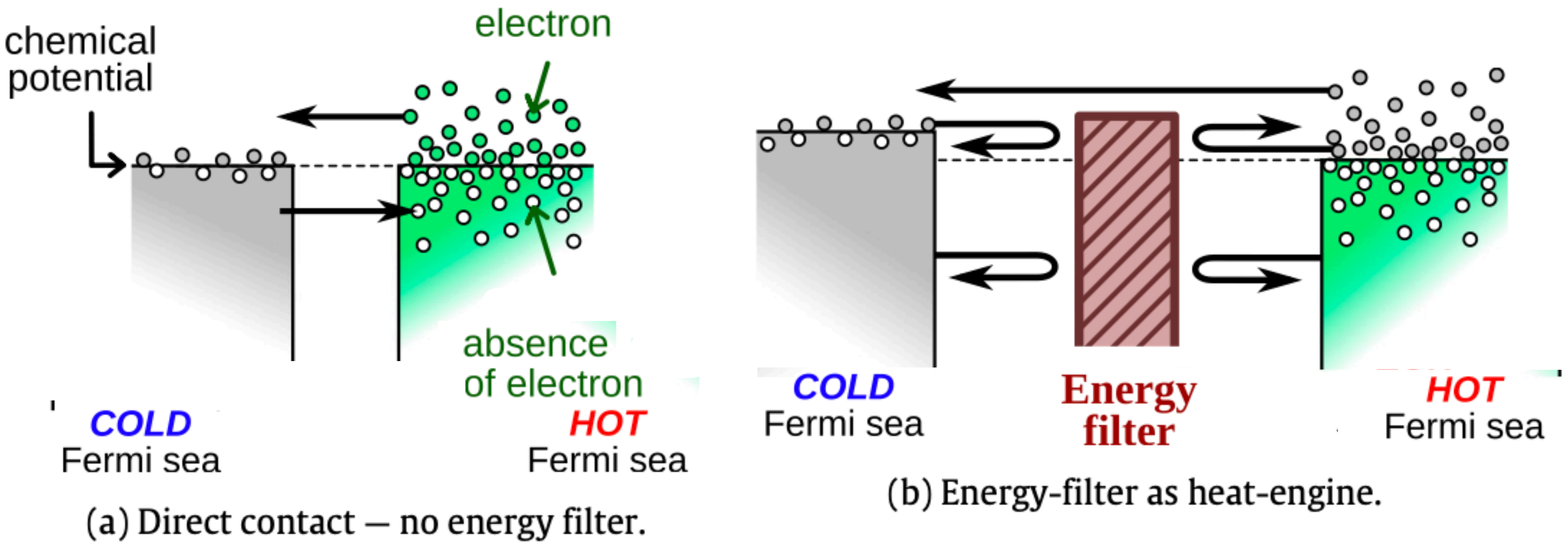

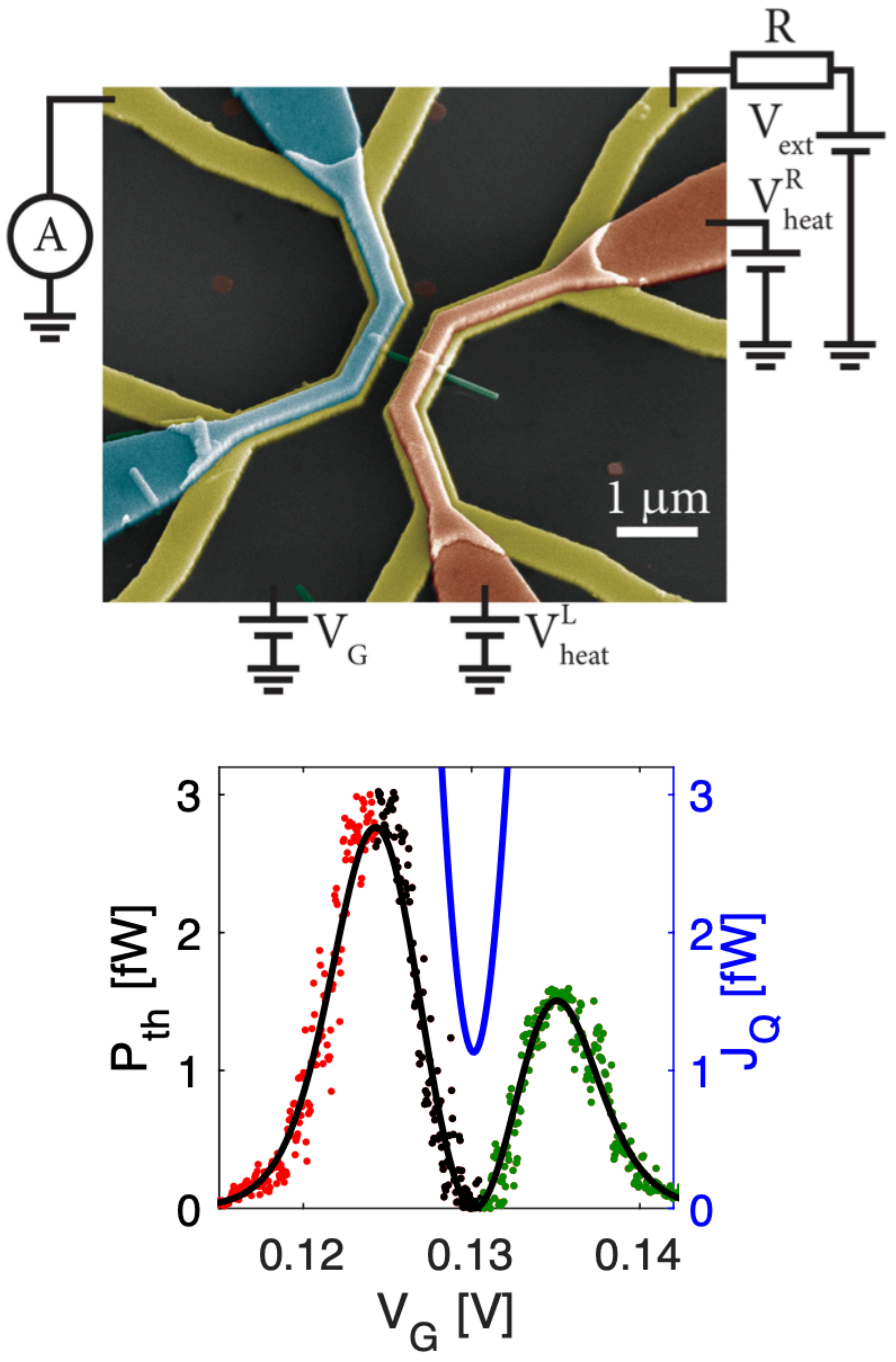

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New opportunities

Refined control over power production at small scales

New challenges

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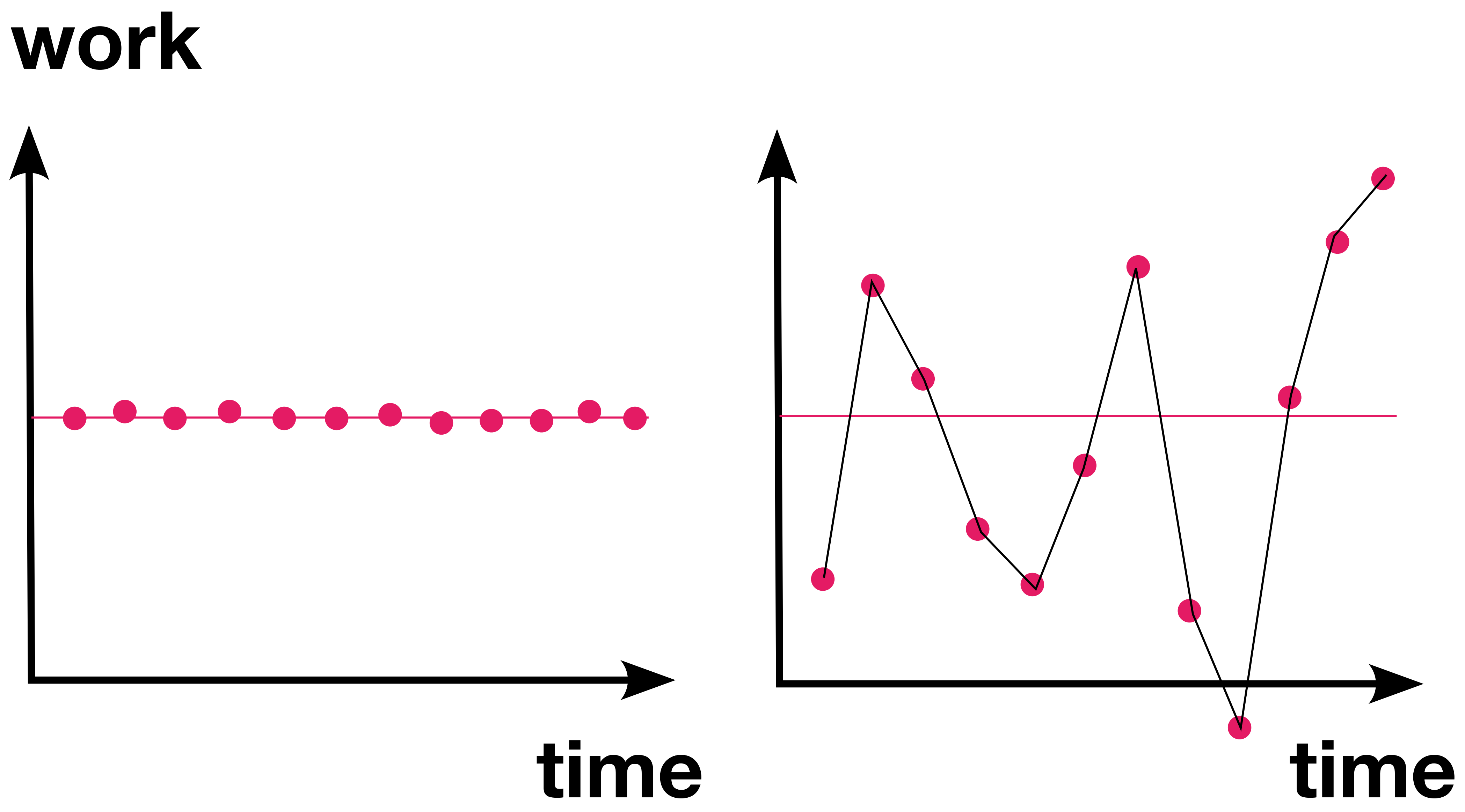
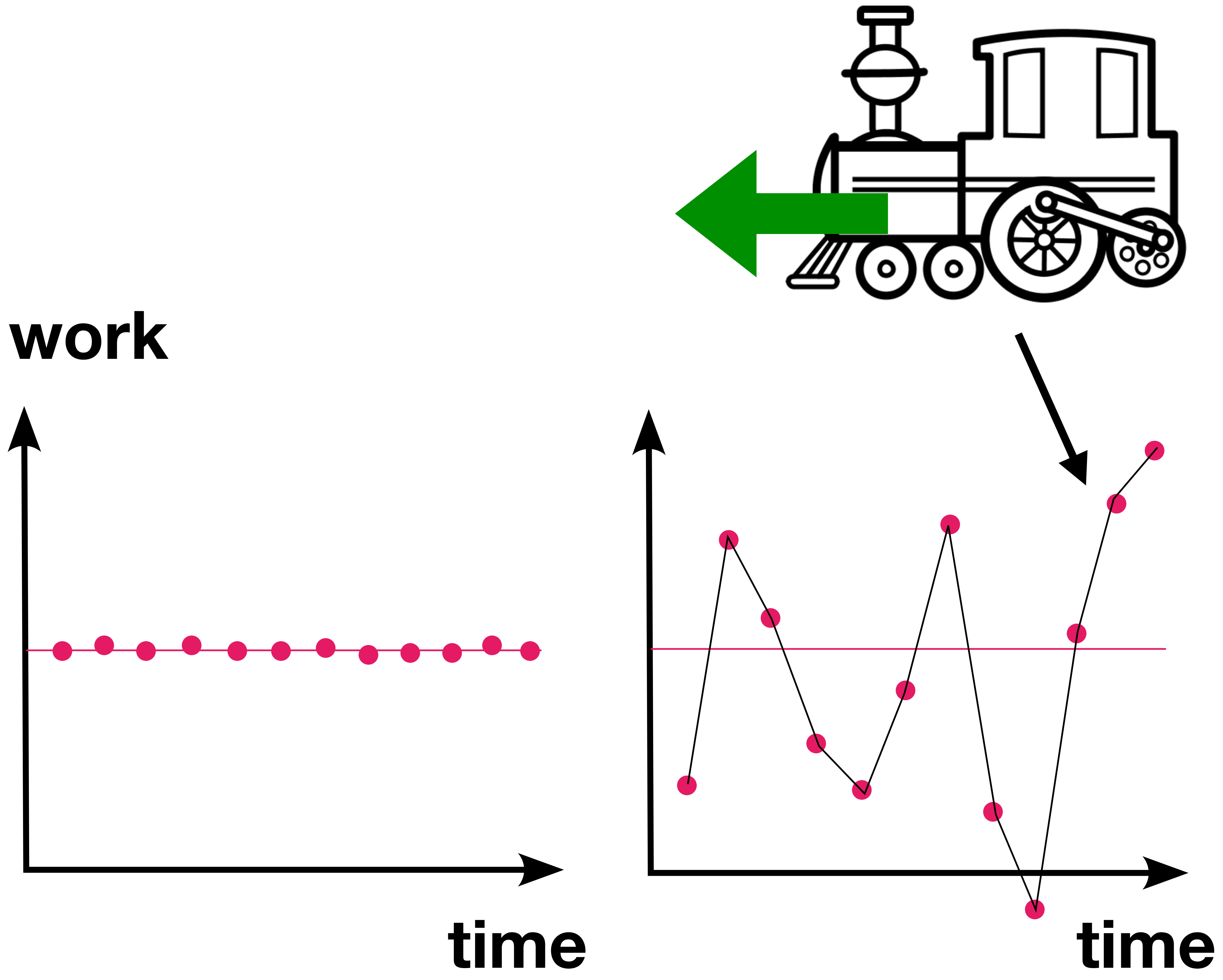
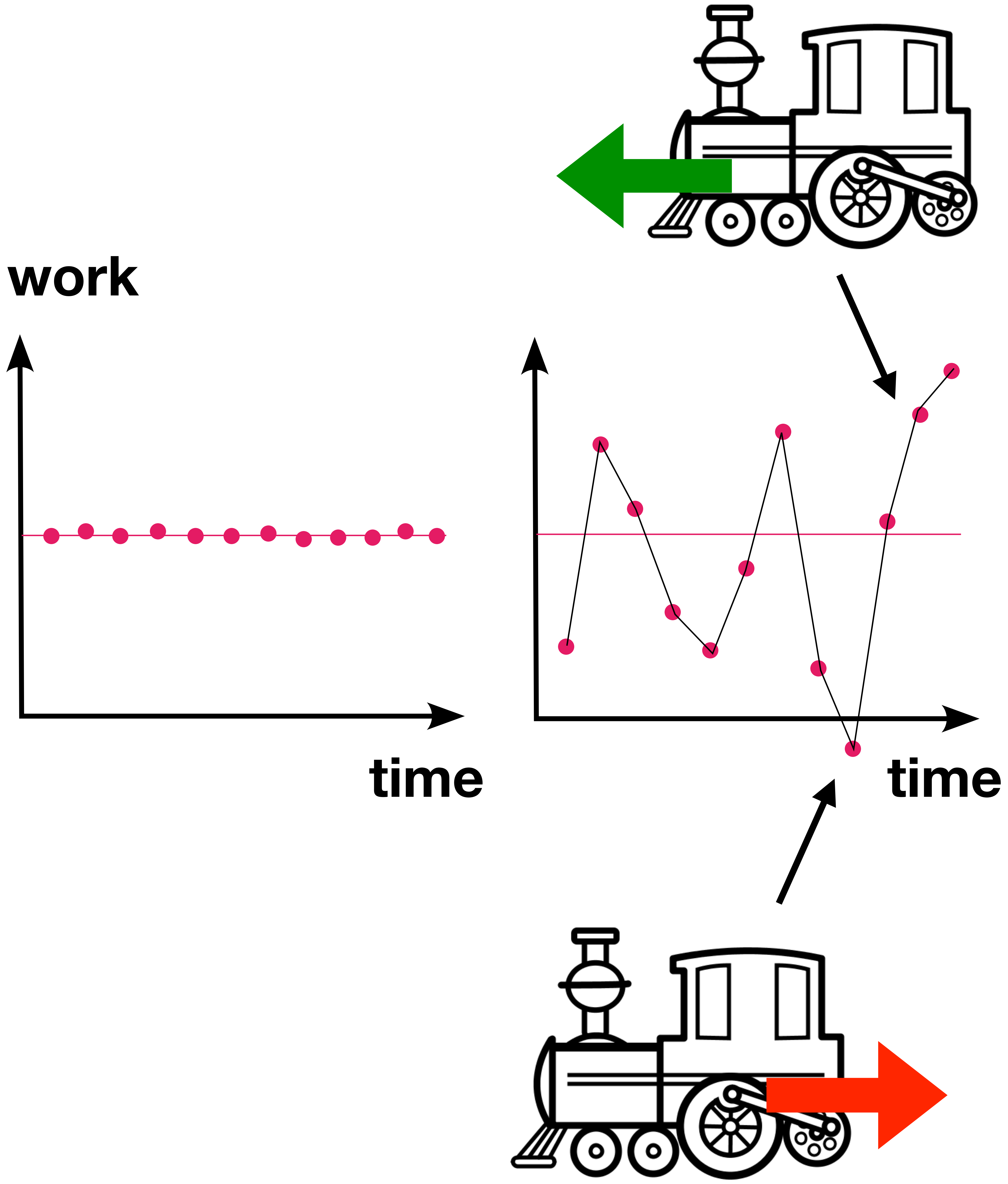
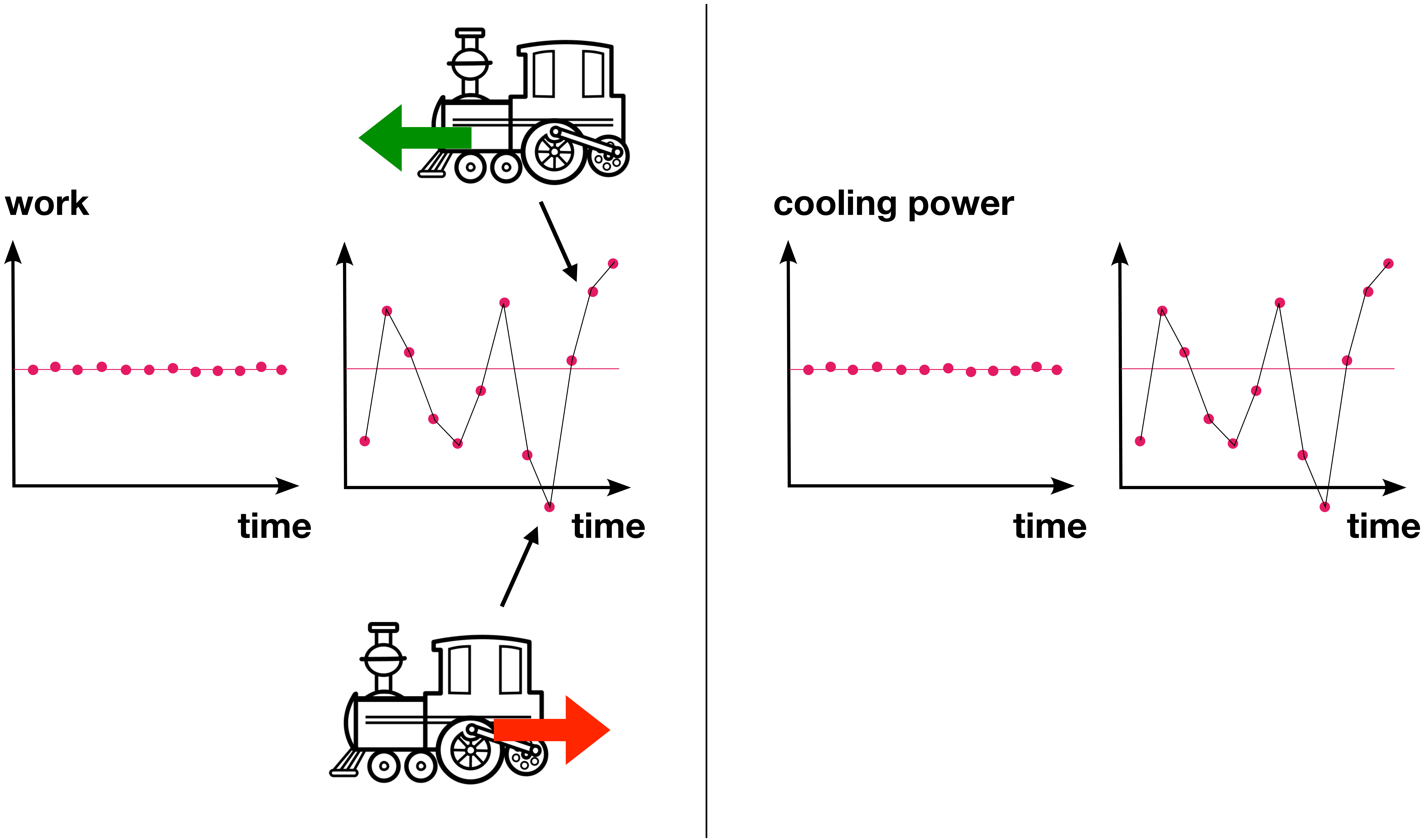
???

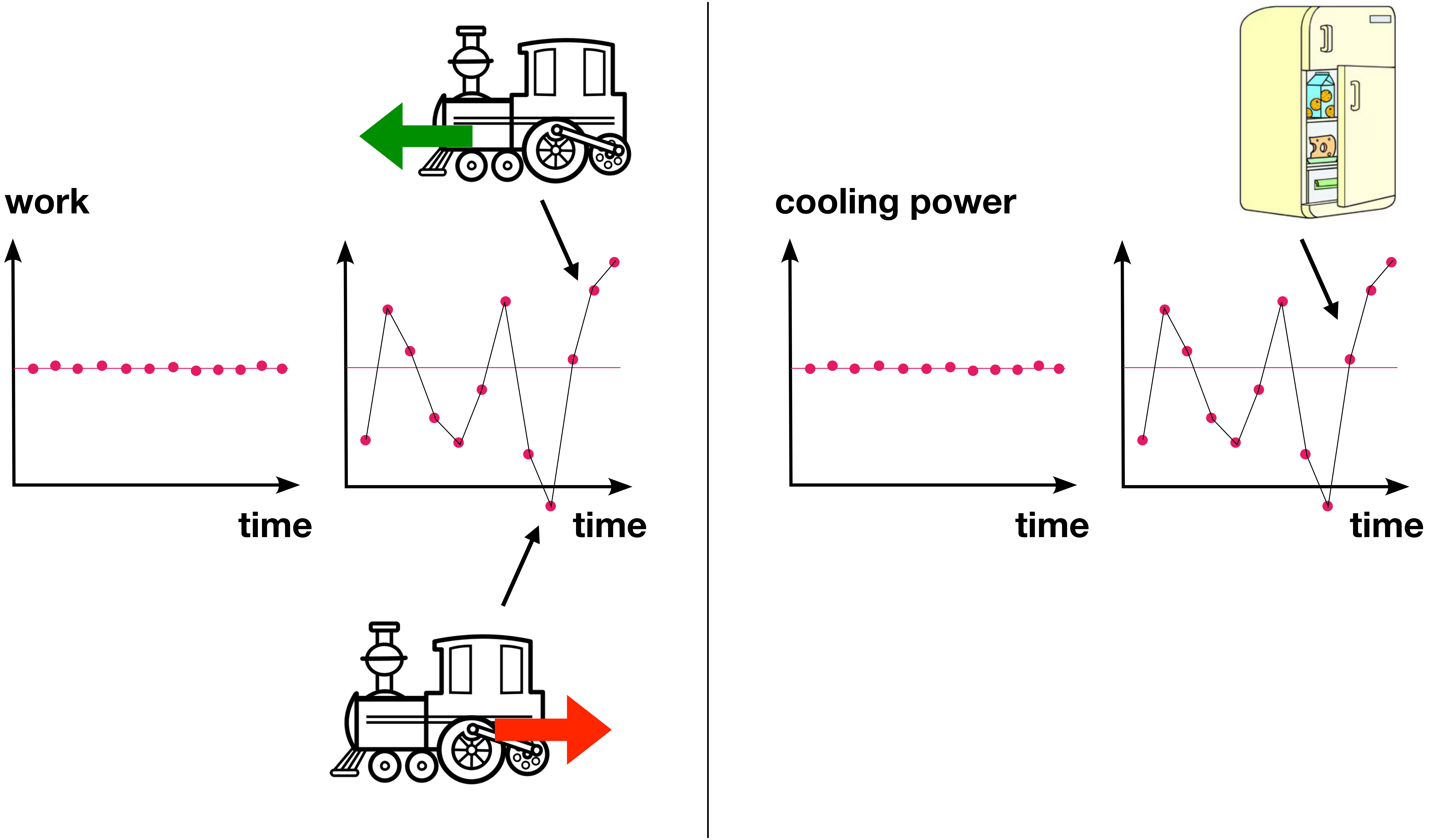
What are the constraints on fluctuations in these nonequilibrium devices?


