EN ESSAYANT CONTINUELLEMENT ON FINIT PAR REUSSIR. DONC: PLUS GA RATE, PLUS ON A DE CHANCES QUE GA MARCHE.

Probabilistic work extraction with singleelectron devices

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Outline

I. Context

- II. Single-electron device energetics
- III. Work extraction bounds with quenches
- IV. Work extraction with « gambling » strategy



> System energy change $\Delta U = W + Q$ (first law)



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2nd law by Kelvin:

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2nd law by Kelvin:

"It is impossible to convert heat completely in a cyclic process." For a system connected to a single heat bath, under an isothermal transformation:

$$W \ge \Delta F = \Delta U - T \Delta S_{sys}$$



System Hamiltonian H_{λ} [{ $q_k(t)$ }], λ : control parameter, { q_k }: system degrees of freedom



- System Hamiltonian H_{λ} [{ $q_k(t)$ }], λ : control parameter, { q_k }: system degrees of freedom
- Nonequilibrium work done by the operator through λ variation in time (« protocol »)

$$W[\{q_k(t)\},\lambda(t)] = \int_{t_i}^{t_f} dt \,\dot{\lambda}(t) \frac{\partial H_{\lambda}[\{q_k(t)\}]}{\partial \lambda}$$

Operator \xrightarrow{W} System

$$W[\{q_k(t)\},\lambda(t)] = \int_{t_i}^{t_f} dt \,\dot{\lambda}(t) \frac{\partial H_{\lambda}[\{q_k(t)\}]}{\partial \lambda}$$







t

- Isolated system: deterministic trajectory, invariant upon time-reversal
- Heat bath: fluctuating degrees of freedom
 - → fluctuating nonequilibrium work
- > Relevant for small (~ $k_B T$) energy scales, few DOF













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 \rightarrow We use a highly fluctuating tunable system, e.g. a single-electron box



> Small metallic island, with small capacitance C_{Σ} dominated by tunnel junctions

NIS tunnel junction



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Use of superconductor: energy gap = low rates

> Thermally activated tunneling at $T_{S,N} \neq 0$, even with $\mu_N = 0$: role of thermal fluctuations



- Small metallic island, with small capacitance C_{Σ} dominated by tunnel junctions
- ► Ultrasmall junctions (area < 100 nm x 100 nm): $C_{\Sigma} \le 1$ fF
- > Energy cost of tunneling: charging energy $E_c = \frac{e^2}{2C_{\Sigma}}$
- > Two junctions: SINIS transistor for transport measurements $\rightarrow E_c$, R_T can be measured



- ► Hamiltonian for equivalent circuit: $H(n, n_g) = E_C (n n_g)^2$
- > Tunable electrostatic energy with gate voltage $n_g = \frac{C_g V_g}{e}$



➢ $E_c \sim 1$ K: occupation of two charge states $N_0 + n, n = 0,1$ for $n_g \in [0; 1]$ below 1K





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➢ Gate driving n_g(t) = work applied
$$W[n(t), n_g(t)] = \int dt \dot{n}_g(t) \frac{\partial H}{\partial n_g}$$





n=1









Master equation:

$$\begin{cases} \partial_t p_0 = -\Gamma^+ p_0 + \Gamma^- p_1 \\ \partial_t p_1 = -\Gamma^- p_1 + \Gamma^+ p_0 \end{cases}$$

Detailed balance at equilibrium:

$$\frac{p_0}{p_1} = \frac{\Gamma^-}{\Gamma^+} = e^{\Delta E(n_g)/k_B T}$$



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\rightarrow For some trajectories, one can have work extraction, $W < \Delta F$







Requirements for the driving protocol over a cycle:

> Bound on minimum work extracted $W^- - \Delta F$ if successful attempt

Work applied
$$W[n(t), n_g(t)] = \int_0^T dt \dot{n}_g(t) \frac{\partial H}{\partial n_g}$$



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- ➤ Maximum probability $p_{W^+}(W \le W^-)$ allowed by Jarzynski's equality

$$p_{W^+}(W \le W^-) \le \frac{e^{\Delta F/k_B T} - e^{-W^+/k_B T}}{e^{-W^-/k_B T} - e^{-W^+/k_B T}}$$
 V. Cavina et al., Sci. Rep. (2016)


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- Quench: fast = no heat exchanged, only work
- Thermalization: no work done, heat removed from/released to the bath