work
time
time

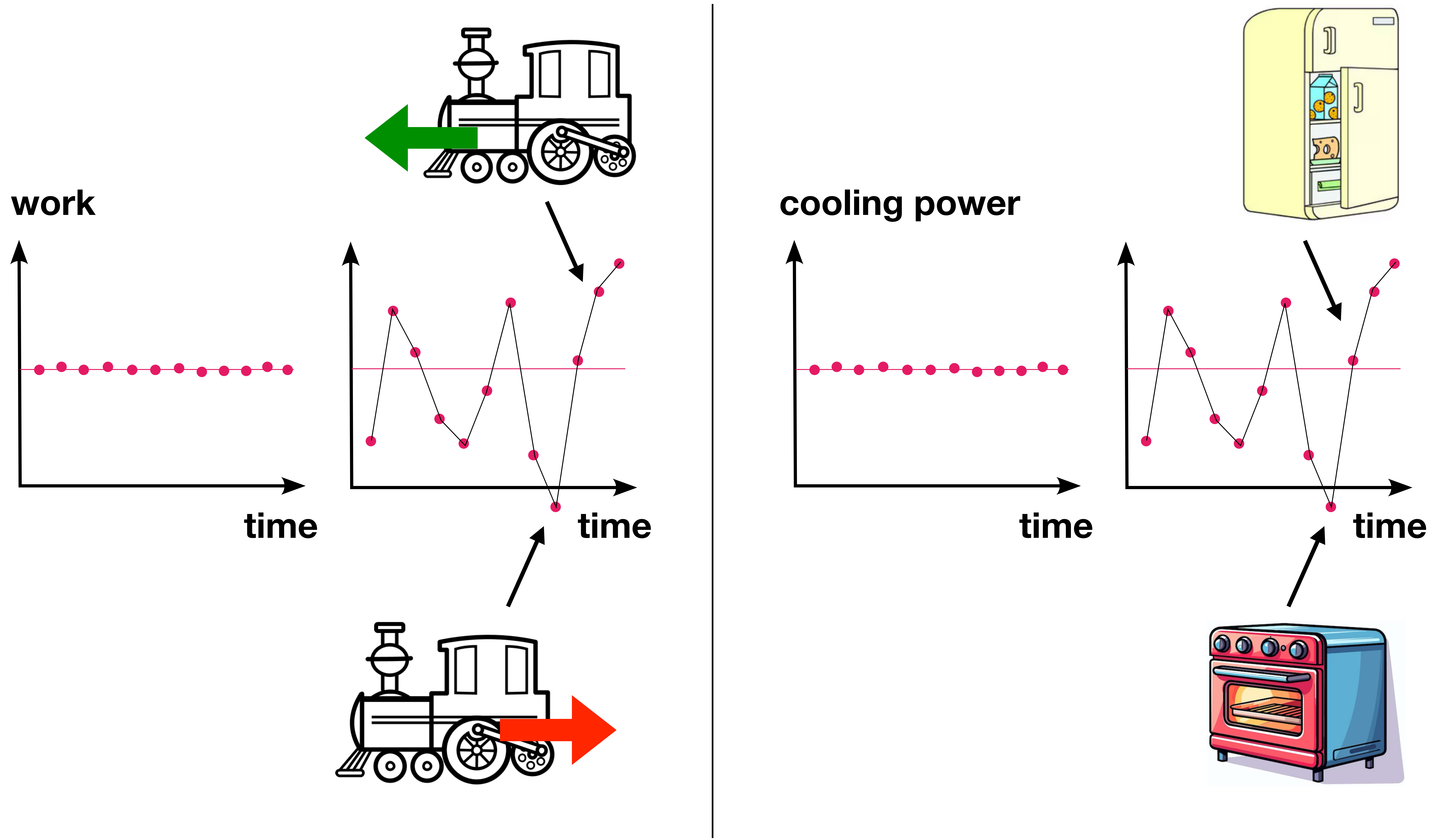

work
time
time


work
time
time


work
time
time
cooling power
time
time



work
time
time
cooling power
time
time

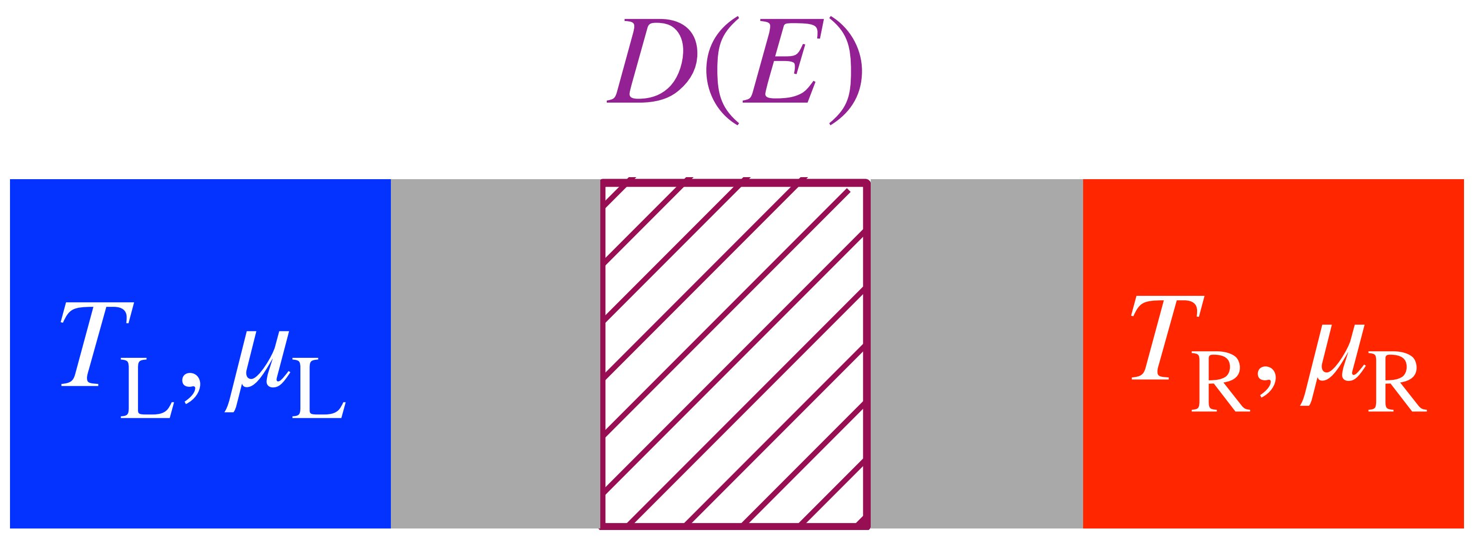

work
time
time
cooling power
time
time

Fluctuations in mesoscopic conductors…

…operating as engines

Charge currents in (thermoelectric) conductors

G. Benenti, G. Casati, K. Saito, R. S. Whitney: Phys. Rep. **694**, 1 (2017).



$$I = \langle \hat{I} \rangle = \frac{-e}{h} \int dE \; D(E) \; [f_R(E) - f_L(E)]$$

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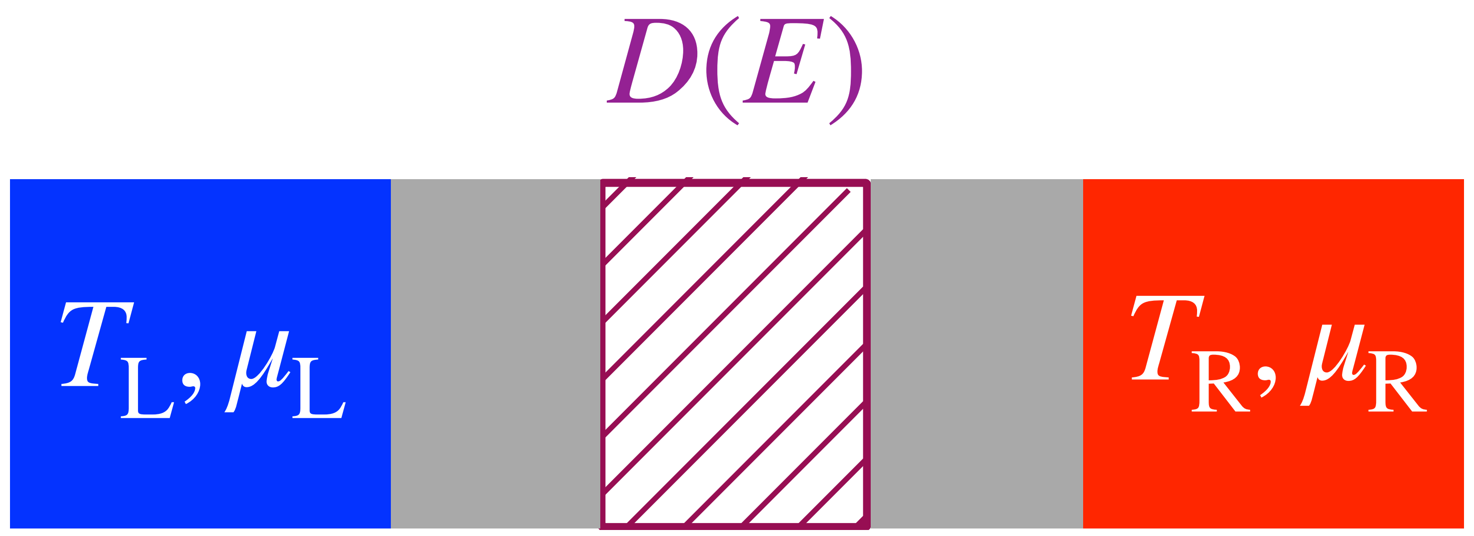

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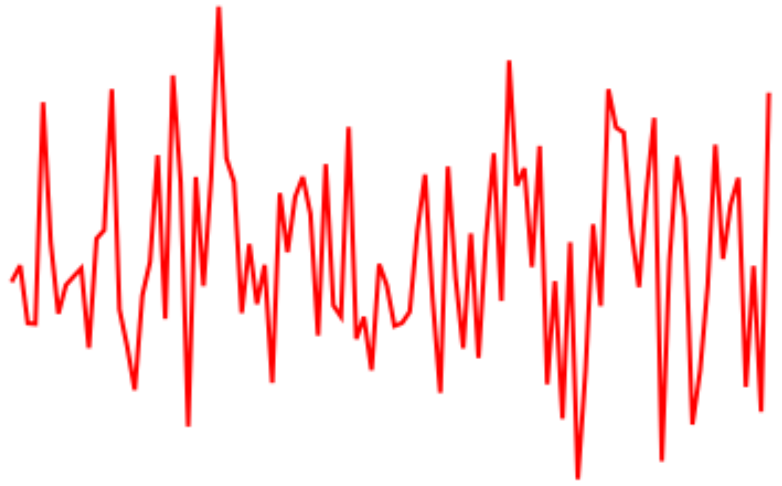
Fluctuations: How noisy are these currents?



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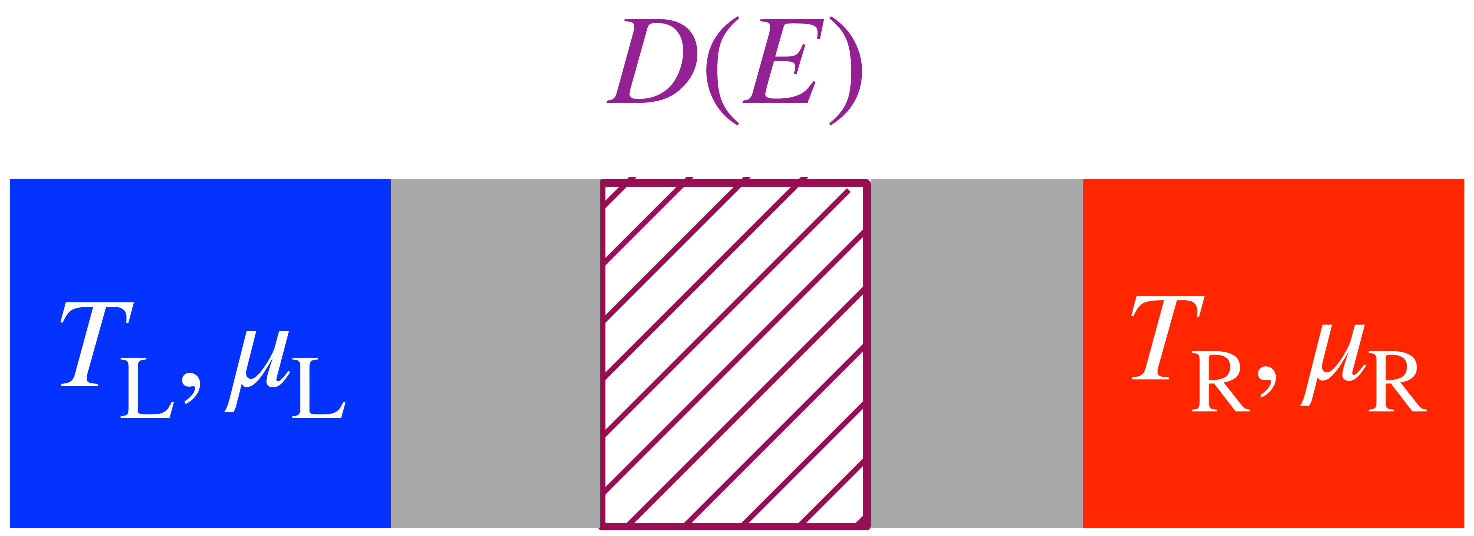

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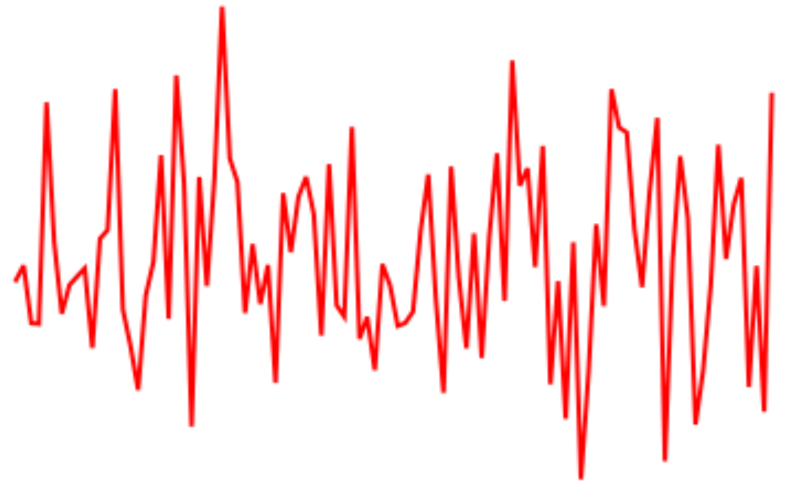

Fluctuation-dissipation theorem

$$S^I = -2ek_B T \left. \frac{\partial I}{\partial \Delta\mu} \right|_{\Delta\mu \equiv 0}$$

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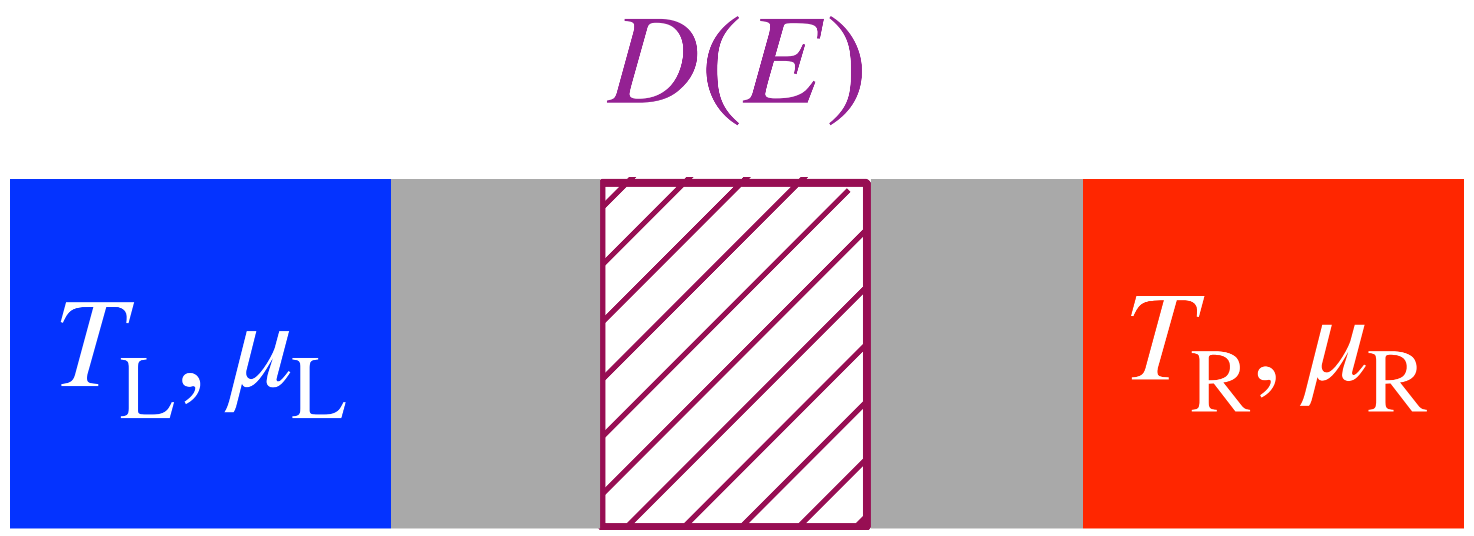

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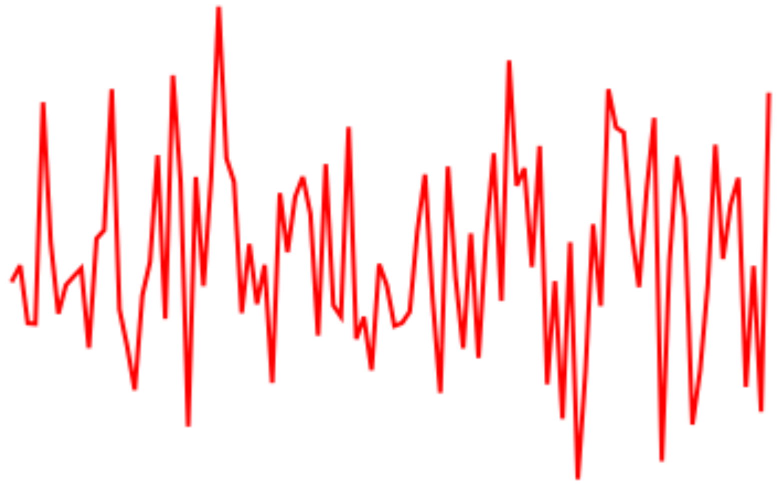

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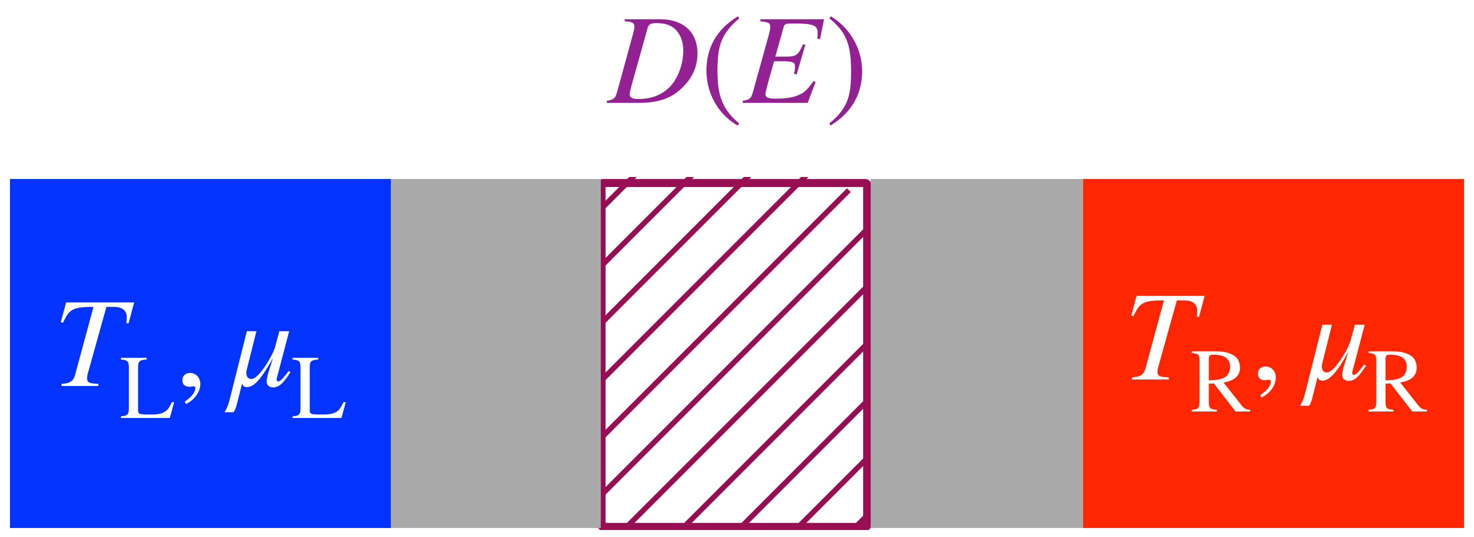
Extension to nonequilibrium?

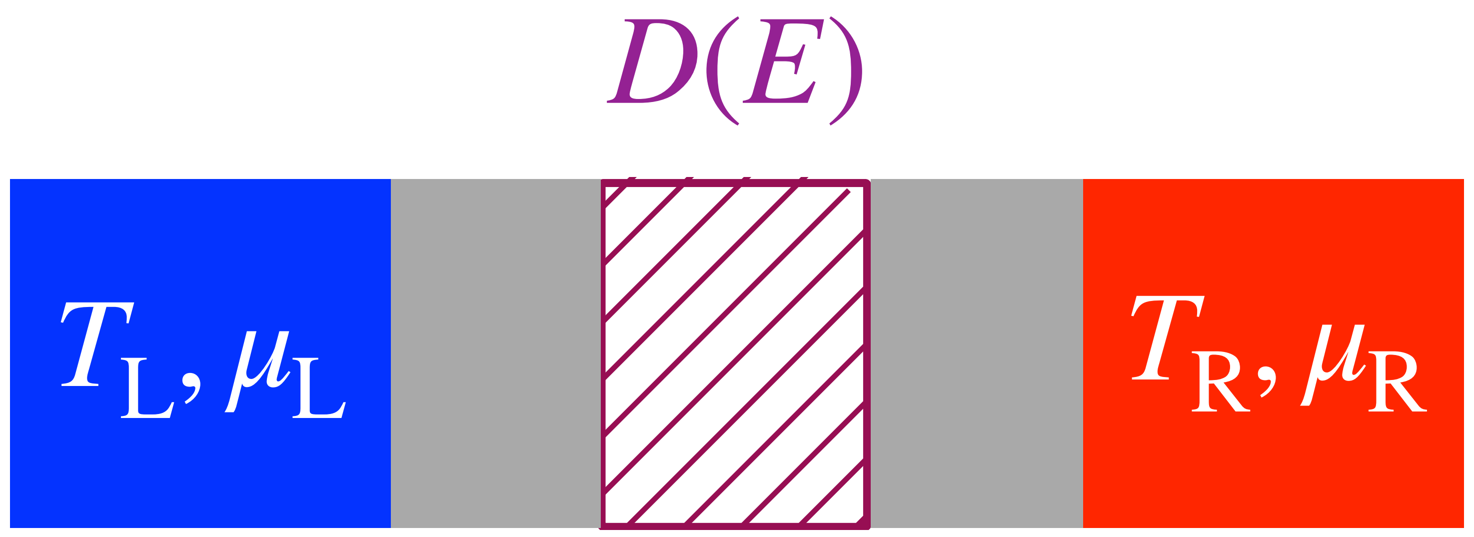
M. Esposito, U. Harbola, S. Mukamel: Rev. Mod. Phys. **81**, 1665 (2009).

Fluctuation-dissipation theorem out of equilibrium



- Temperature difference is crucial
- Goal: produce power (requires good, energy-dependent transmission)

Fluctuation-dissipation theorem out of equilibrium



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Nonequlibrium FDT

Extension to potential bias

$D \ll 1,\ \Delta T = 0$

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L. S. Levitov, M. Reznikov: Phys. Rev. B **70**, 115305 (2004).
I. Safi: Phys. Rev. B **102**, 041113 (2020).
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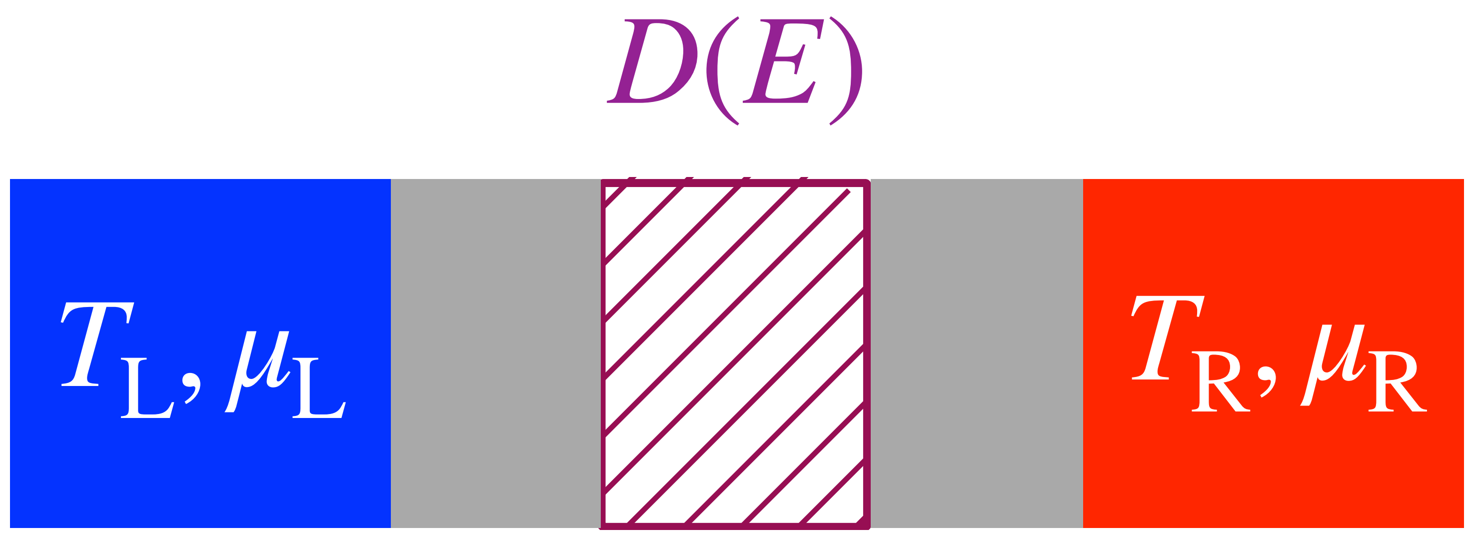

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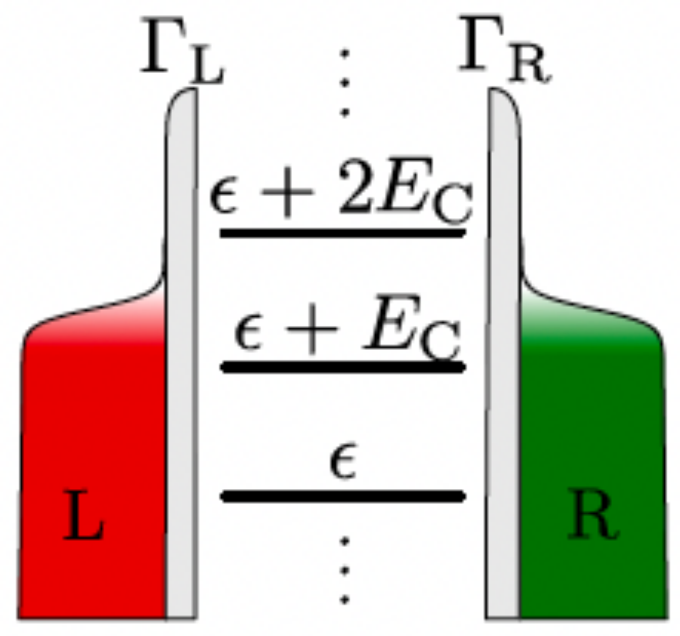
M. Esposito, U. Harbola, S. Mukamel: Rev. Mod. Phys. **81**, 1665 (2009).

With $\Gamma_\mathrm{L}, \Gamma_\mathrm{R}, \epsilon, E_\mathrm{C}$

slowly time-dependent

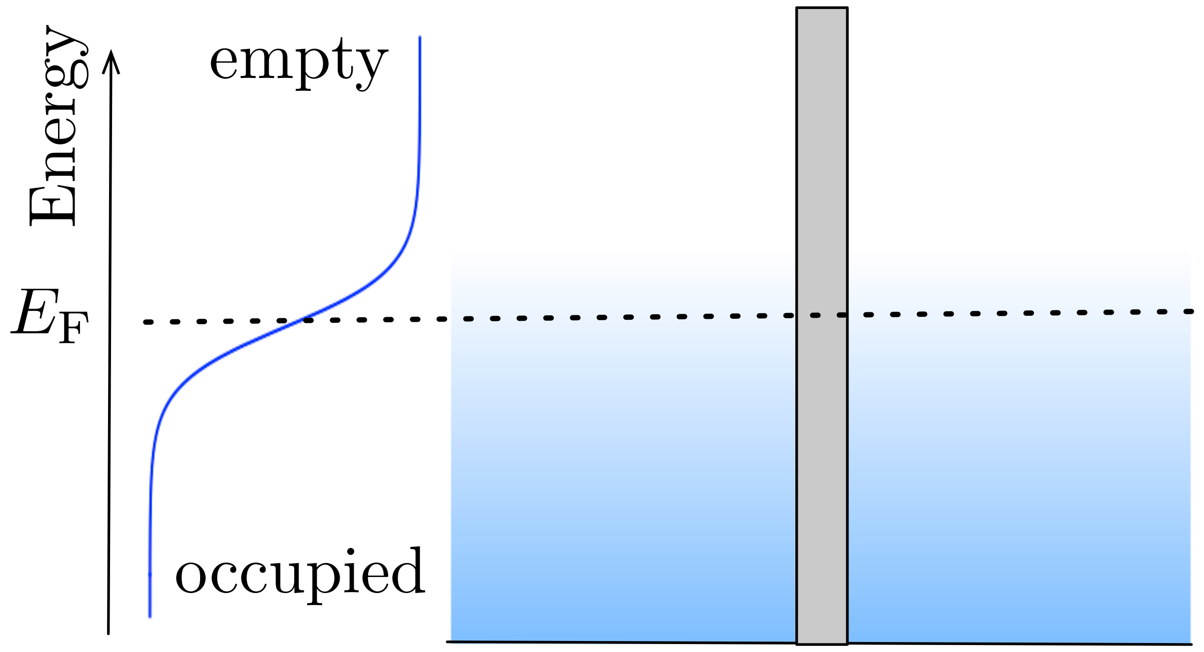


$$S_{\alpha\gamma}\Big|_{\{\mu_\alpha\}\to\mu} = k_\mathrm{B}T \frac{\partial^2 P}{\partial\mu_\alpha \partial\mu_\gamma}\Big|_{\{\mu_\alpha\}\to\mu}$$

R.-P. Riwar, J. Splettstoesser: New J. Phys. **23**, 013010 (2021)

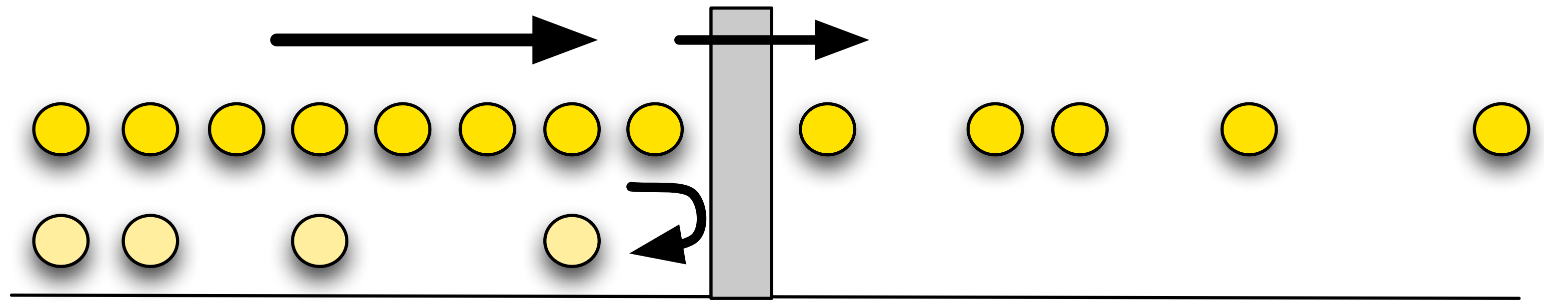
Noise in (thermoelectric) conductors

Thermal noise



$$\Theta^I_\alpha \equiv \frac{e^2}{h}\int_{-\infty}^{\infty} dE\ D(E)\ f_\alpha(E)[1 - f_\alpha(E)]$$

Shot noise



$$S^I_{\mathrm{sh}} \equiv \frac{e^2}{h}\int_{-\infty}^{\infty} dE\ D(E)[1 - D(E)]\ [f_{\mathrm{L}}(E) - f_{\mathrm{R}}(E)]^2$$

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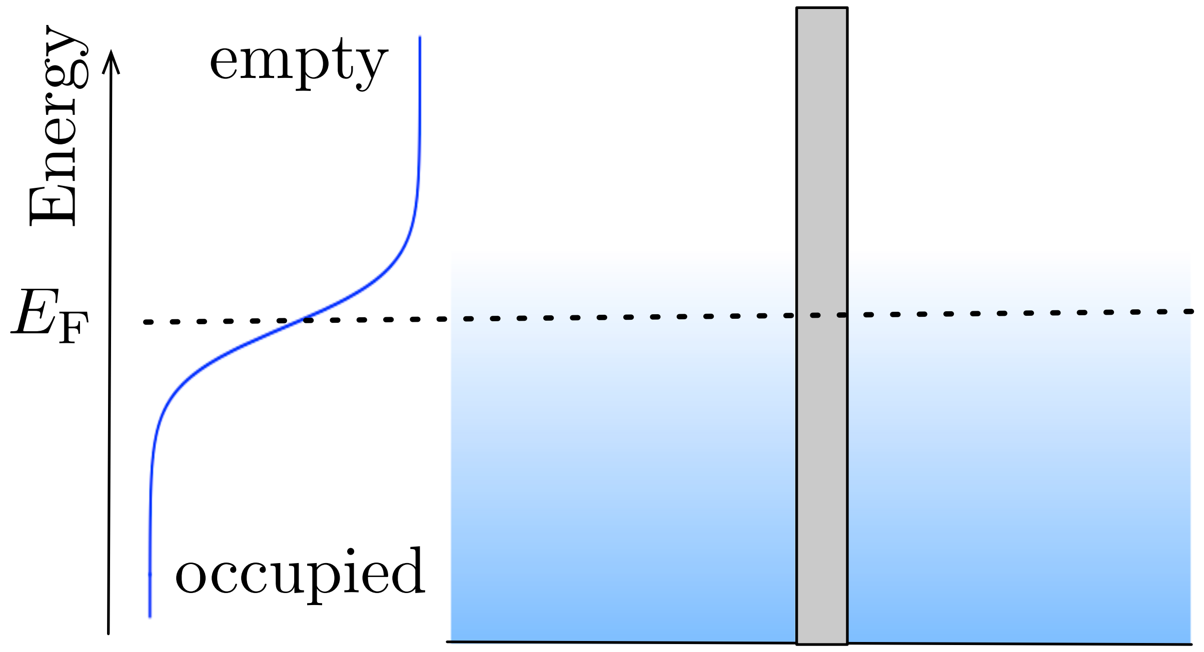

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Persists at equilibrium!

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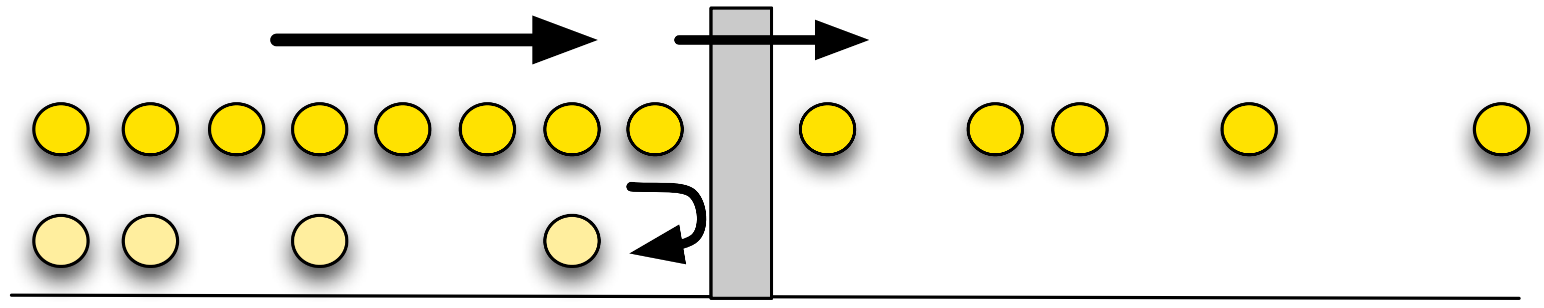

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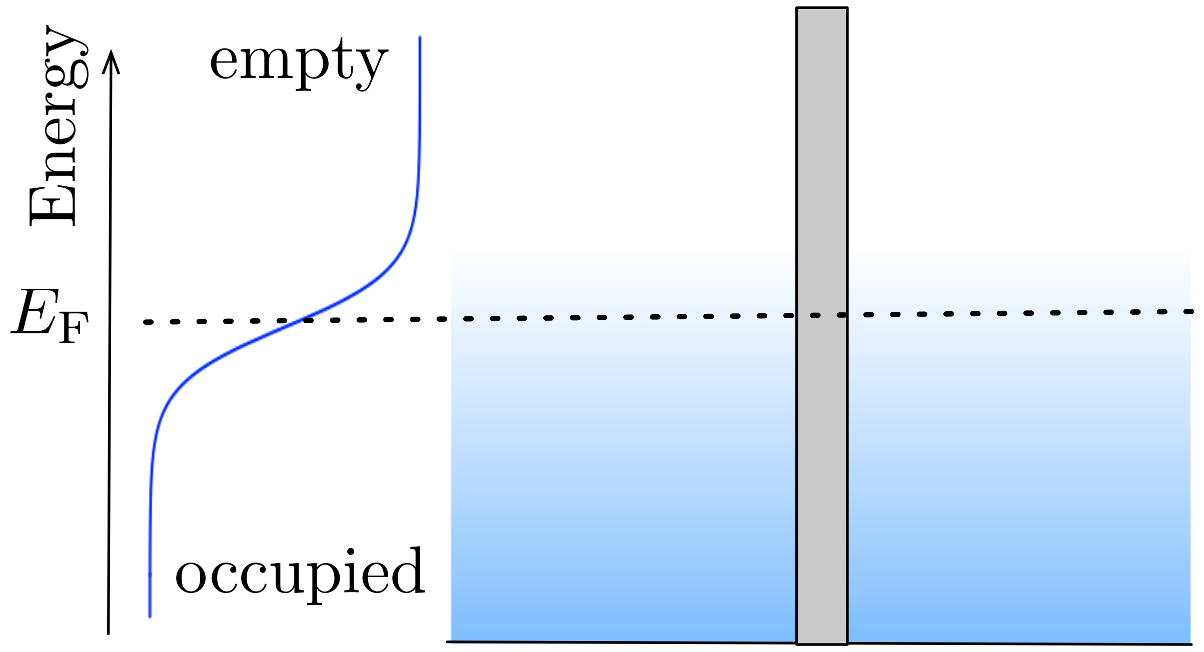

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→ Fulfils fluctuation dissipation theorem!

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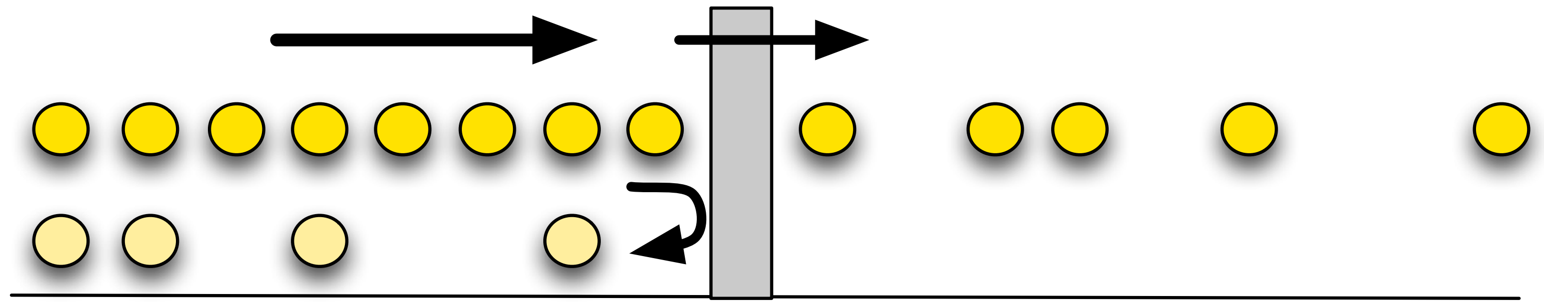

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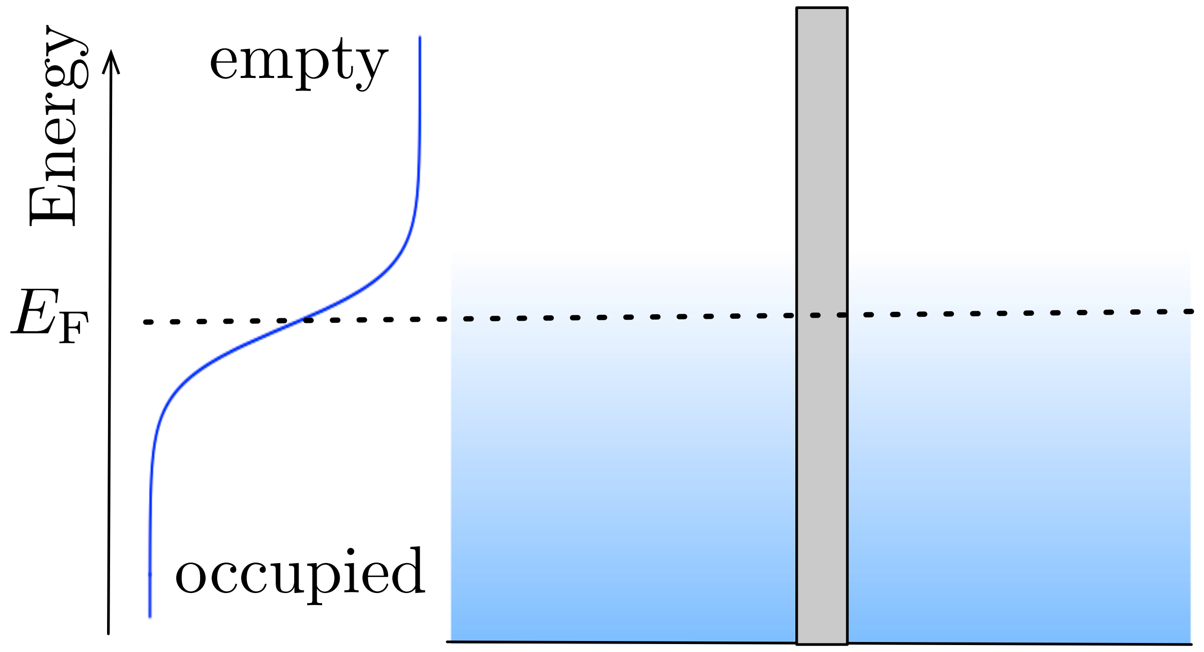

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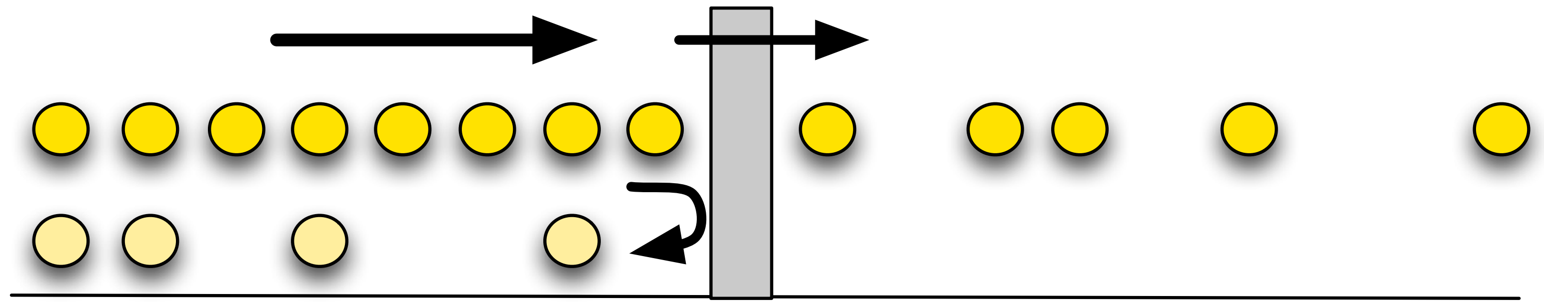

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Nonequilibrium effect!

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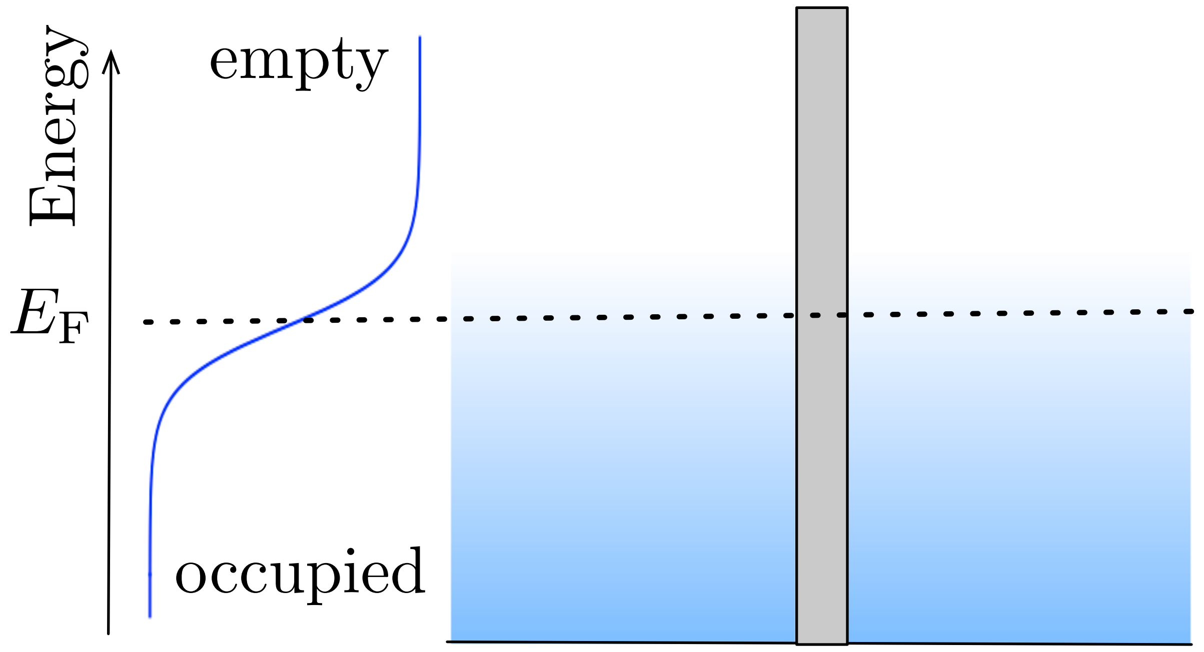

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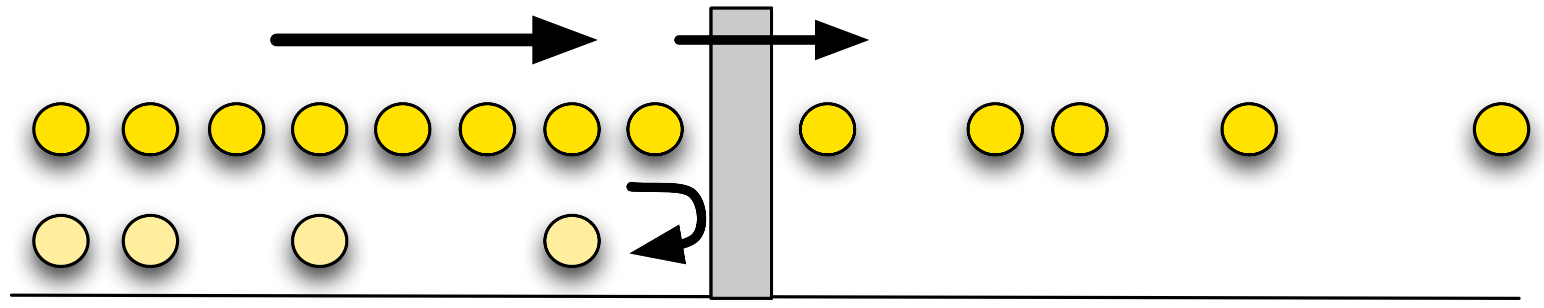

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Nonequilibrium effect!

$D \ll 1$

$\Delta T = 0$

→ Contributes to extension of fluctuation dissipation theorem!