 $(\omega_{H_a}, E_b|1\rangle\langle 1|)$

 E_{h}

 $E_{\rm fin}$

DTT

E

 $E_{\rm in}$

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- Quench: fast = no heat exchanged, only work
- Thermalization: no work done, heat removed from/released to the bath
- Large quench = splits in two the work distribution

$$p(W) = p_0 \delta(W - W^+) + (\mathbf{1} - \mathbf{p_0}) \delta(W - W^-)$$



 \succ Two « reversible » ramps, time interval \gg tunneling time

Quench time « tunneling time: no heat exchange



 \succ Two « reversible » ramps, time interval \gg tunneling time

- Quench time « tunneling time: no heat exchange
- ► Closed driving cycle: $W = -Q = -\sum_i \Delta E[n_g(t_i)]\Delta n(t_i)$, with *Q* obtained from the jump record \rightarrow work done over one trajectory can be inferred
- ca. 1000 repetitions = distribution of work fluctuations



Theory for truly QS ramps: $W(n_q) = (1 - 2n_q)\Delta E(n_g^*)$





$$W^{\pm} = \mp \Delta E(n_g^*)$$



> Depends only on state n_q at quench onset:





$$W^{\pm} = \mp \Delta E(n_g^*)$$

after

> Depends only on state n_q at quench onset: win if excited state





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> Depends only on state n_q at quench onset:







- \succ Δ $n_g = n_g^* 1/2 = 0.11$ → small quench amplitude
- Finite peak width: imperfect quasistatic ramp



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- Finite peak width: imperfect quasistatic ramp



- \succ Δ n_g = 0.17 → large quench amplitude
- Larger « violations », but smaller probability



- > Probability of violation decreases with quench amplitude $\Delta n_g = n_g^* 1/2$
- ➢ Weights: Gibbs functions p₀, 1 − p₀ for favorable/unfavorable state just before the quench



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- > Average work performed on system positive: in agreement with second law ($\Delta F = 0$ for our closed cycle)
- Increases with quench amplitude: more irreversibility introduced

O. Maillet et al., Phys. Rev. Lett. 122, 150604 (2019)



- Heat dissipated by imperfect quasi-static driving: broadening
- Additional irreversibility increases with a steeper ramp slope



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- > Master equation approach: time evolution of work probability distribution $\rho(W, t)$
- Good agreement with data, no free parameter



$$W(n_q) = (1 - 2n_q)\Delta E(n_g^*)$$

Depends only on state at quench onset: win if excited state = less likely

- \succ Work extracted can be arbitrarily close to E_C if successful
- \succ The larger, the less probable



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The larger, the less probable = how to make extraction more probable ?

A neat limit



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V. Cavina et al., Sci. Rep. (2016)

A neat limit



If we loosen requirements: $W^- \to \Delta F$, $W^+ \to \infty$

> Maximum probability $p_{\infty}(W \leq W^{-})$ allowed by Jarzynski's equality

$$p_{\infty}(W \le W^{-}) \le e^{-\frac{W^{-} - \Delta F}{k_{B}T}} \xrightarrow[W^{-} \to \Delta F]{1}$$

A neat limit



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$$p_{\infty}(W \le W^{-}) \le e^{-\frac{W^{-} - \Delta F}{k_B T}} \xrightarrow[W^{-} \to \Delta F]{1}$$

 \rightarrow « Violation » probability can be made arbitrarily close to 1!



 $W(n_q) = k_B T(\Delta S)_{quench} + \Sigma(n_q)$ n_q : charge state at quench onset



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 $(\Delta S)_{quench} = S(n_{g,b}) - S(n_{g,a})$



$$W(n_q) = k_B T(\Delta S)_{quench} + \Sigma(n_q)$$

 $(\Delta S)_{quench} = S(n_{g,b}) - S(n_{g,a})$

S: Shannon entropy for a TLS:

 $S(n_g) = -(1 - p_0(n_g)) \ln(1 - p_0(n_g)) - p_0(n_g) \ln p_0(n_g)$





$$W(n_q) = \underbrace{k_B T(\Delta S)_{quench}}_{<0!} + \Sigma(n_q)$$

 \succ Σ(n_q) > 0 if $n_{g,b} > n_{g,a} > 1/2$

 $\succ \Sigma(n_q = 1) < -k_B T(\Delta S)_{quench}$







$$W(n_q) = \underbrace{k_B T(\Delta S)_{quench}}_{<0!} + \Sigma(n_q)$$

≻ Σ $(n_q) > 0$ if $n_{g,b} > n_{g,a} > 1/2$

$$\succ \Sigma(n_q = 1) < -k_B T(\Delta S)_{quench} \rightarrow \text{win}$$

$$\succ \Sigma(n_q = 0) > -k_B T(\Delta S)_{quench}$$





$$V < 0$$

$$n_q = 0 \qquad 1$$



- Ist term: Shannon entropy decreases away from degeneracy
- ➤ W(n_q = 1) < 0: probability of work extraction = ground state probability at the quench onset = favorable!</p>



O. Maillet et al., Phys. Rev. Lett. 122, 150604 (2019)

> 65 % of successful, « violation » events $(W < \Delta F)$





O. Maillet et al., Phys. Rev. Lett. 122, 150604 (2019)

▶ 65 % of successful, « violation » events $(W < \Delta F)$

- Efficiency limited by irreversible driving
- No theoretical bound to 99,999... % probable work extraction: optimization required (RF SET, longer ramps...)!
- Statistics matters!