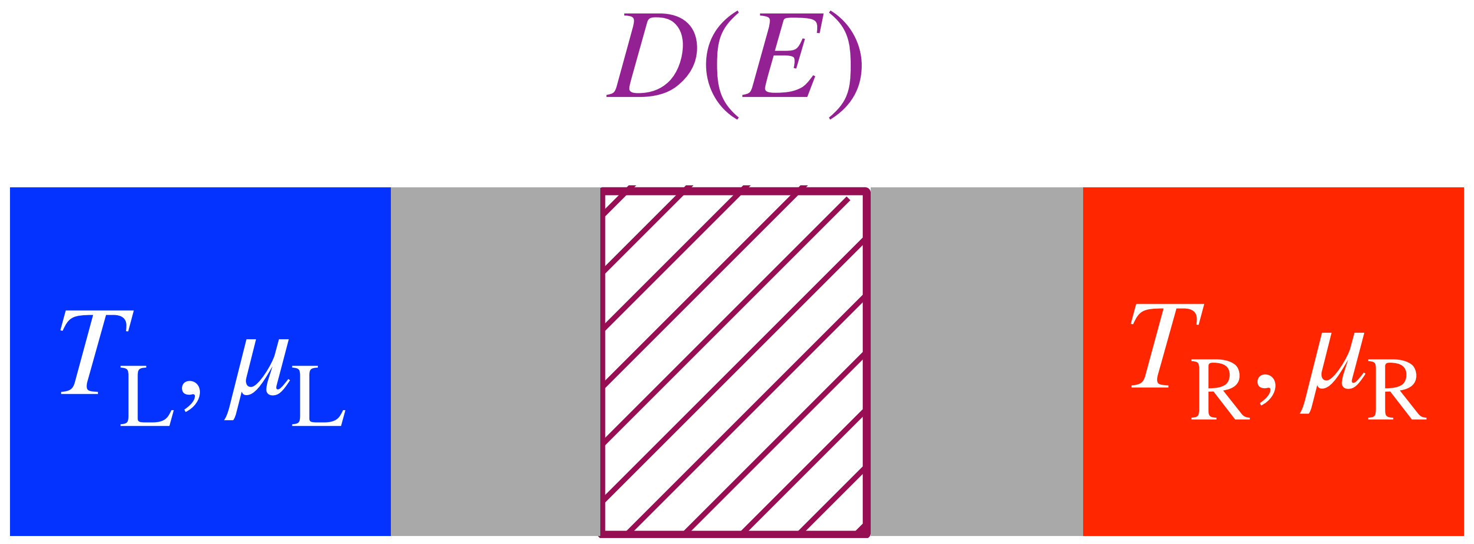
Ya. M. Blanter, M. Büttiker: Phys. Rep. **336**, 1 (2000).

Fluctuation dissipation bounds

Rates for transport

$$\Gamma_{\rightarrow} \equiv \int \frac{dE}{h} D(E) \left[f_{\mathrm{L}}(E) \left[1 - f_{\mathrm{R}}(E) \right] \right]$$

$$\Gamma_{\leftarrow} \equiv \int \frac{dE}{h} D(E) \left[f_{\mathrm{R}}(E) \left[1 - f_{\mathrm{L}}(E) \right] \right]$$



L. S. Levitov, M. Reznikov: Phys. Rev. B **70**, 115305 (2004).

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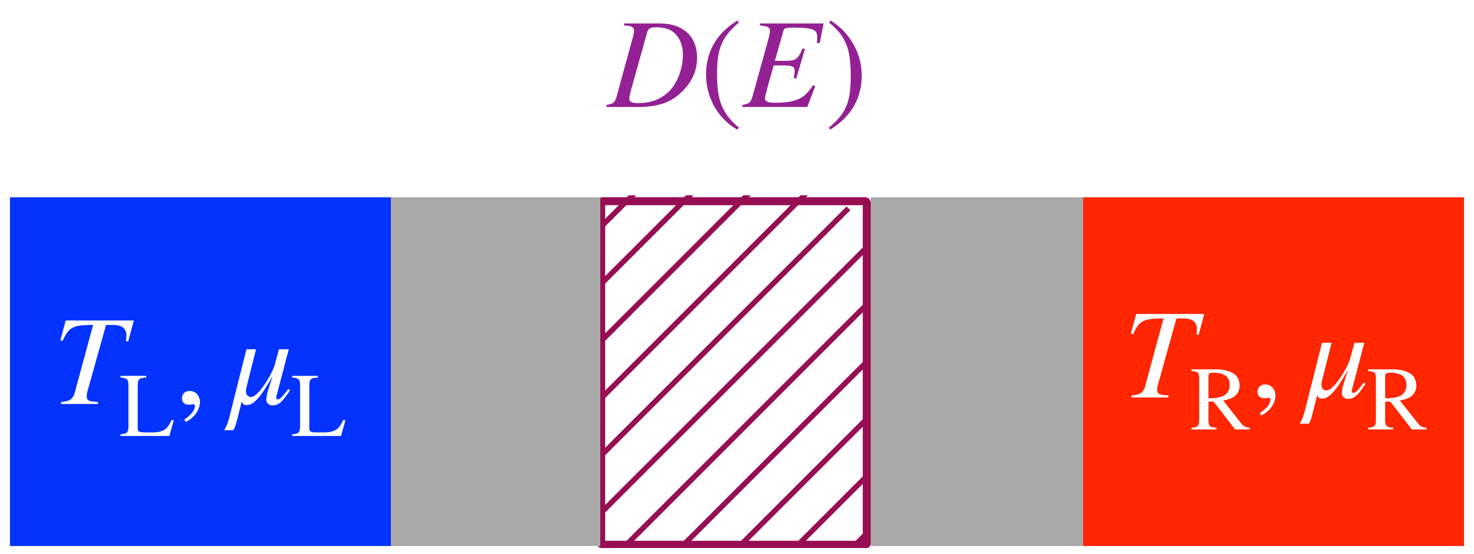

Determine current…

$$I = -e \left(\Gamma_{\rightarrow} - \Gamma_{\leftarrow} \right)$$

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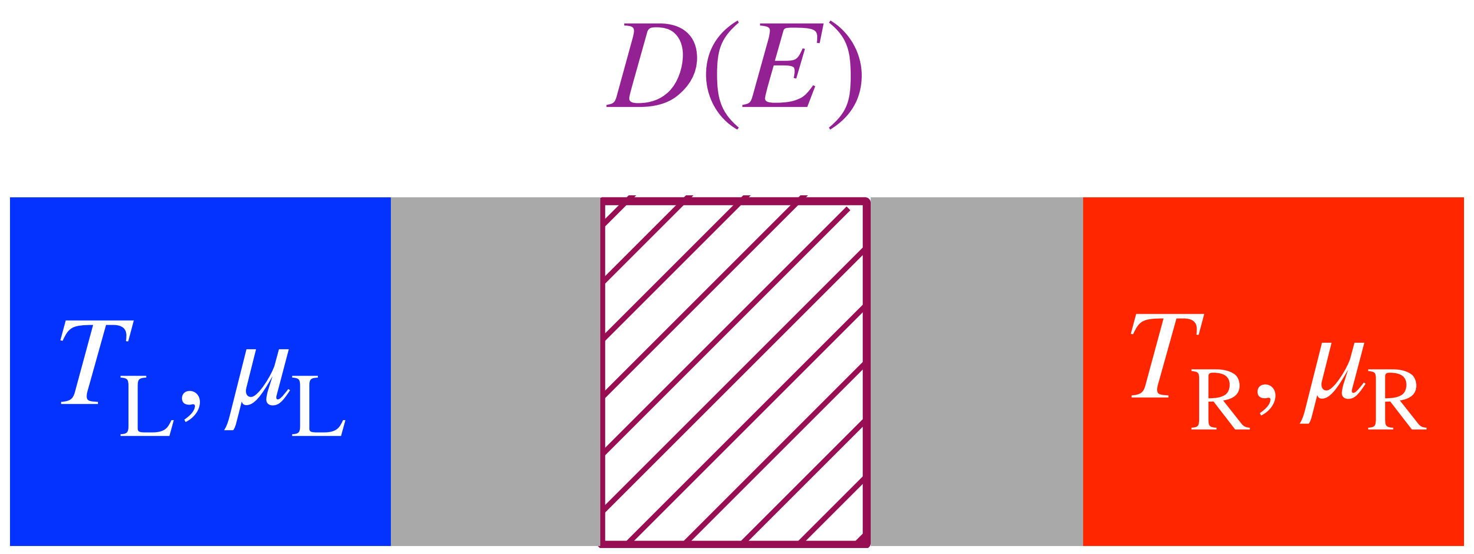
Determine current…

$$I = -e \left(\Gamma_{\rightarrow} - \Gamma_{\leftarrow} \right)$$

…AND fluctuations

$$S^I \leq e^2 \left(\Gamma_{\rightarrow} + \Gamma_{\leftarrow} \right)$$

$$D \ll 1, \qquad S^I \rightarrow e^2 \left(\Gamma_{\rightarrow} + \Gamma_{\leftarrow} \right)$$



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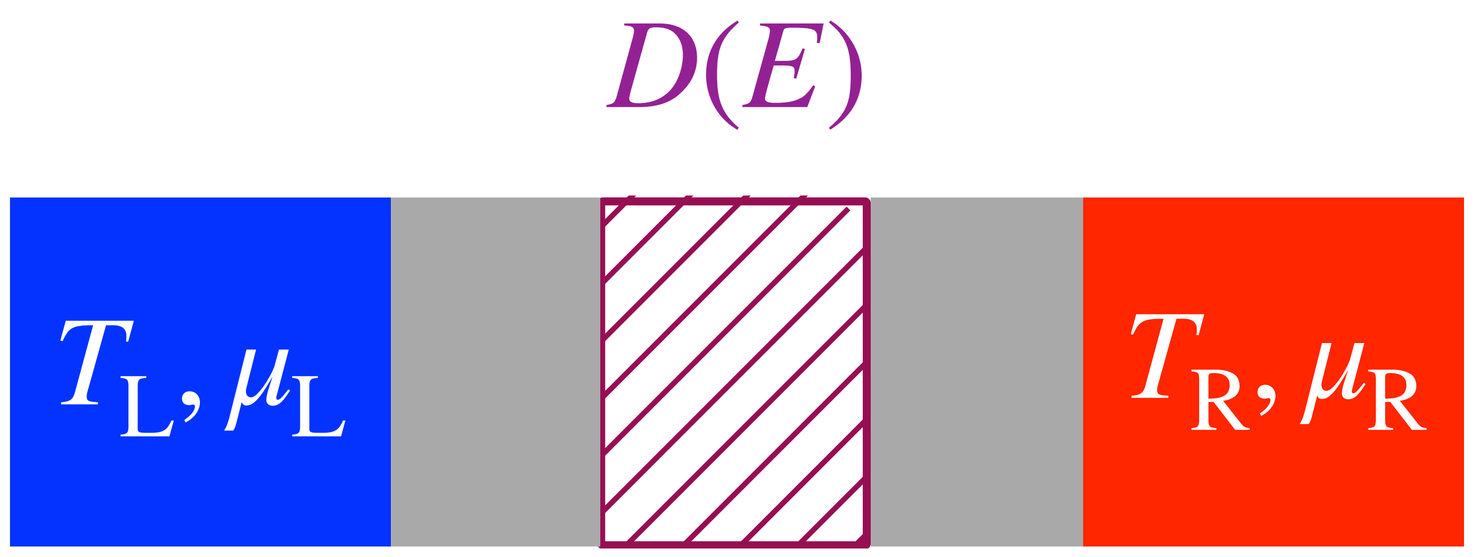

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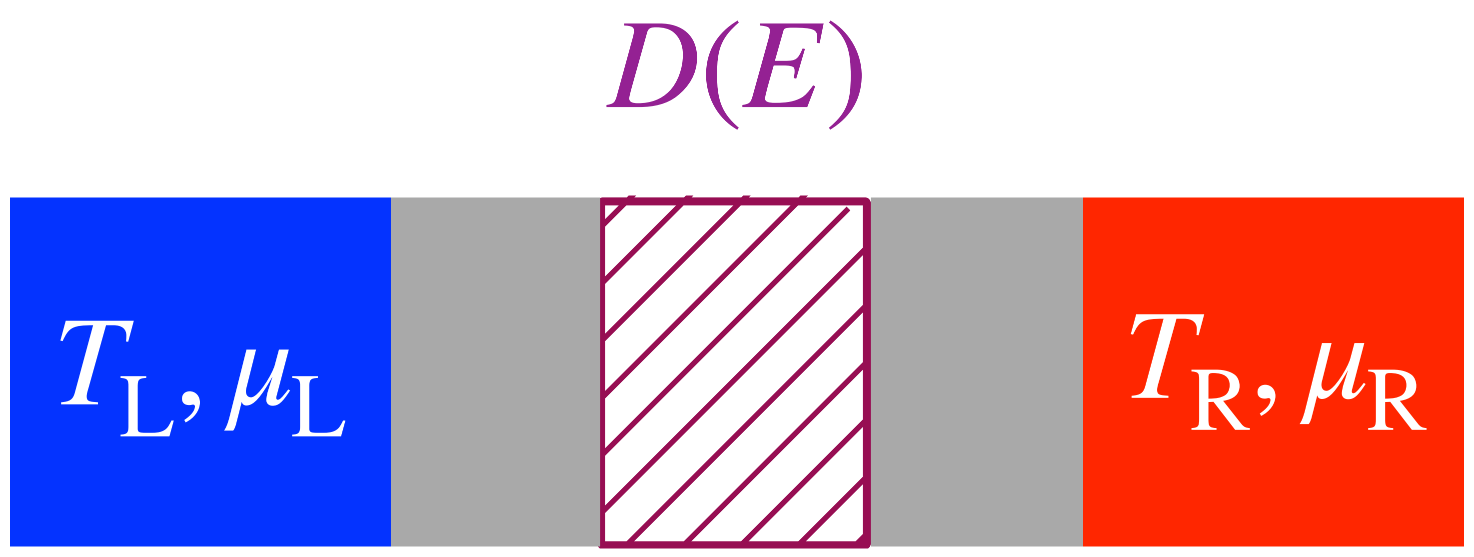
Interested in a general nonequilibrium situation!



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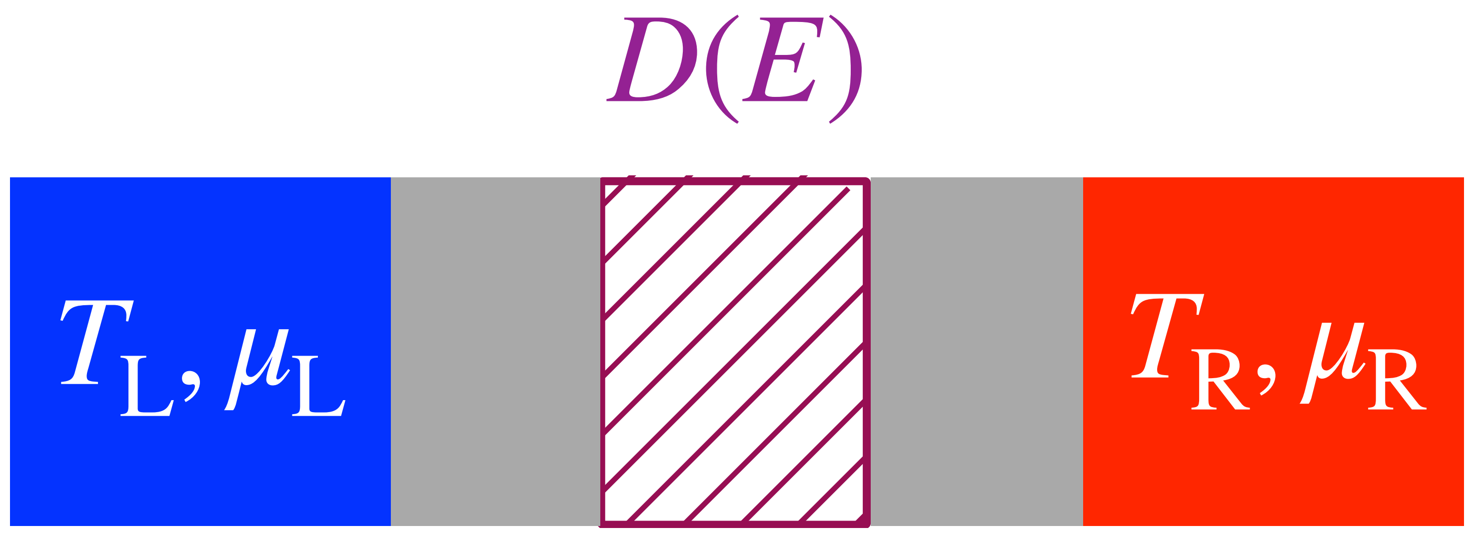
Excess noise

$$S^{I} - 2\Theta_{\mathrm{hot}}$$



L. Tesser, J. Splettstoesser: Phys. Rev. Lett. **132**, 186304 (2024)

Fluctuation dissipation bounds

Excess Rates for transport

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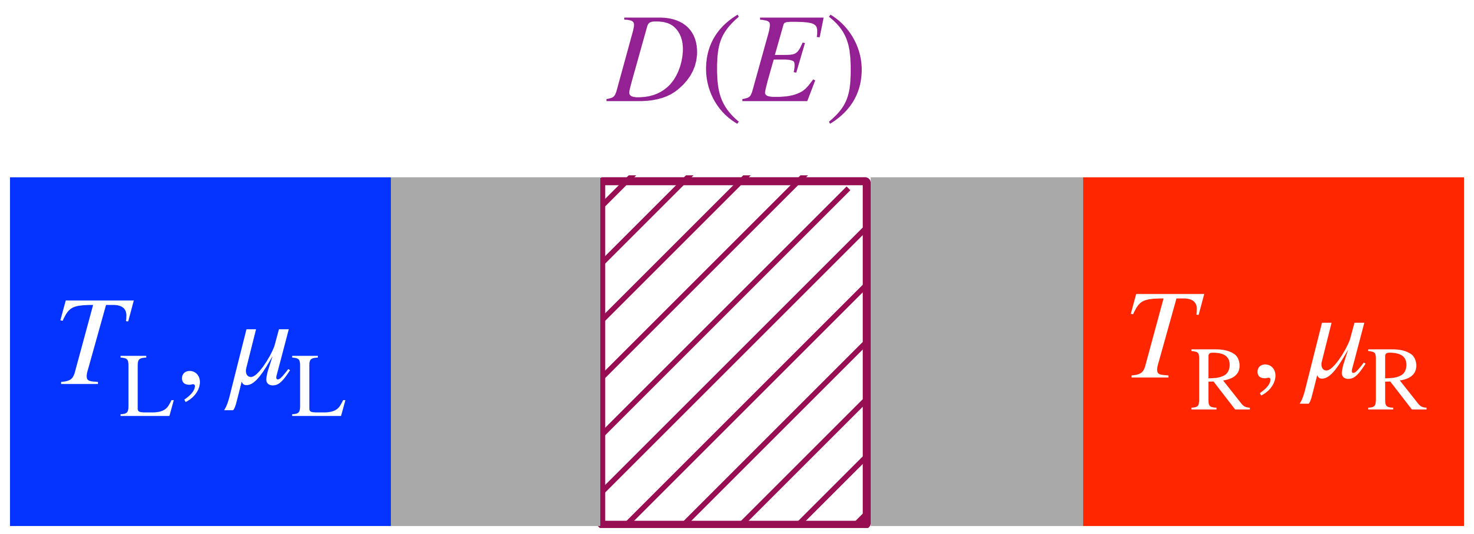
$$S^I - 2\Theta_{\rm hot}$$



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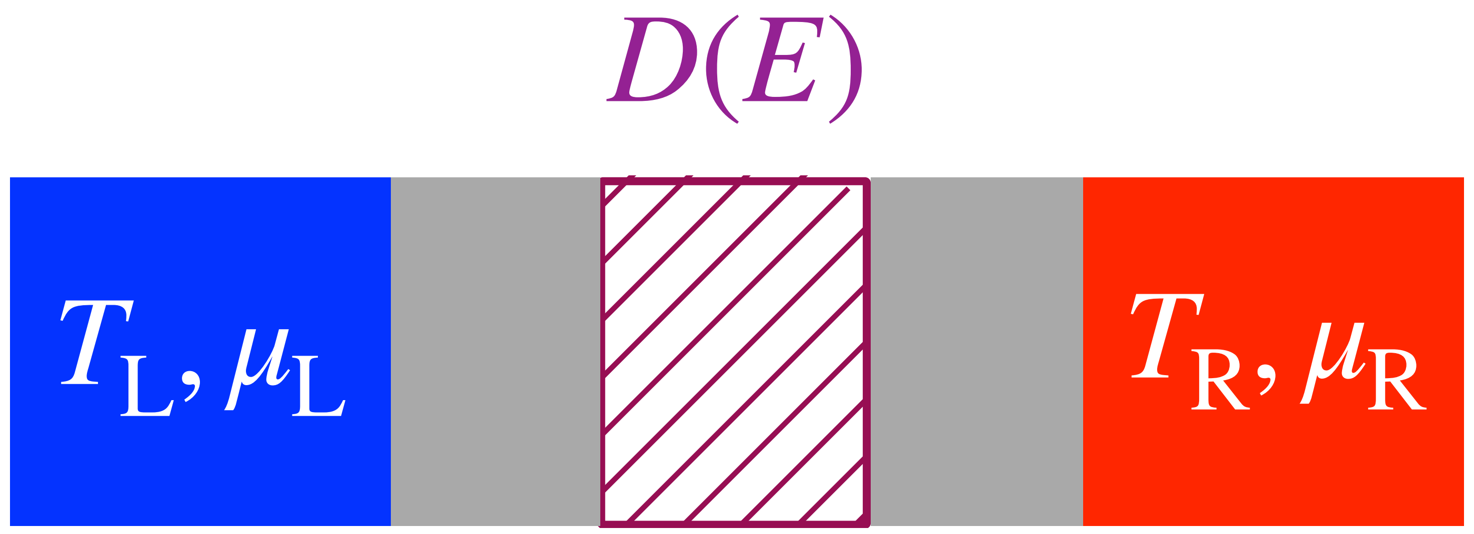

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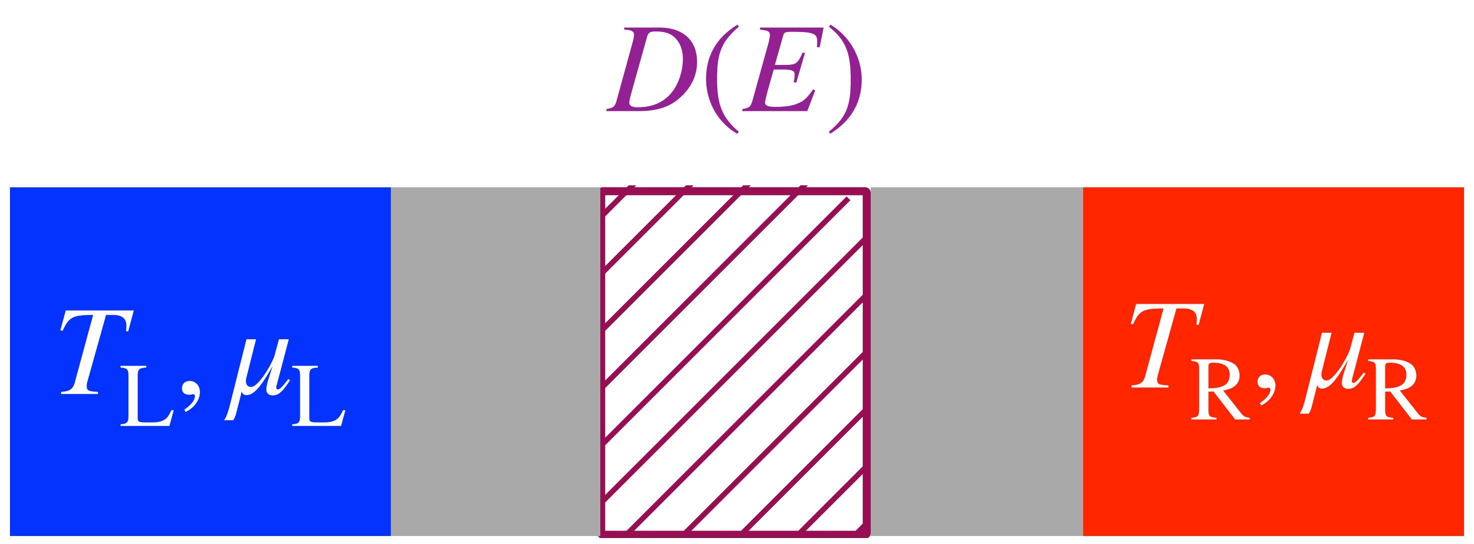
$$S^I - 2\Theta_{\mathrm{hot}}$$



Determine current…

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$$S^I - 2\Theta_{\mathrm{hot}} \leq e^2\left(\tilde{\Gamma}_{\rightarrow} + \tilde{\Gamma}_{\leftarrow}\right)$$

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Excess Rates for transport

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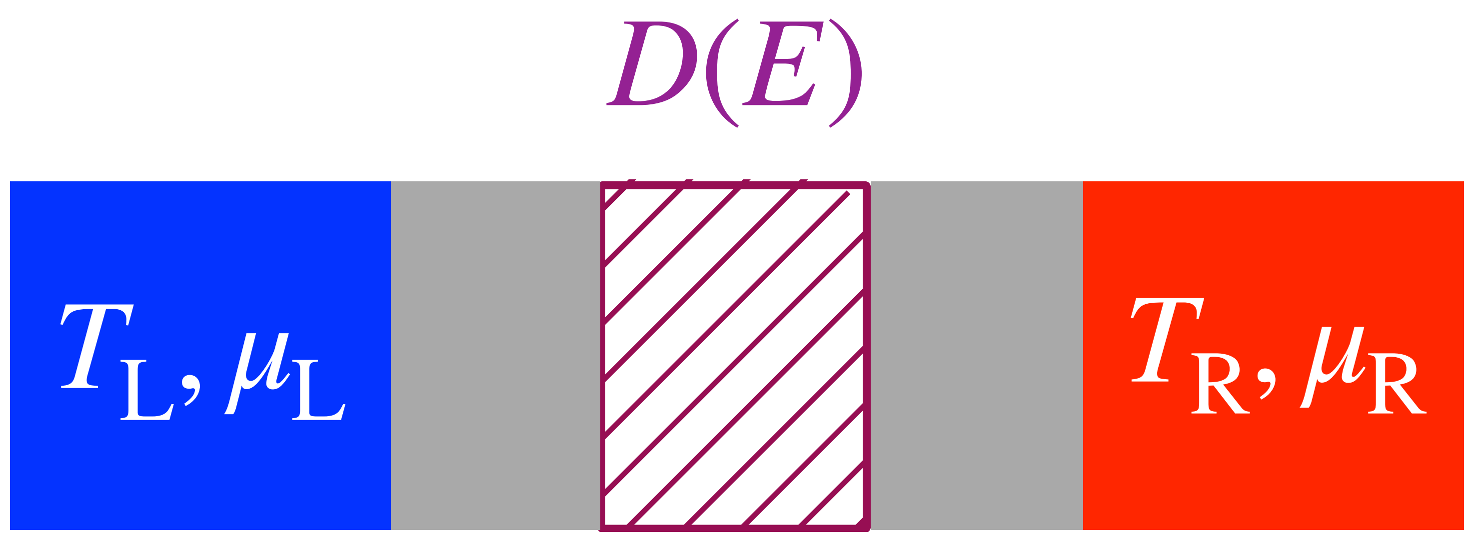
$$\tilde{\Gamma}_{\leftarrow} \equiv \int \frac{dE}{h} D(E)[f_{\mathrm{R}}(E)[1-f_{\mathrm{L}}(E)] - D(E)\Big[f_{\mathrm{R}}(E)\big[1-f_{\mathrm{R}}(E)\big]\Big]$$

Excess noise

$$S^I - 2\Theta_{\mathrm{hot}}$$



Determine current…

$$I = -e\left(\tilde{\Gamma}_{\rightarrow} - \tilde{\Gamma}_{\leftarrow}\right)$$

…AND fluctuations

$$S^I - 2\Theta_{\mathrm{hot}} \leq e^2\left(\tilde{\Gamma}_{\rightarrow} + \tilde{\Gamma}_{\leftarrow}\right) \leq eI \tanh\left(\frac{\Delta\mu}{2k_{\mathrm{B}}\Delta T}\right)$$

L. Tesser, J. Splettstoesser: Phys. Rev. Lett. **132**, 186304 (2024)

Fluctuation dissipation bounds

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Excess noise

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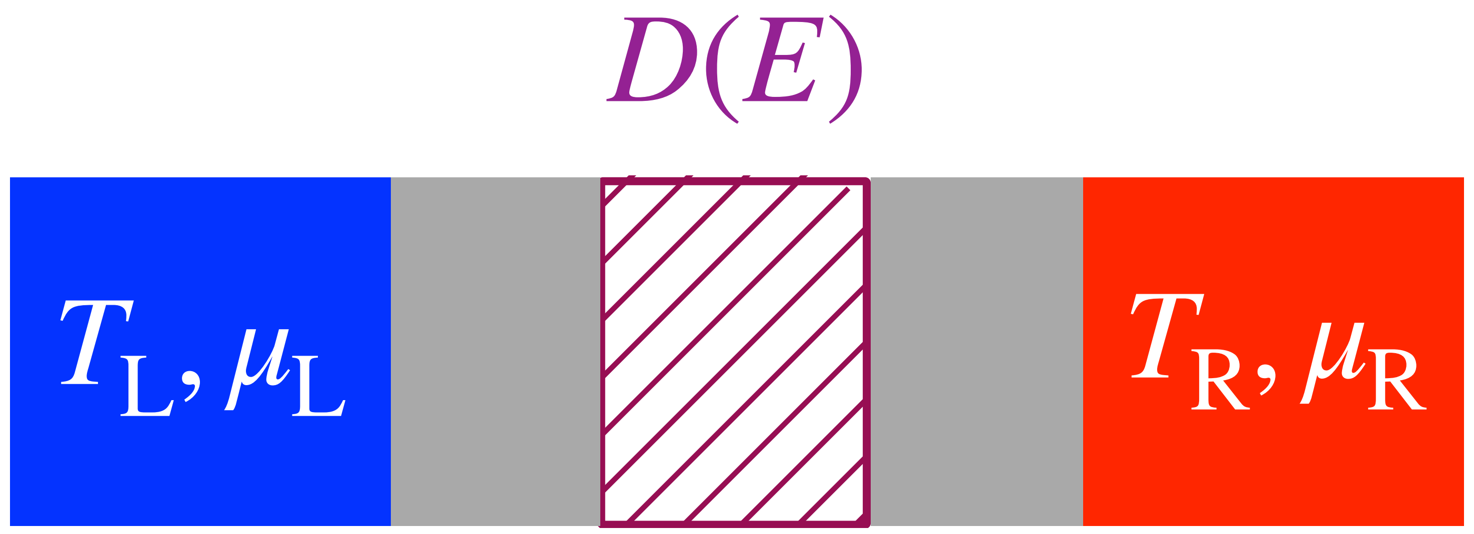

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General fluctuation dissipation bound

L. Tesser, J. Splettstoesser: Phys. Rev. Lett. **132**, 186304 (2024)

Fluctuation dissipation bounds (total noise)

$$-eI\tanh\left(\frac{\Delta\mu}{2k_\mathrm{B}\Delta T}\right) \leq S^I \leq \frac{e^2k_\mathrm{B}}{h}\left(T_\mathrm{L}+T_\mathrm{R}\right)+\left(-eI+\frac{e^2\Delta\mu}{h}\right)\tanh\left(\frac{\Delta\mu}{2k_\mathrm{B}\Delta T}\right)$$

L. Tesser, J. Splettstoesser: Phys. Rev. Lett. **132**, 186304 (2024)

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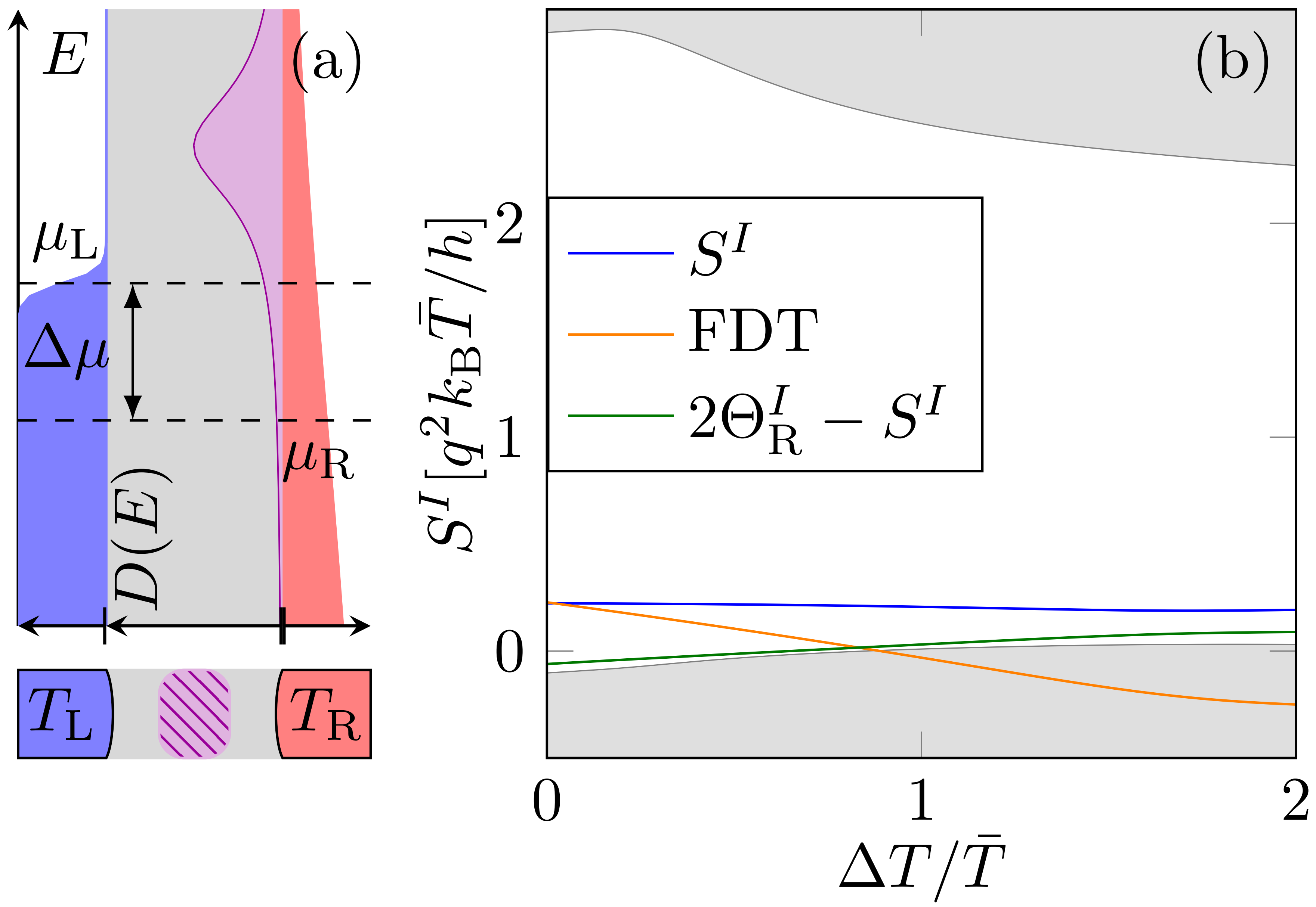

Approaches lower bound in tunnelling regime

L. Tesser, J. Splettstoesser: Phys. Rev. Lett. **132**, 186304 (2024)

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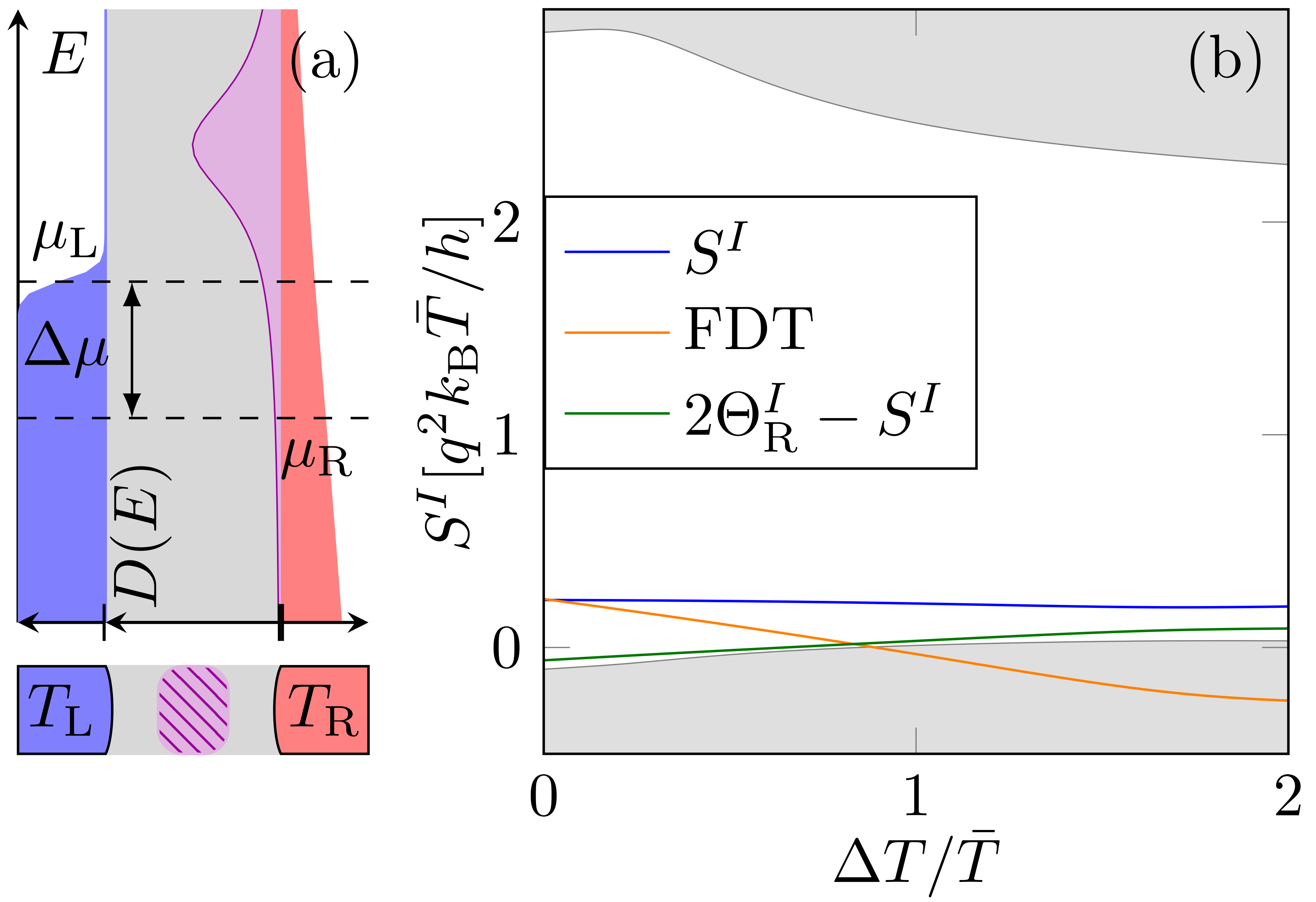
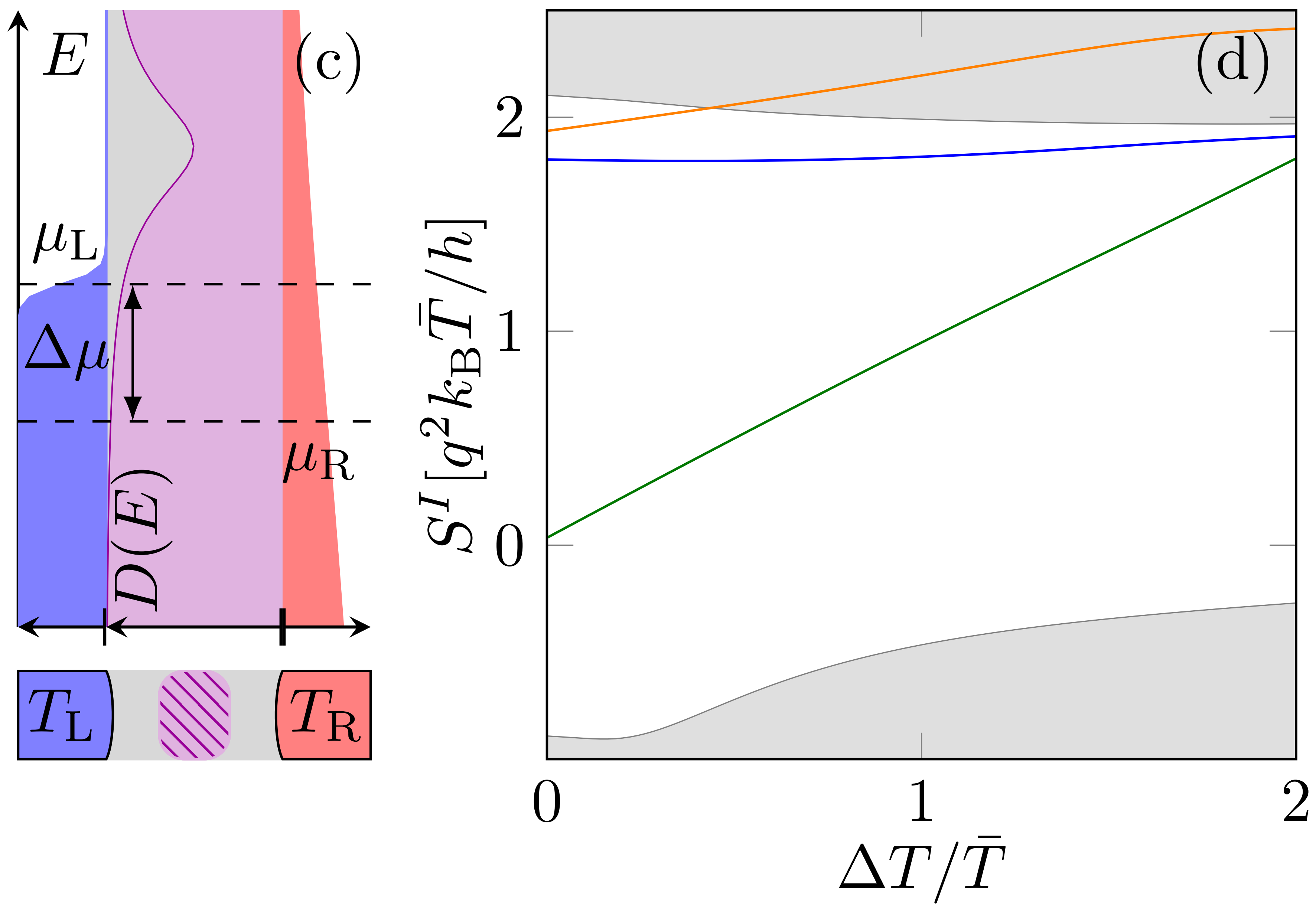


Approaches lower bound in tunnelling regime

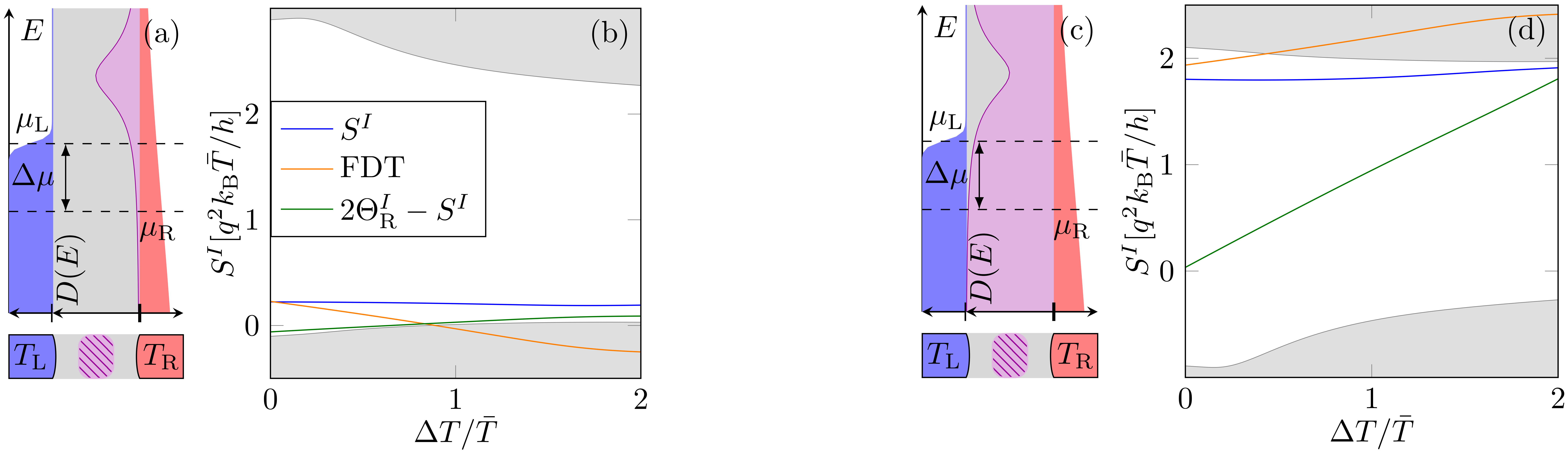


….upper bound in anti-tunnelling regime

L. Tesser, J. Splettstoesser: Phys. Rev. Lett. **132**, 186304 (2024)

Fluctuation dissipation bounds (total noise)

$$-eI\tanh\left(\frac{\Delta\mu}{2k_B\Delta T}\right) \le S^I \le \frac{e^2k_B}{h}(T_L+T_R) + \left(-eI + \frac{e^2\Delta\mu}{h}\right)\tanh\left(\frac{\Delta\mu}{2k_B\Delta T}\right)$$



Approaches lower bound in tunnelling regime ....upper bound in anti-tunnelling regime

Approaches equality for LARGE ΔT → Opposite of FDT

L. Tesser, J. Splettstoesser: Phys. Rev. Lett. **132**, 186304 (2024)

Fluctuations in mesoscopic conductors…

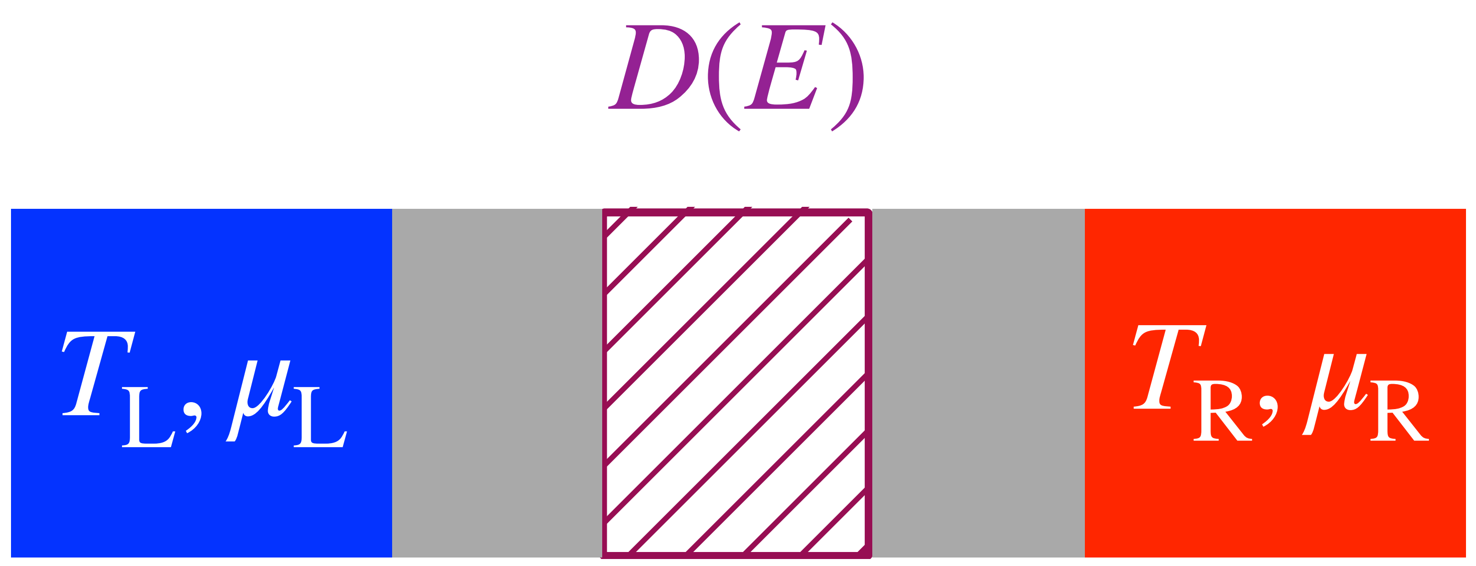
…operating as engines

Fluctuation bounds for output power

Thermodynamic uncertainty relation (TUR)

$$S^P \geq 2\frac{k_B P^2}{\dot{\Sigma}}$$

A. C. Barato, U. Seifert: Phys. Rev. Lett. **114**, 158101 (2015).



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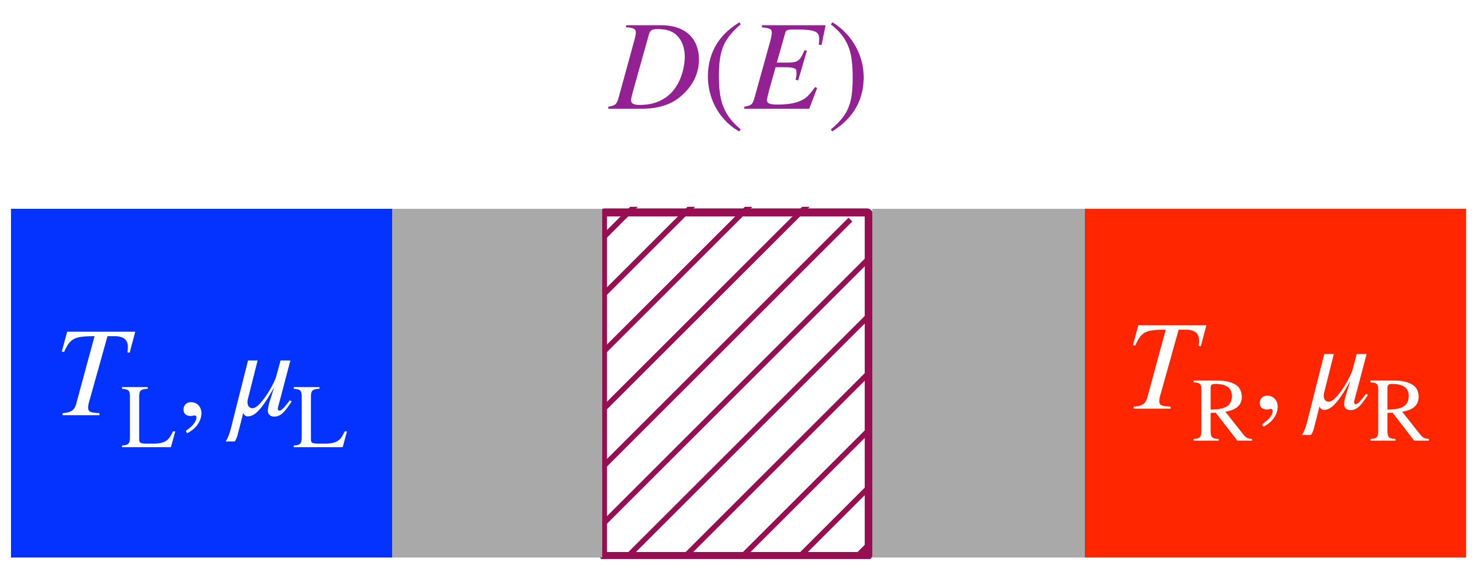
Valid if
“local detailed balance” holds