Partial summary

Hybrid normal-superconducting single-electron box with simple energetics as a model system for stochastic thermodynamics

- O. Maillet *et al.*, Phys. Rev. Lett. **122**, 150604 (2019)
- Optimal protocol to extract work from thermal fluctuations far beyond the 2nd law prescription



Average work ?



• $\langle W \rangle \ge \langle \Delta F \rangle$ if all protocols have the same duration...

Average work ?



- $\langle W \rangle \ge \langle \Delta F \rangle$ if all protocols have the same duration...
- What if we stop driving when we are happy about the work output ? (« A gambler who knows when to walk away »)





Average work ?



- $\langle W \rangle \ge \langle \Delta F \rangle$ if all protocols have the same duration...
- What if we stop driving when we are happy about the work output ? (« A gambler who knows when to walk away »)



 \rightarrow for all trajectories, $W \leq W_{th}$. What about $\langle W \rangle_{\rm T}$?



Gambling demon



• Stochastic free energy incl. $p_{n(t)}(t)$ (measured + ME)

 $\Delta F(\mathbf{T}) = \Delta U(\mathbf{T}) + k_B T \log p_{n(\mathbf{T})} (\mathbf{T}) / p_{n(0)}(0)$

Gambling demon



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- Small work threshold:
 - Many full trajectories with negative work


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 $\Delta F(\mathbf{T}) = \Delta U(\mathbf{T}) + k_B T \log p_{n(\mathbf{T})}(\mathbf{T}) / p_{n(0)}(\mathbf{0})$

- Small work threshold:
 - Many full trajectories with negative work
 - > Many trajectories stopped before ramp completed, at times $T \sim \Gamma_d^{-1}$



Gambling demon



• For low thresholds, $\langle W \rangle_{\rm T} < \langle \Delta F \rangle_{\rm T}$

→ redefinition of 2nd law for stopping-time trajectories: $\langle W \rangle_{\rm T} \ge \langle \Delta F \rangle_{\rm T} - k_B T \langle \delta \rangle_{\rm T}$

$$\langle \delta \rangle_{\mathrm{T}} = \frac{p_{n(\mathrm{T}-\tau)}(\mathrm{T}-\tau)_{n_g(\mathrm{T}-\tau)}}{n(\mathrm{T})_{n_{g(\mathrm{T})}}}$$

Stochastic distinguishability between forward and backward-under-time-reversed-protocol trajectory

Gambling demon



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Stochastic distinguishability between forward and backward-under-time-reversed-protocol trajectory

Longer ramp time (= more quasi-static): smaller
« violation »

G. Manzano et al., Phys. Rev. Lett. 126, 060803 (2021)

Generalized Jarzynski's Equality



- Violation of standard JE for stopping-time trajectories
- JE restored when including stochastic distinguishability :

$$\left\langle e^{-\beta(W-\Delta F)-\delta}\right\rangle_{\mathrm{T}}=1$$

G. Manzano et al., Phys. Rev. Lett. 126, 060803 (2021)

Summary

- Hybrid normal-superconducting singleelectron box for stochastic thermodynamics
- Optimal protocol to extract work from thermal fluctuations far beyond the 2nd law prescription

O. Maillet et al., Phys. Rev. Lett. 122, 150604 (2019)

- Stopping-time « gambling » strategy in fast ramps to favor average work extraction
- Extended Jarzynski equality and 2nd law bound including stochastic stopping time

G. Manzano et al., Phys. Rev. Lett. 126, 060803 (2021)









Thank you !





Jukka Pekola





Paolo Erdman



Christopher Jarzynski







Gonzalo Manzano Edgar Roldán Rosario Fazio

- Colloidal particle in an harmonic trap
- Experimental demonstration of second law « violations » at short timescales (black data)



Double (quantum) dot = direction of electron tunneling = entropy measurements



B. Küng *et al.*, Phys. Rev. X (2012)

S. Singh *et al.*, Phys Rev. B (2019)



SET

b

ne

- Test of Jarzynski equality with a single electron box
- Gate driving cycle, measurement of heat exchange (tunneling events) during the cycle







- Szilard engine: feedback on system driving applied using the information gained by a detector (= Maxwell Demon operation)
- Work extracted from the system on average, close to Landauer limit (-kTlog2)
- > Not a true violation of 2nd law: entropy created in the Demon («cost of information»)