Can be violated
in quantum systems

A. C. Barato, U. Seifert: Phys. Rev. Lett. **114**, 158101 (2015).



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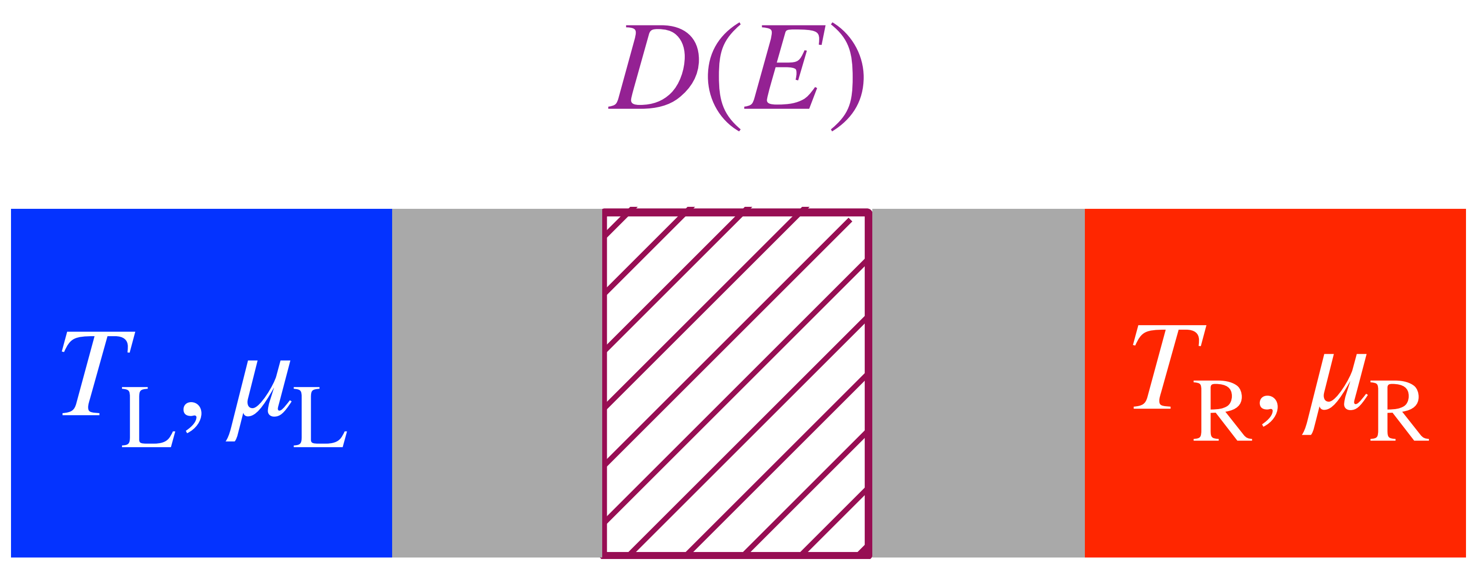

Fluctuation-dissipation bound

$$S^P \geq P\Delta\mu \tanh\left(\frac{\Delta\mu}{2k_B\Delta T}\right)$$

L. Tesser, J. Splettstoesser: Phys. Rev. Lett. **132**, 186304 (2024)

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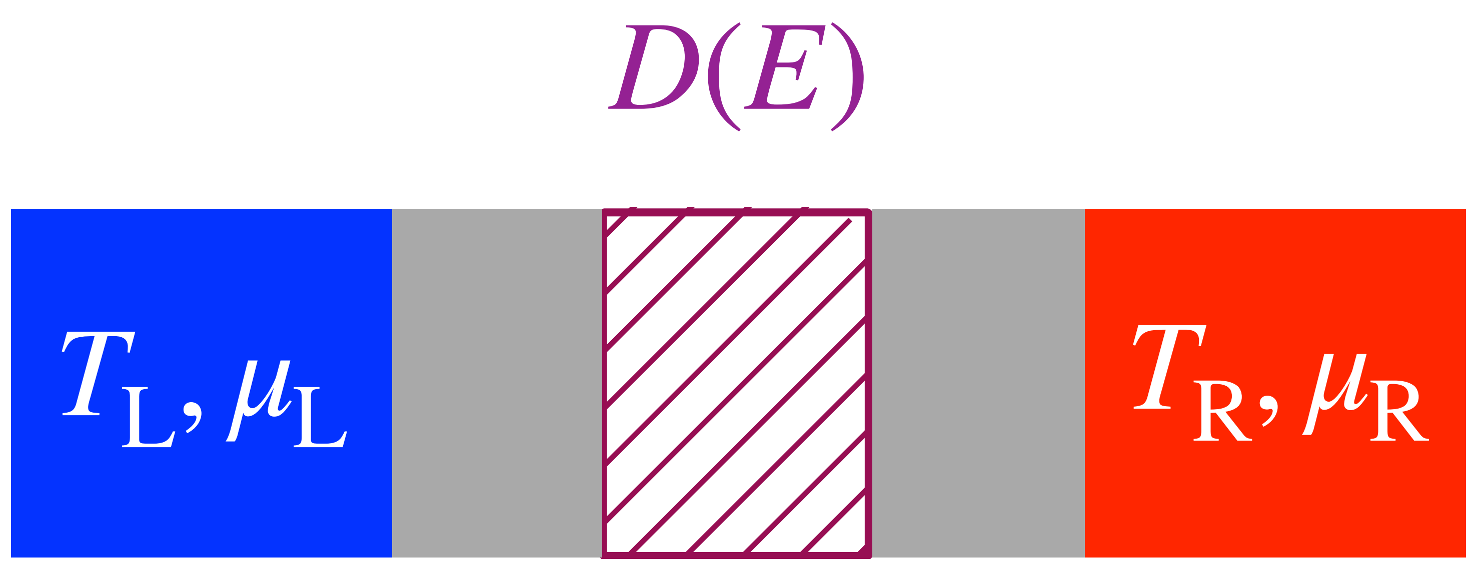

Fluctuation-dissipation bound

$$S^P \geq P\Delta\mu \tanh\left(\frac{\Delta\mu}{2k_B\Delta T}\right)$$

Different from TUR
(entropy production not explicitly included)

Holds for any weakly interacting quantum system

L. Tesser, J. Splettstoesser: Phys. Rev. Lett. **132**, 186304 (2024)

Fluctuation bounds for output power

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$$S^P \geq 2\frac{k_B P^2}{\dot{\Sigma}}$$

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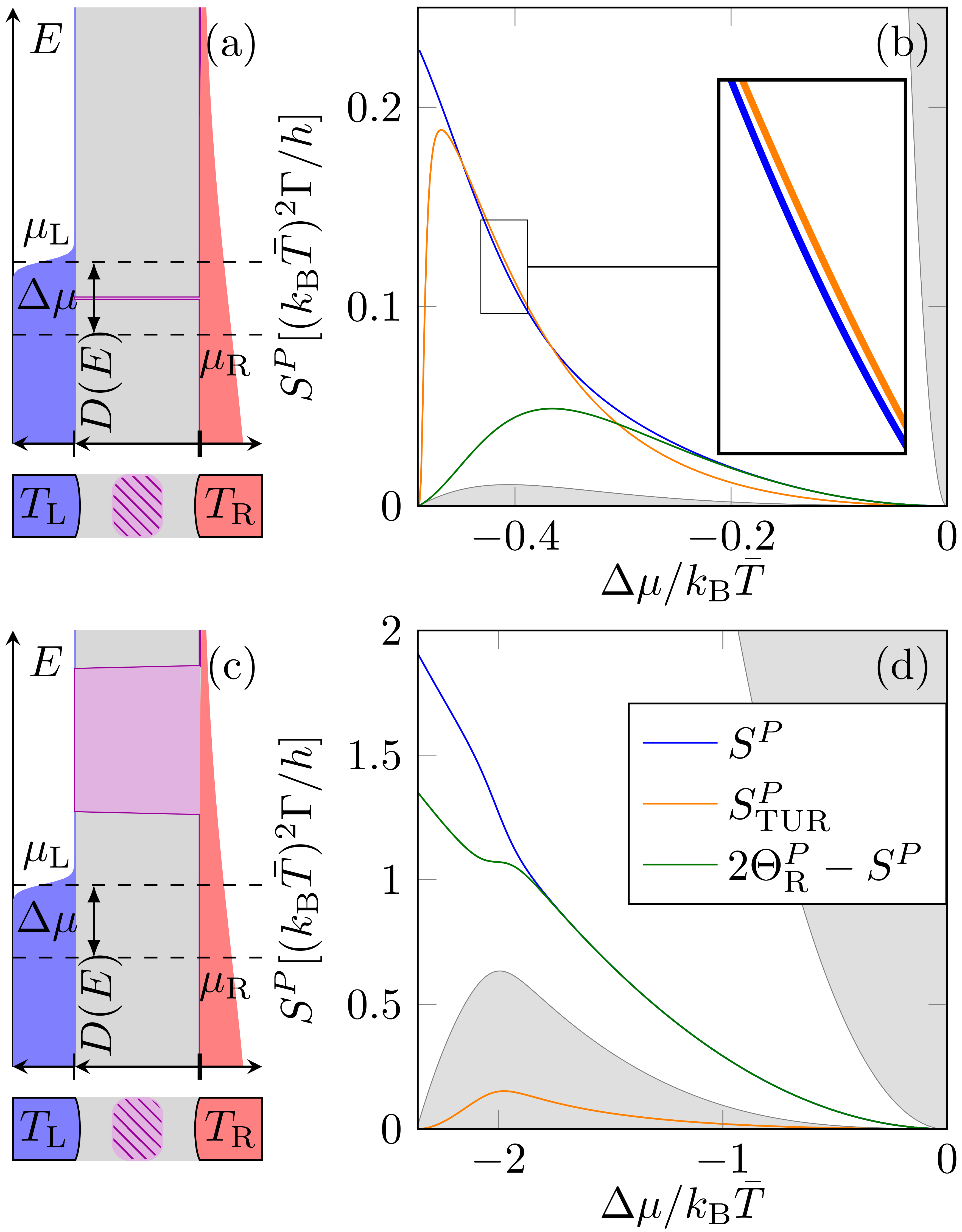

Holds when TUR is violated

Can be more restrictive than TUR

(Also sets an upper bound)

L. Tesser, J. Splettstoesser: Phys. Rev. Lett. **132**, 186304 (2024)

Validity regime and further steps

Fluctuation dissipation theorem

$$S^I = qI \coth\left(\frac{\Delta\mu}{2k_\mathrm{B}T}\right)$$

Fluctuation dissipation bound

$$-eI \tanh\left(\frac{\Delta\mu}{2k_\mathrm{B}\Delta T}\right) \leq S^I$$

$$\leq \frac{e^2 k_\mathrm{B}}{h}\left(T_\mathrm{L} + T_\mathrm{R}\right) + \left(-eI + \frac{e^2\Delta\mu}{h}\right)\tanh\left(\frac{\Delta\mu}{2k_\mathrm{B}\Delta T}\right)$$

Thermodynamic uncertainty relations

$$S^P \geq 2\frac{k_\mathrm{B}P^2}{\dot{\Sigma}}$$

Validity regime and further steps

<u>Fluctuation dissipation theorem</u>

$$S^I = qI\coth\left(\frac{\Delta\mu}{2k_\mathrm{B}T}\right)$$

+ Equality

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- Inequality, upper AND lower bound

<u>Thermodynamic uncertainty relations</u>

$$S^P \geq 2\frac{k_\mathrm{B}P^2}{\dot{\Sigma}}$$

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Validity regime and further steps

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$$S^I = qI \coth\left(\frac{\Delta\mu}{2k_B T}\right)$$

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+ arbitrarily strong Coulomb interaction

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Validity regime and further steps

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+ arbitrary transmission

Thermodynamic uncertainty relations

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- detailed balance

Validity regime and further steps

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→ **Interacting** nonequilibrium systems? L. Tesser, M. Acciai, C. Spånslätt, I. Safi, J. Splettstoesser: arXiv:2409.00981 (2024)

→ What happens if resources are nonthermal? Or in bosonic systems?

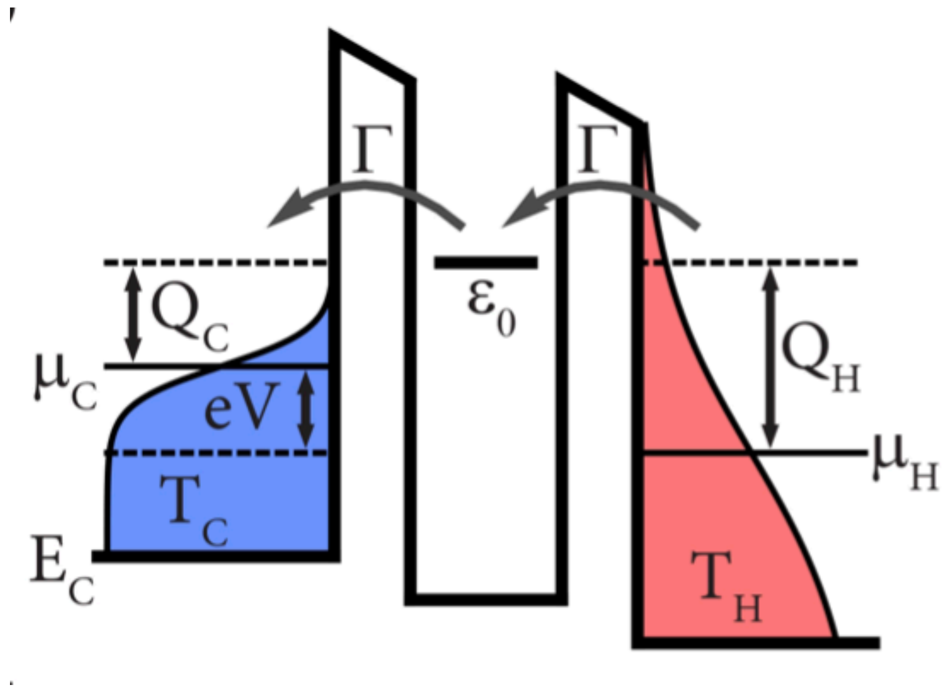
Precision bounds for quantum transport of generic transport observables

Precision bounds for generic transport observables

…and more generic systems.

Precision bounds for generic transport observables

Nonthermal distributions

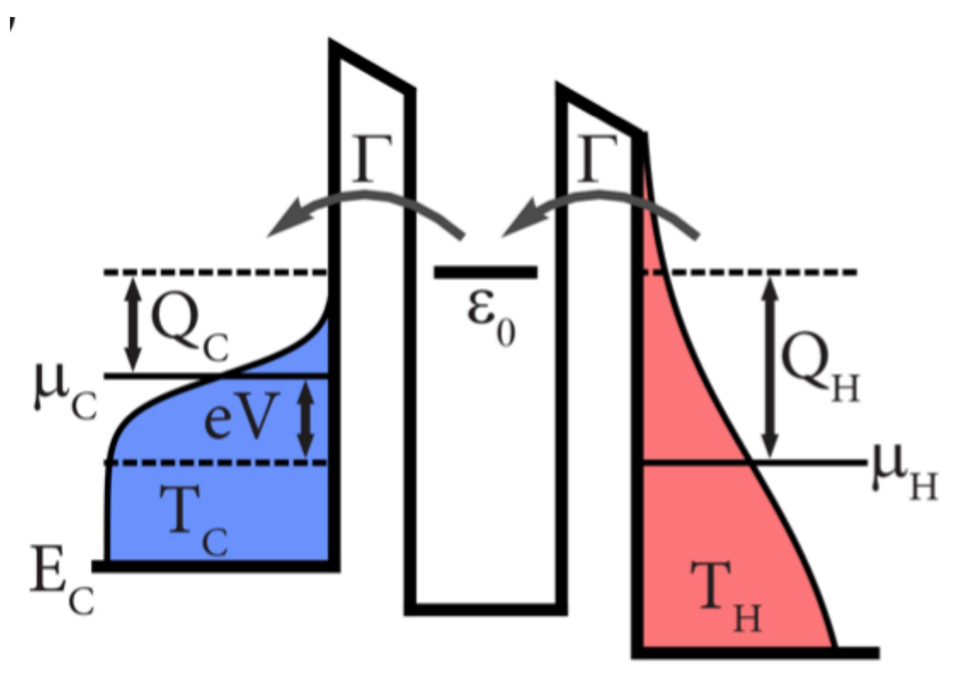


Contact with two reservoirs?

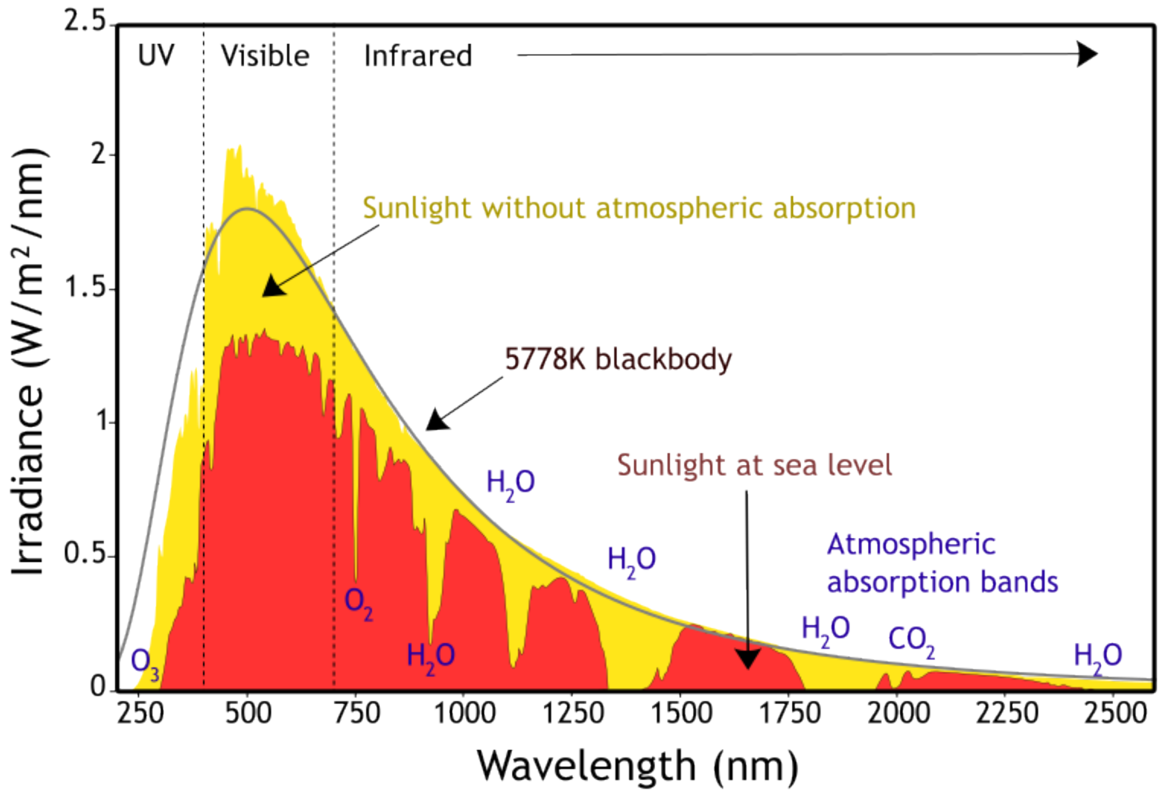
…and more generic systems.

Precision bounds for generic transport observables

Nonthermal distributions



Contact with two reservoirs?

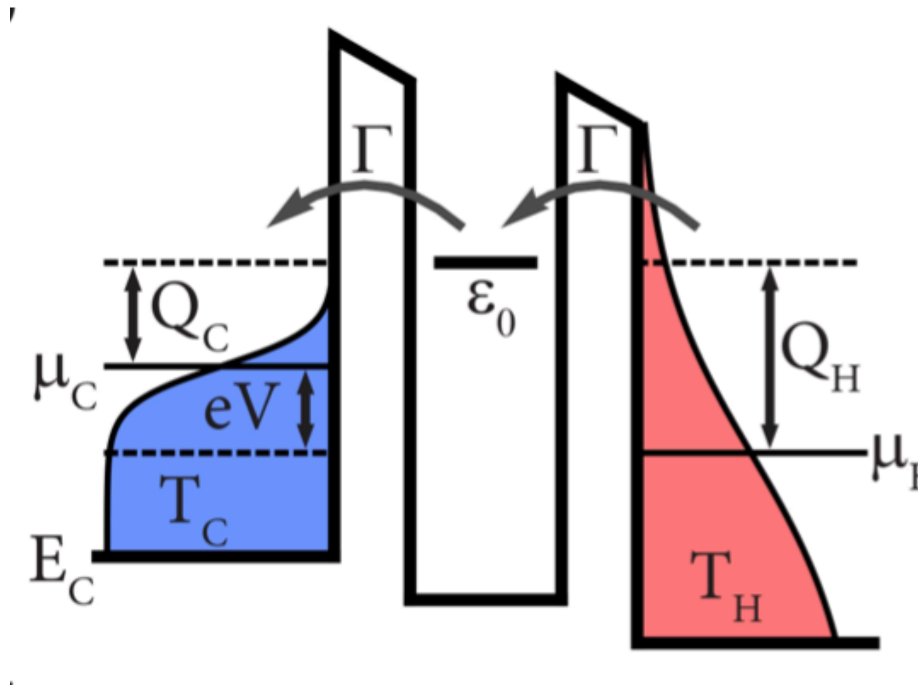


Filtering or resonant effects

…and more generic systems.

Precision bounds for generic transport observables

Nonthermal distributions



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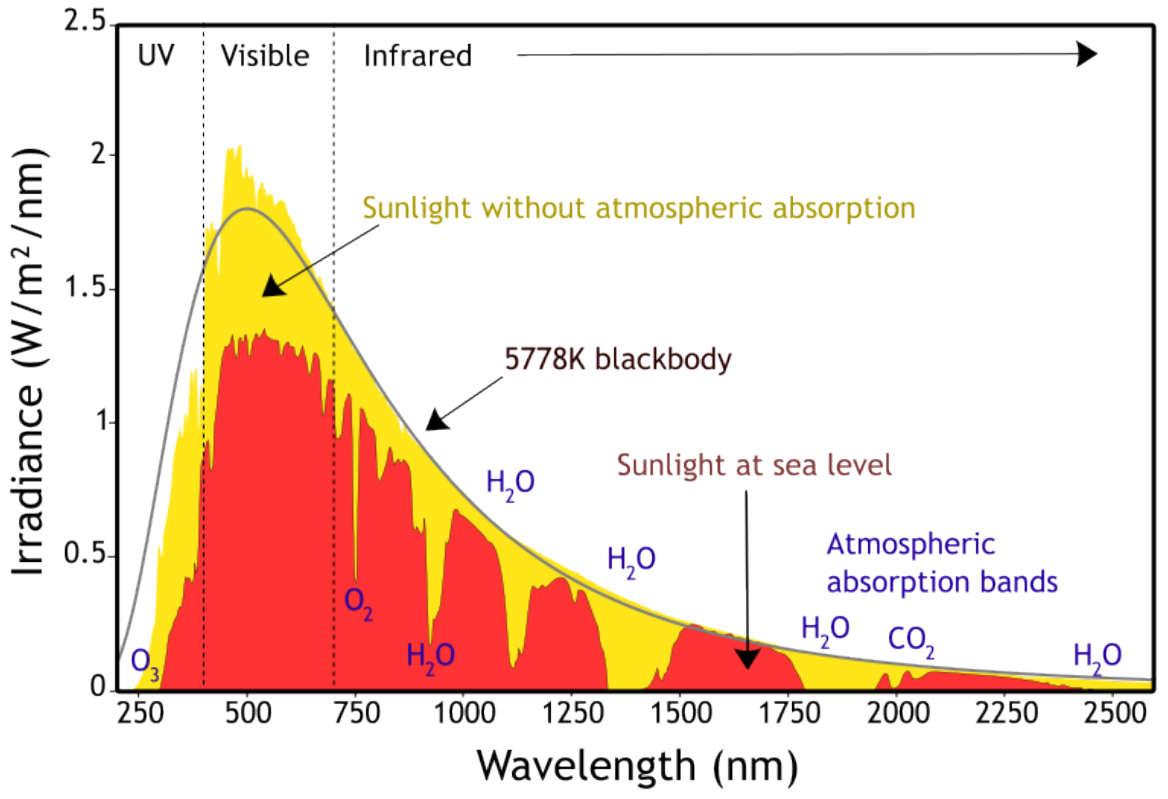

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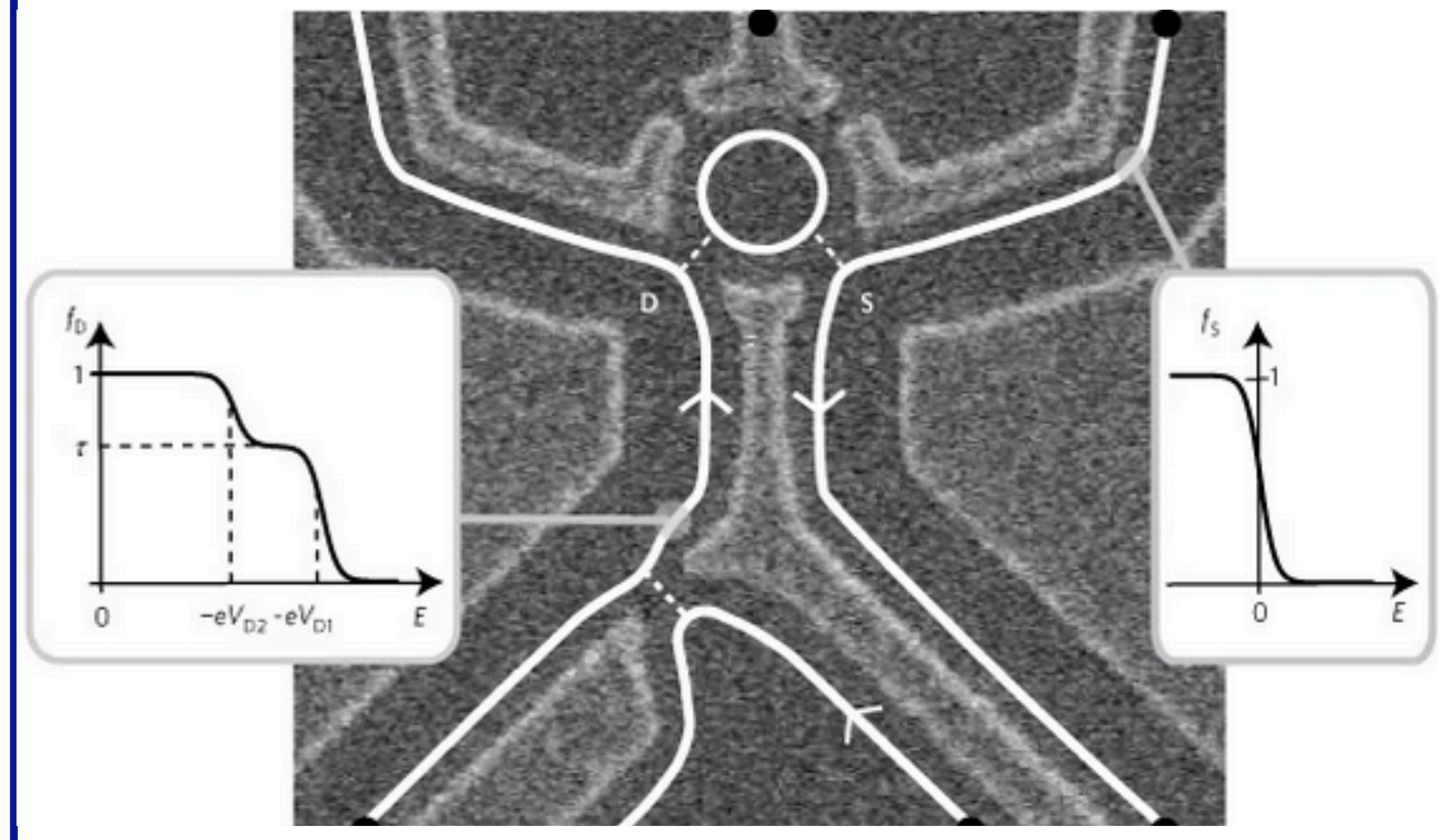

C. Altimiras, H. Le Sueur, U. Gennser, A. Cavanna, D. Mailly, F. Pierre: Nat. Phys. **6**, 34 (2009).

…and more generic systems.

Precision bounds for generic transport observables

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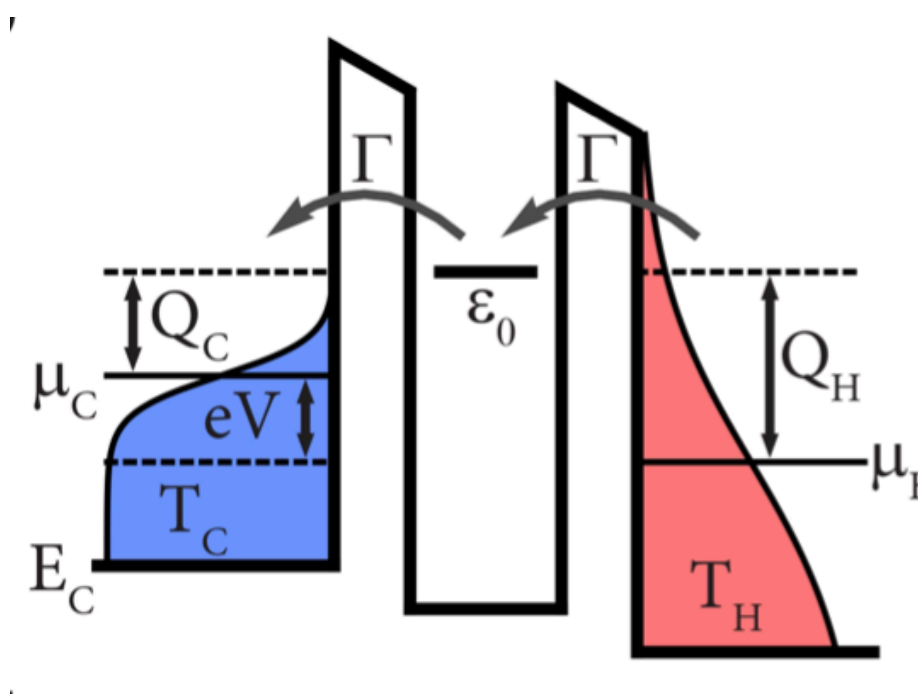

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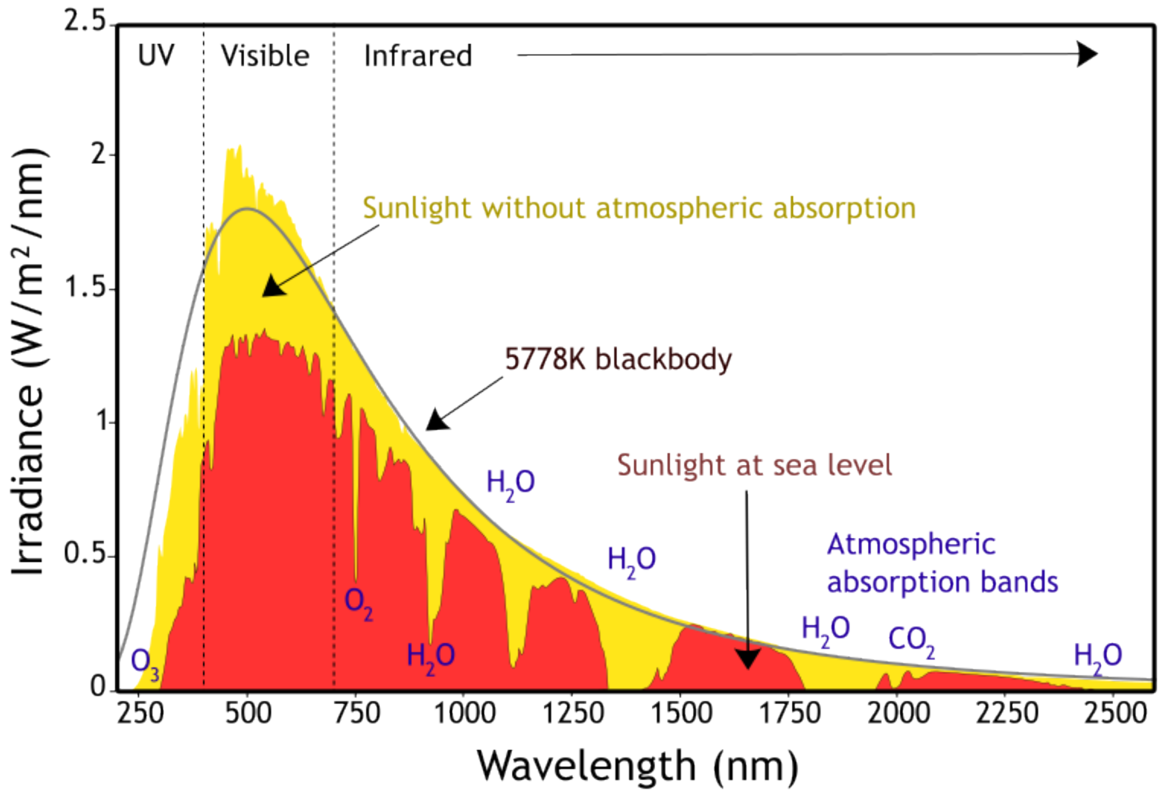

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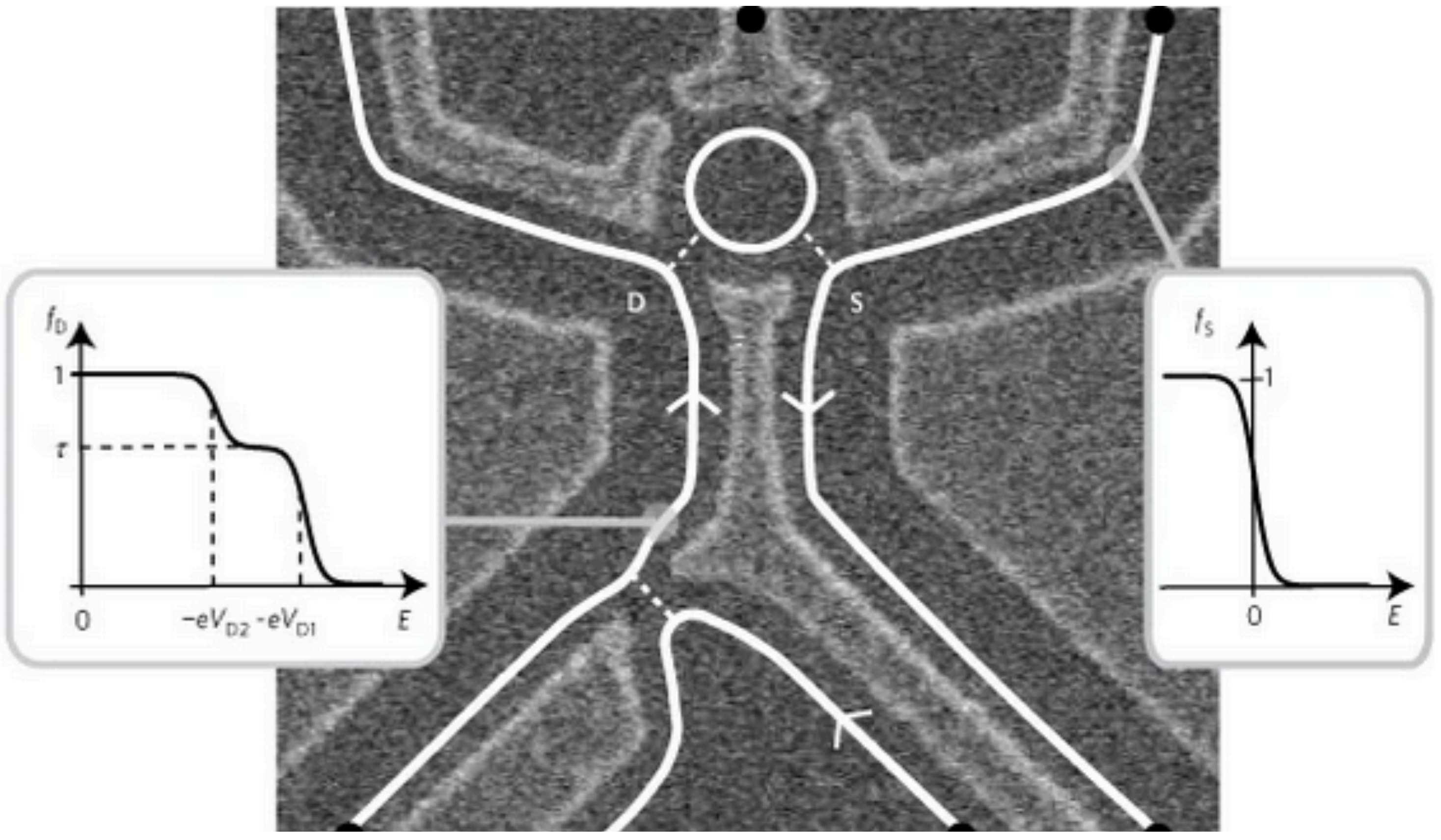

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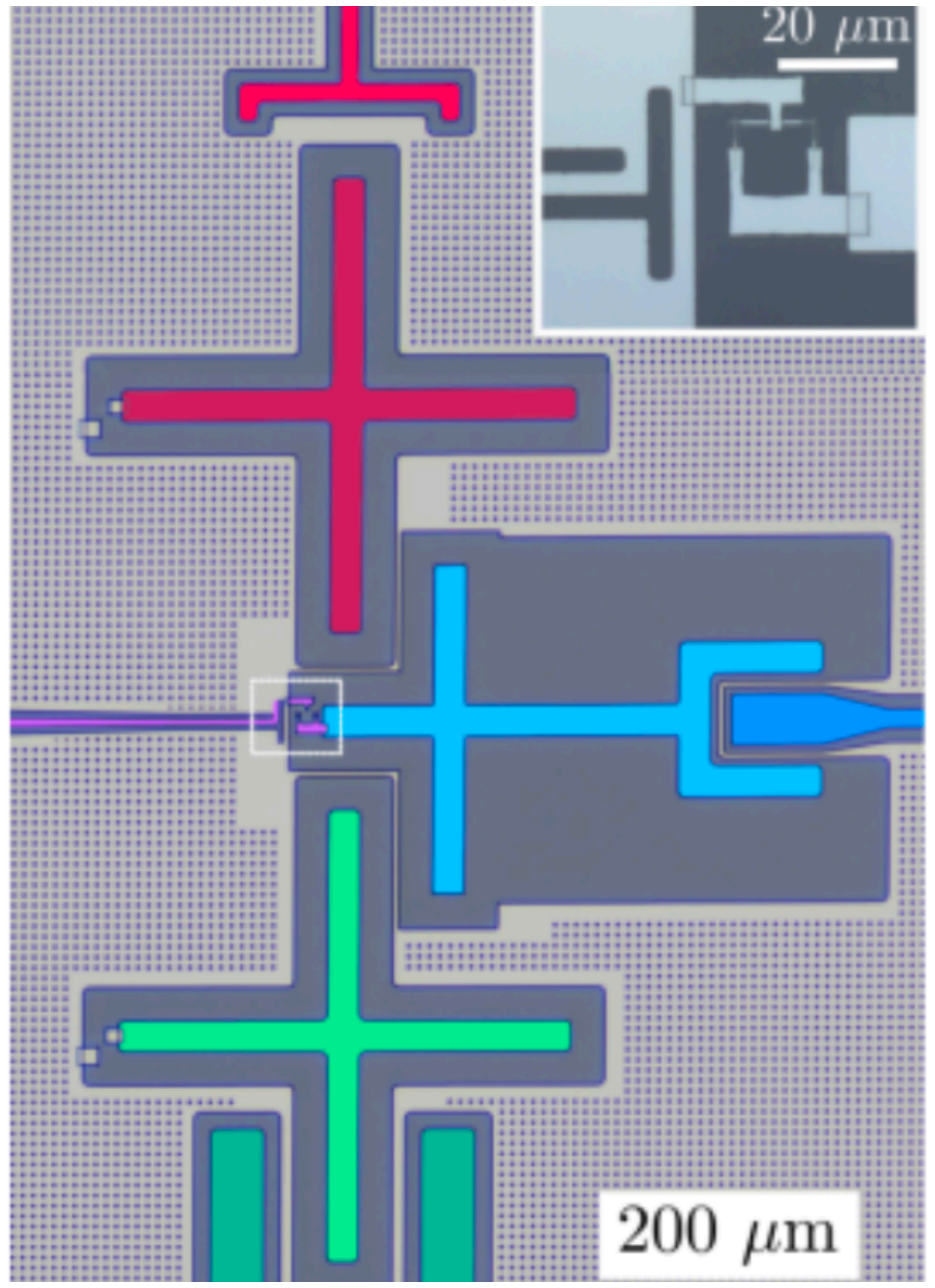
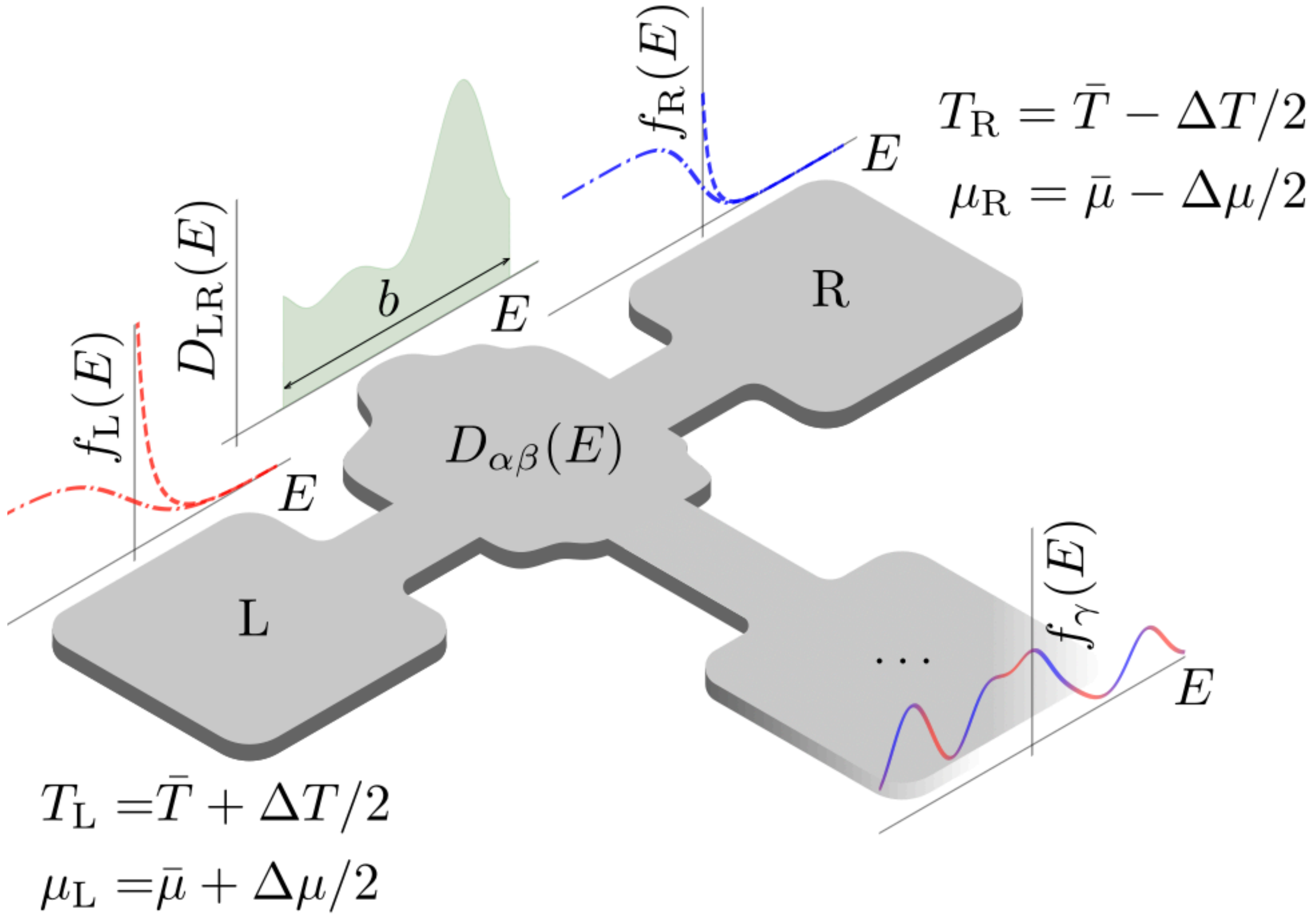
Bosonic heat transport



M. A. Aamir, P. J. Suria, J. A. M. Guzmán, C. Castillo-Moreno, J. M. Epstein, N. Yunger Halpern, and S. Gasparinetti, arkiv:2305.16710

Thermodynamics of steady-state quantum transport

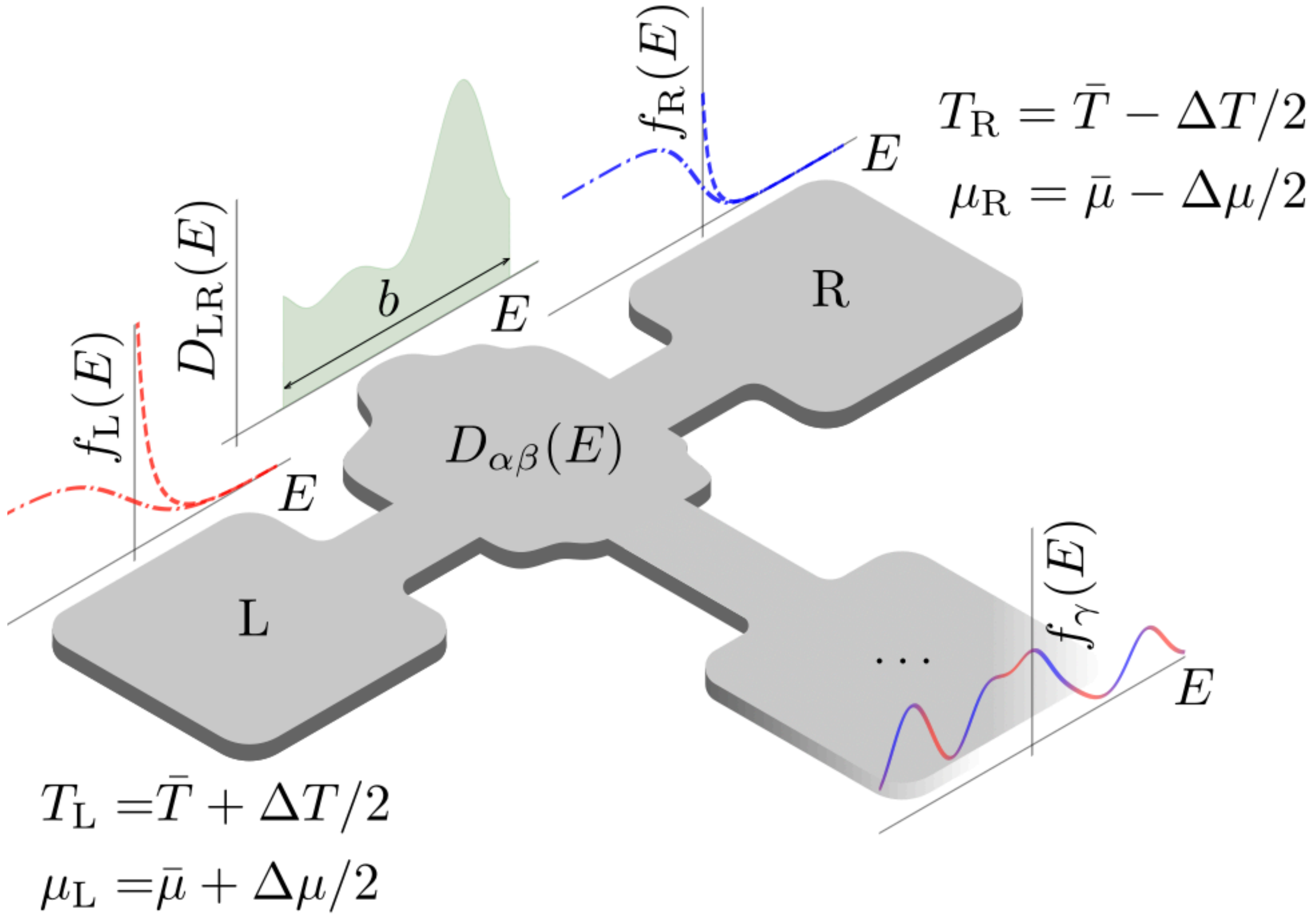
Multi-terminal setup with particle/energy reservoirs



Thermodynamics of steady-state quantum transport

Multi-terminal setup with particle/energy reservoirs

Differences of potentials, temperatures, occupations…



Thermodynamics of steady-state quantum transport

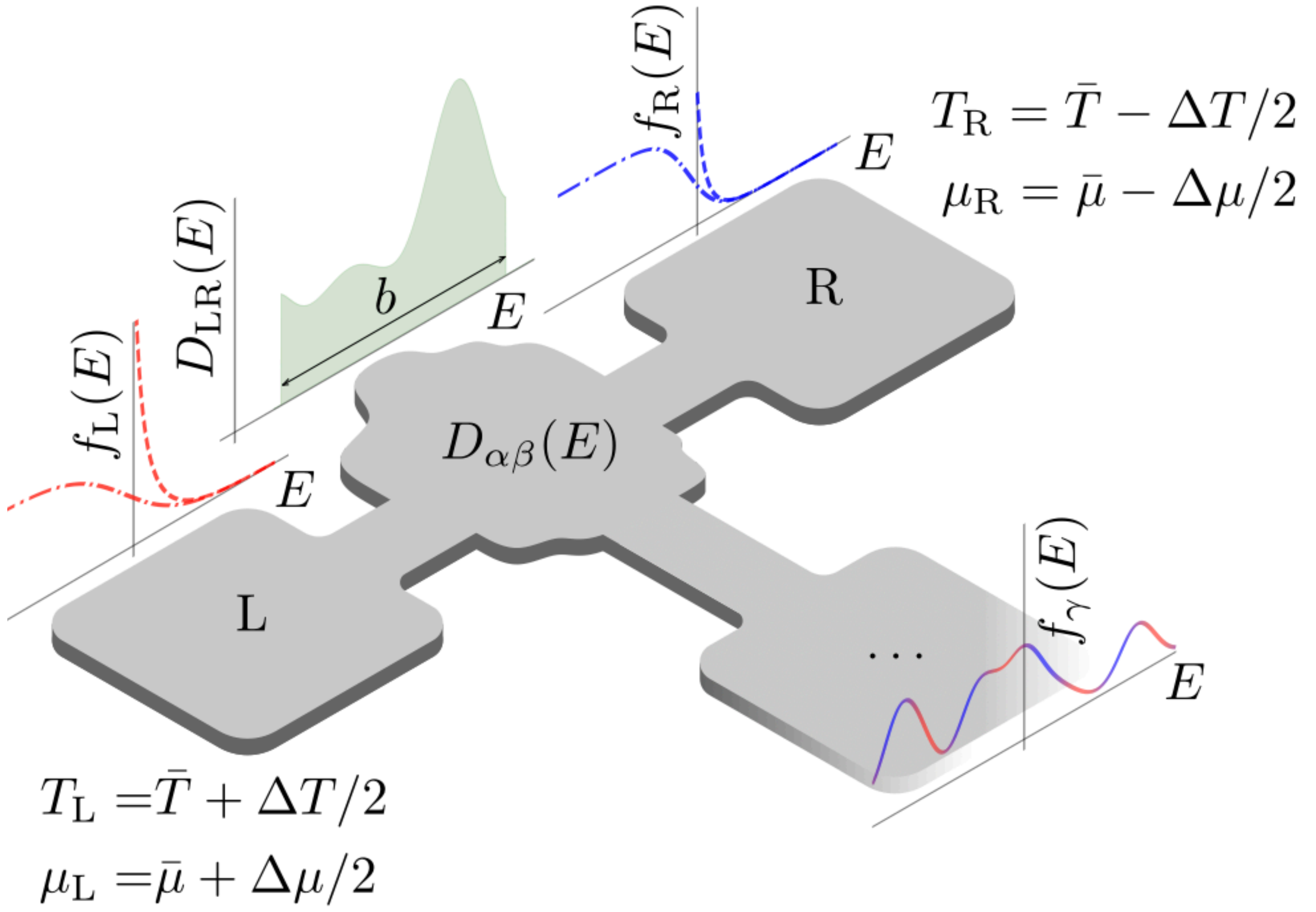
Multi-terminal setup with particle/energy reservoirs

Differences of potentials, temperatures, occupations…

Induce …particle currents → Do work

…heat currents → Cooling

reduce entropy — produce entropy…



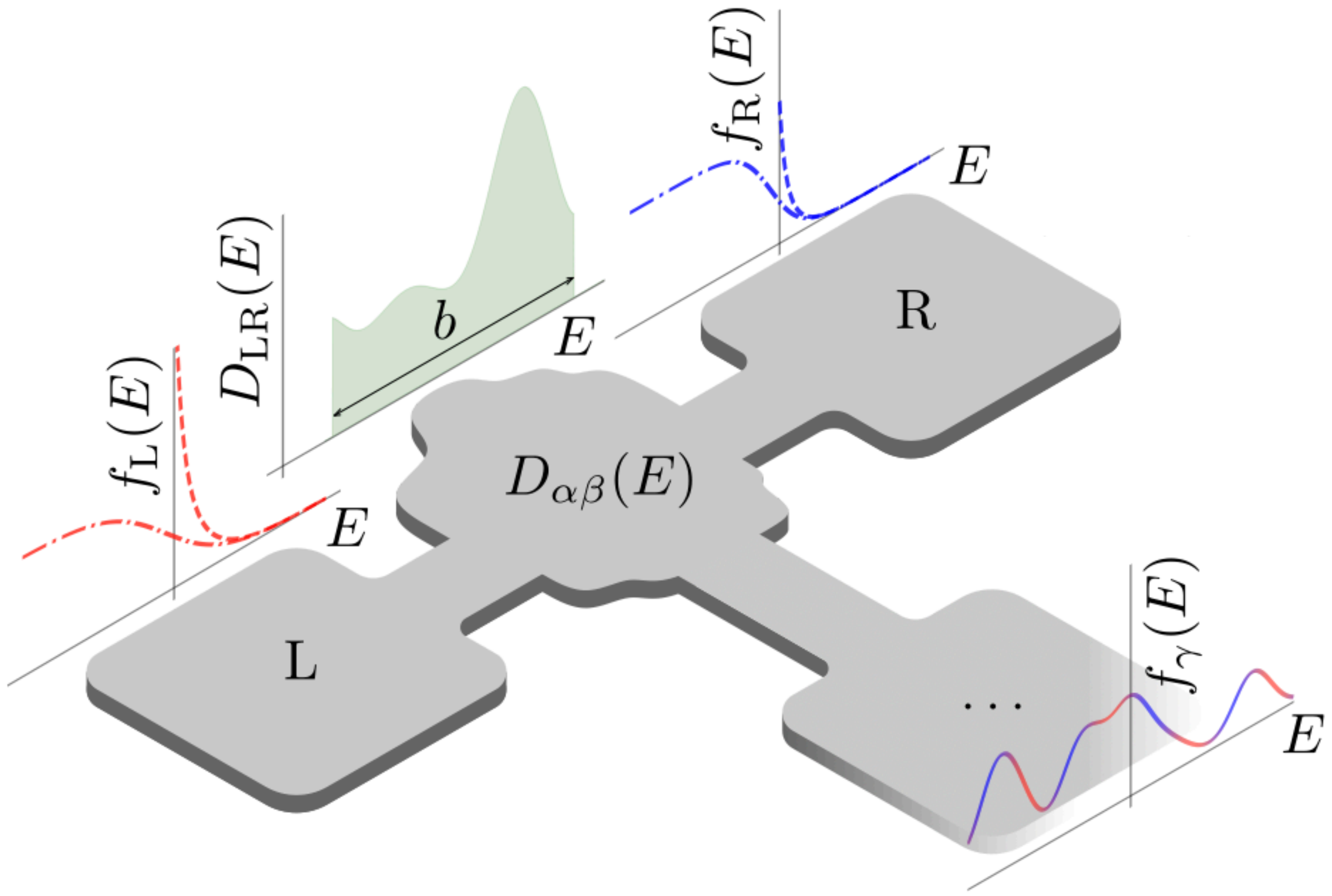
Generic currents and their fluctuations

Generic current

$$I_\alpha^{(\nu)} = \frac{1}{h}\int dE\; x_\alpha^{(\nu)} \sum_\beta D_{\alpha\beta}(E)[f_\beta(E) - f_\alpha(E)]$$

with

$x_\alpha^{(n)} \equiv 1$	$x_\alpha^{(E)} \equiv E$	$x_\alpha^{(\sigma)} \equiv \log\left[\frac{f_\alpha(E)}{1-f_\alpha(E)}\right]$
Particle	Energy	Entropy



M. Acciai*, L. Tesser*, J. Eriksson, R. Sánchez, R. S. Whitney, J. Splettstoesser: Phys. Rev. B **109**, 075405 (2024).

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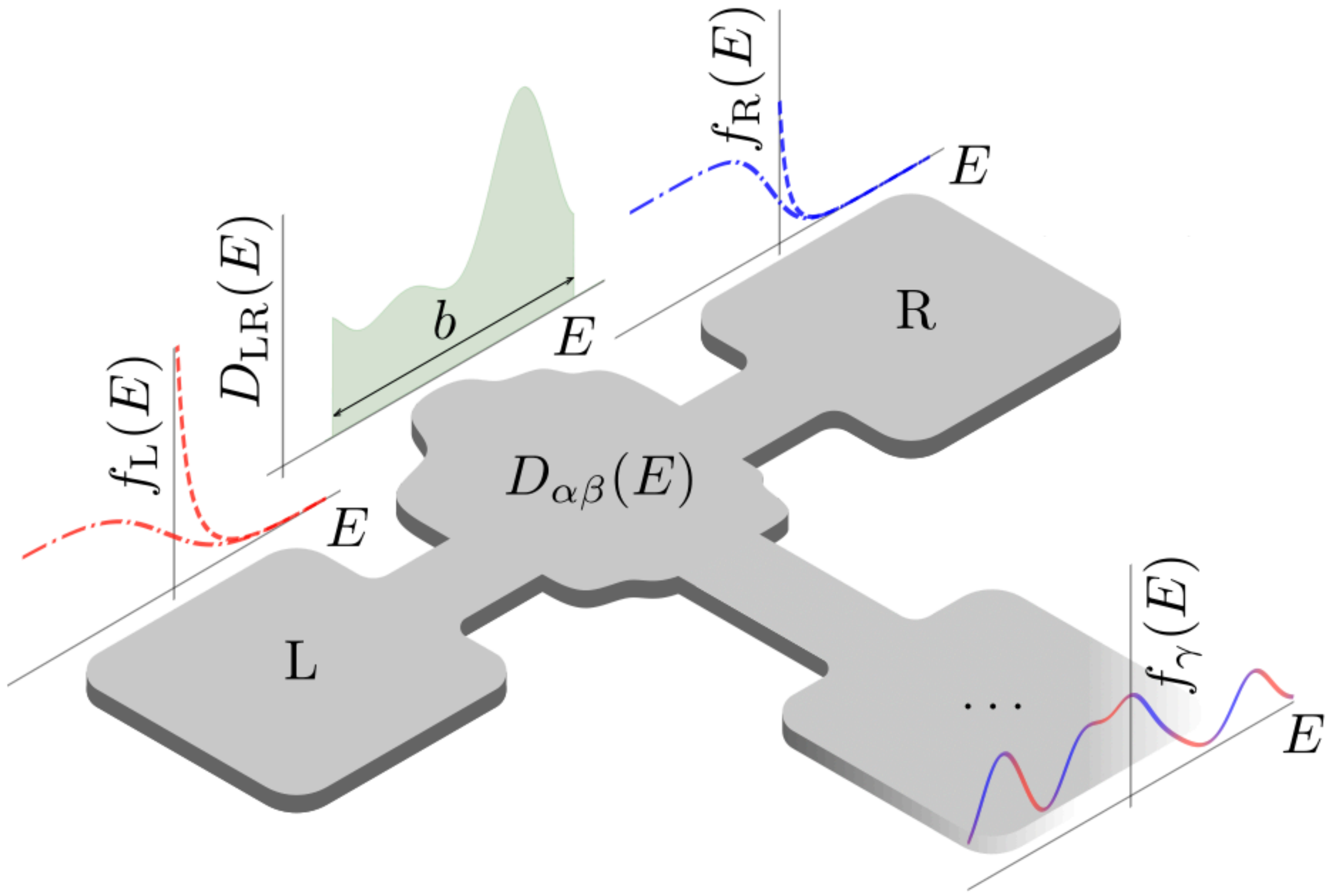
with $x_\alpha^{(n)} \equiv 1$ — Particle; $x_\alpha^{(E)} \equiv E$ — Energy; $x_\alpha^{(\sigma)} \equiv \log\left[\frac{f_\alpha(E)}{1-f_\alpha(E)}\right]$ — Entropy



→ Fluctuations split into classical and quantum part

$$S_{\alpha\alpha,\mathrm{cl}}^{(\nu)} = \frac{1}{h}\int dE\; \left[x_\alpha^{(\nu)}\right]^2 \sum_{\beta\neq\alpha} D_{\alpha\beta}\left[f_\alpha\left(1 \pm f_\beta\right) + f_\beta\left(1 \pm f_\alpha\right)\right]$$

$$S_{\alpha\alpha,\mathrm{qu}}^{(\nu)} = \pm\frac{1}{h}\int dE\; \left[x_\alpha^{(\nu)}\right]^2 \left[\sum_{\beta\neq\alpha} D_{\alpha\beta}(f_\alpha - f_\beta)\right]^2$$

$\pm$, Bosons/fermions

any distribution…

M. Acciai*, L. Tesser*, J. Eriksson, R. Sánchez, R. S. Whitney, J. Splettstoesser: Phys. Rev. B **109**, 075405 (2024).

Bound on classical fluctuations/precision

Generic current

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(Classical) fluctuations

$$S_{\alpha\alpha,\mathrm{cl}}^{(\nu)} = \frac{1}{h}\int dE\ \left[x_\alpha^{(\nu)}\right]^2 \sum_{\beta\neq\alpha} D_{\alpha\beta}\left[f_\alpha\left(1 \pm f_\beta\right) + f_\beta\left(1 \pm f_\alpha\right)\right]$$

D. Palmqvist, L. Tesser, J. Splettstoesser: arXiv:2410.10793 (2024).
M. Acciai*, L. Tesser*, J. Eriksson, R. Sánchez, R. S. Whitney, J. Splettstoesser: Phys. Rev. B **109**, 075405 (2024).

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$$S_{\alpha\alpha,\text{cl}}^{(N)} \geq \frac{\left(I_\alpha^{(\nu)}\right)^2}{S_{\alpha\alpha,\text{cl}}^{(\nu)}}$$

D. Palmqvist, L. Tesser, J. Splettstoesser: arXiv:2410.10793 (2024).

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(Classical) fluctuations

$$S_{\alpha\alpha,\text{cl}}^{(\nu)} = \frac{1}{h}\int dE\, \left[x_\alpha^{(\nu)}\right]^2 \sum_{\beta\neq\alpha} D_{\alpha\beta}\left[f_\alpha\left(1 \pm f_\beta\right) + f_\beta\left(1 \pm f_\alpha\right)\right]$$

$$\Rightarrow \quad S_{\alpha\alpha,\text{cl}}^{(N)} \geq \frac{\left(I_\alpha^{(\nu)}\right)^2}{S_{\alpha\alpha,\text{cl}}^{(\nu)}}$$

- $f_\alpha - f_\beta \leq f_\alpha\left(1 \pm f_\beta\right) + f_\beta\left(1 \pm f_\alpha\right)$

  Reaches **equality**
  when contact β is "empty"

D. Palmqvist, L. Tesser, J. Splettstoesser: arXiv:2410.10793 (2024).
M. Acciai*, L. Tesser*, J. Eriksson, R. Sánchez, R. S. Whitney, J. Splettstoesser: Phys. Rev. B **109**, 075405 (2024).

Bound on classical fluctuations/precision

Generic current

$$I_\alpha^{(\nu)} = \frac{1}{h}\int dE\, x_\alpha^{(\nu)} \sum_\gamma D_{\alpha\beta}[f_\alpha(E) - f_\beta(E)]$$

(Classical) fluctuations

$$S_{\alpha\alpha,\mathrm{cl}}^{(\nu)} = \frac{1}{h}\int dE\, \left[x_\alpha^{(\nu)}\right]^2 \sum_{\beta\neq\alpha} D_{\alpha\beta}\left[f_\alpha\left(1 \pm f_\beta\right) + f_\beta\left(1 \pm f_\alpha\right)\right]$$

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  Reaches **equality**
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- $|x| \leq |x|^2 + \frac{1}{4}$

D. Palmqvist, L. Tesser, J. Splettstoesser: arXiv:2410.10793 (2024).
M. Acciai*, L. Tesser*, J. Eriksson, R. Sánchez, R. S. Whitney, J. Splettstoesser: Phys. Rev. B **109**, 075405 (2024).

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Kinetic uncertainty relation for quantum transport

I. Di Terlizzi, M. Baiesi: J. Phys. A: Math. Theor. **52**, 2 (2018)

D. Palmqvist, L. Tesser, J. Splettstoesser: arXiv:2410.10793 (2024).

M. Acciai*, L. Tesser*, J. Eriksson, R. Sánchez, R. S. Whitney, J. Splettstoesser: Phys. Rev. B **109**, 075405 (2024).

What about the quantum corrections?

D. Palmqvist, L. Tesser, J. Splettstoesser: arXiv:2410.10793 (2024).

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For bosons, quantum contributions to noise are positive: bound holds!

Bosons: $S_{\alpha\alpha} = S_{\alpha\alpha,\mathrm{cl}} + S_{\alpha\alpha,\mathrm{qu}}$

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Precision is limited by bandwidth B_α^ν

D. Palmqvist, L. Tesser, J. Splettstoesser: arXiv:2410.10793 (2024).

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D. Palmqvist, L. Tesser, J. Splettstoesser: arXiv:2410.10793 (2024).

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New tight bound — KURL:

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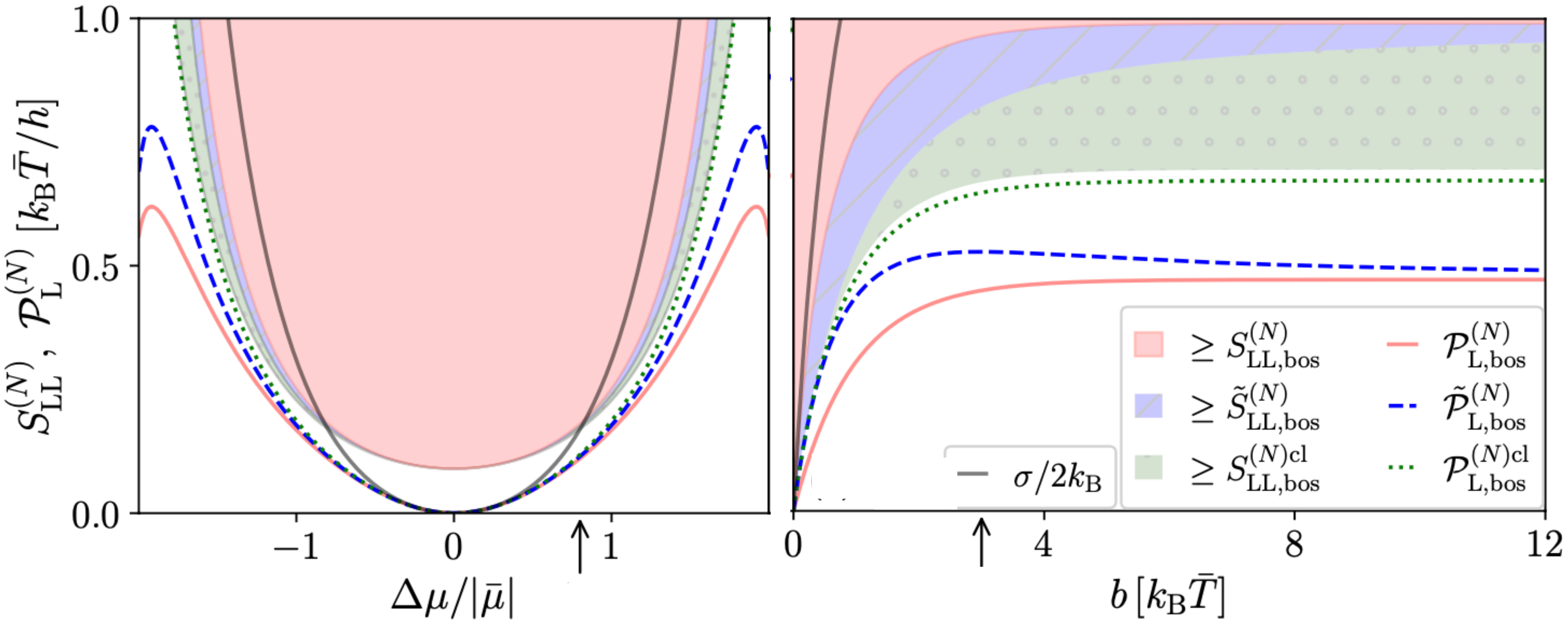
D. Palmqvist, L. Tesser, J. Splettstoesser: arXiv:2410.10793 (2024).

Precision bounds for bosons



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- Tight bounds for large bias (high precision) and small bandwidth (low precision)
- Tight when TUR is not and vice versa

D. Palmqvist, L. Tesser, J. Splettstoesser: arXiv:2410.10793 (2024).

Quantum noise and full fluctuations in fermionic systems

Fermions: $S_{\alpha\alpha} = S_{\alpha\alpha,\mathrm{cl}} - S_{\alpha\alpha,\mathrm{qu}}$

Noise can be fully supressed!!

D. Palmqvist, L. Tesser, J. Splettstoesser: arXiv:2410.10793 (2024).

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Precision bounds for fermions:

$$\frac{1}{R_\alpha^2} S^{(N)}_{\alpha\alpha,\mathrm{fer}} \geq \frac{\left(I_\alpha^{(\nu)}\right)^2}{S^{(\nu)}_{\alpha\alpha,\mathrm{fer}}} \equiv \mathscr{P}^{(\nu)}_{\alpha,\mathrm{fer}}$$

With minimum reflection:

$$R_\alpha \equiv \inf_{E \in A} D_{\alpha\alpha}(E)$$

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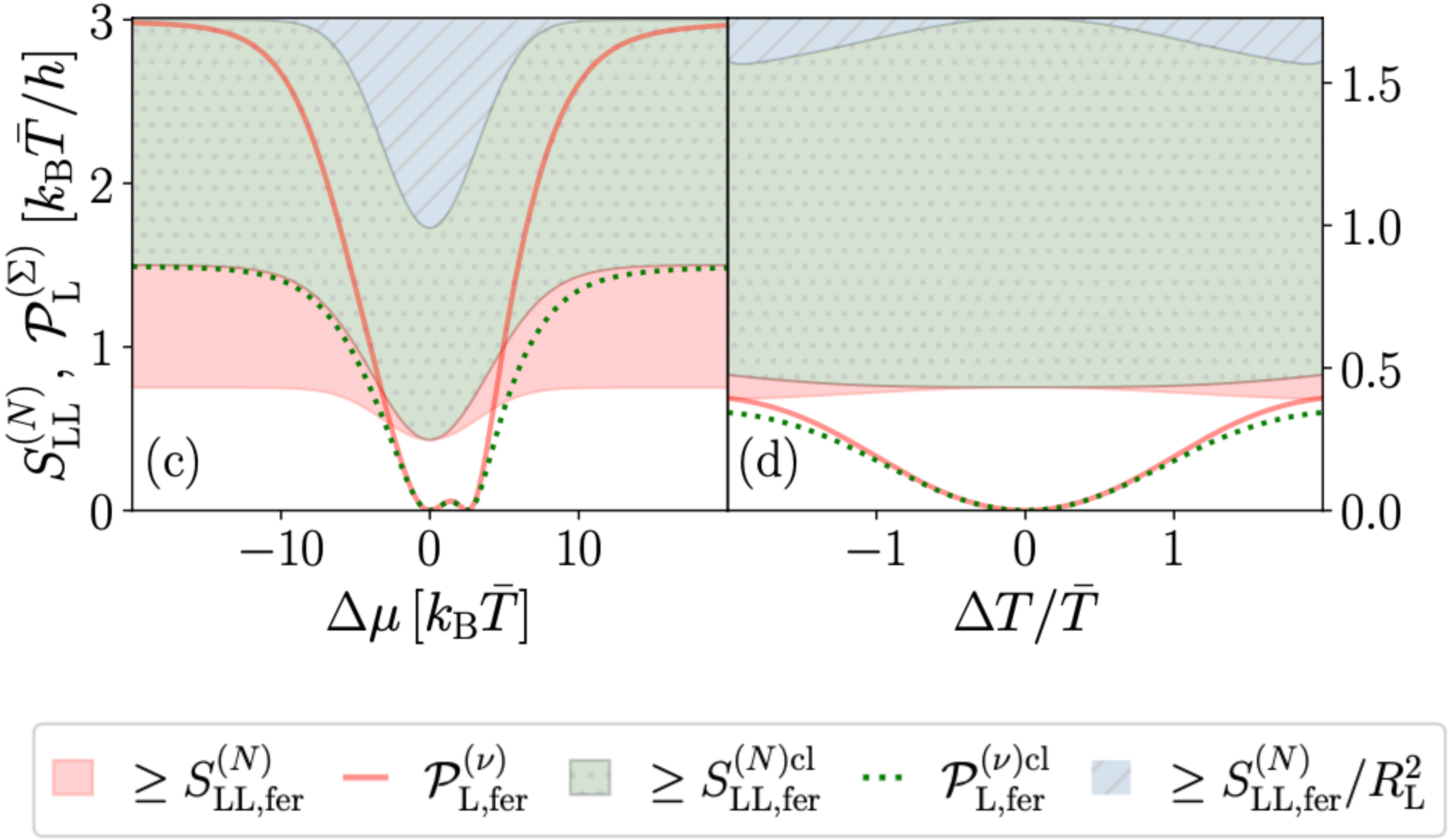

D. Palmqvist, L. Tesser, J. Splettstoesser: arXiv:2410.10793 (2024).

Acknowledgements/Announcements

Ludovico Tesser   Didrik Palmqvist

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WALLENBERG ACADEMY FELLOWS

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WALLENBERG ACADEMY FELLOWS

Master projects in the group

…possible travel/housing support

Conclusions

- New challenges and new opportunities in nanoscale heat engines
- Novel goals (precision) and resources (nonthermal/ fluctuations)
- **Fluctuation-dissipation bound** for nonequilibrium steady-state heat engines

L. Tesser, J. Splettstoesser: Phys. Rev. Lett. **132**, 186304 (2024).

- **Kinetic uncertainty relations** - valid for generic currents, nonthermal resources
- Strongly different quantum modifications for fermions/bosons — expressed in terms of measurable quantities

D. Palmqvist, L. Tesser, J. Splettstoesser: arXiv:2410.10793 (2024).

popular science summary:

noise in steady-state heat engines