QUANTUM ENGINEERING TOULOUSE

LCAR Laboratoire Collisions Agrégats Réactivité

Université de Toulouse

UNIVERSITÉ TOULOUSE III PAUL SABATIER

cnrs

in collaboration:

icb Laboratoire Interdisciplinaire Carnot de Bourgogne

Experiments with Bose-Einstein condensates

From atom cooling to quantum control and simulation

QuanTEEM Winter School
Dijon, 24/02/2025

Bruno Peaudecerf (CNRS)

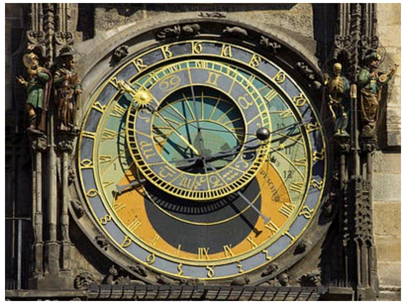
Lecture III

Quantum simulation

Introduction

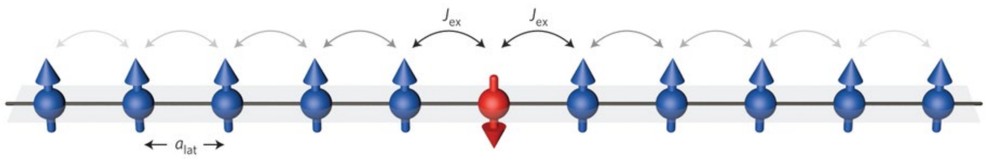
- In general, we can try to **calculate** or **simulate** a phenomenon to gain understanding

  “Classical” simulation:
  astronomical clocks



Identify mechanisms
+ implement them
in analog device

- Many quantum phenomena are **hard to calculate** on a (classical) computer.



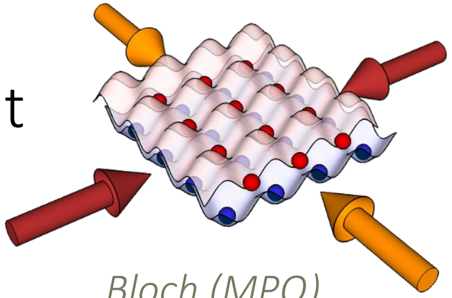
T. Fukuhara *et al., Nature Physics* **9**, 235–241 (2013)

Quantum simulation
Nature isn’t classical, (...), and if you want to make a simulation of nature, you’d better make it quantum mechanical.

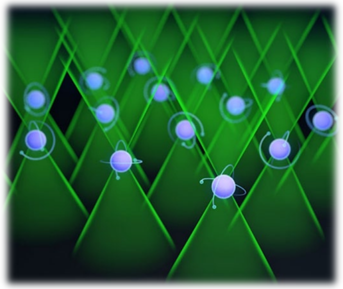
R.P. Feynman, *Int. J. Theor. Phys.* 21, 467488 (1982)

Introduction

- Bose-Einstein condensates (ultracold atoms generally) are a **versatile platform for quantum simulation :**
  - **Potentials** can be engineered with light (e.g. lattice without defects)



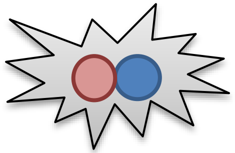
Bloch (MPQ)



Optical lattices
Tweezer arrays

Kaufman (JILA)

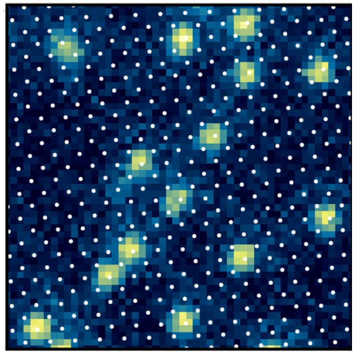
  - **Interactions** can be controlled



Vary density,
Magnetic (Feshbach) resonances

  - **State** can be prepared and measured with precision

Single fermions
on a 2D lattice



Kuhr (Strathclyde, Glasgow)

Outline

- Examples of quantum simulation with cold atoms
- Localization and coherent scattering
- Realization with a quantum pendulum
- Experiments with BEC

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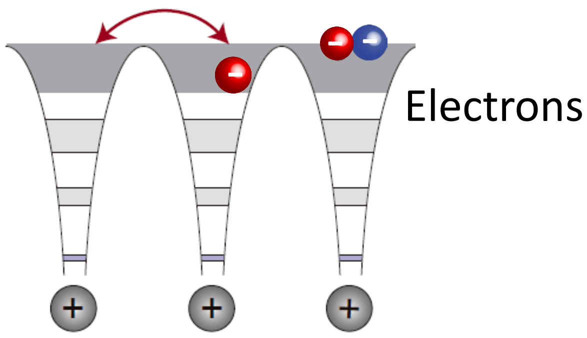
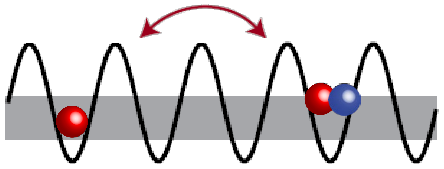
Many-body physics : the Mott transition

Mott insulator: a conducting material becomes insulating at low temperature due to electron-electron interactions.

Simulation: cold atoms in an optical lattice

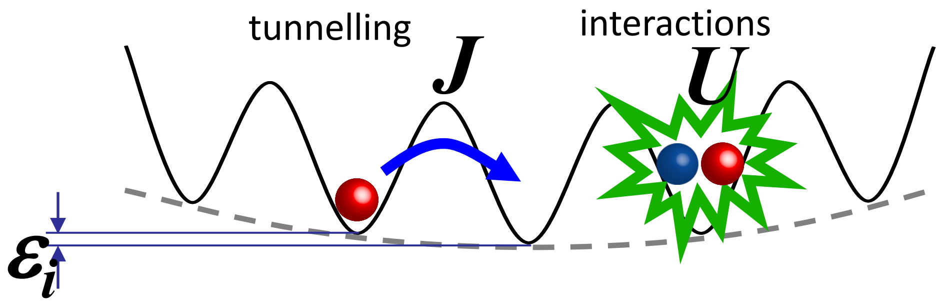


Potential from ions



: Atoms

Potential from lasers

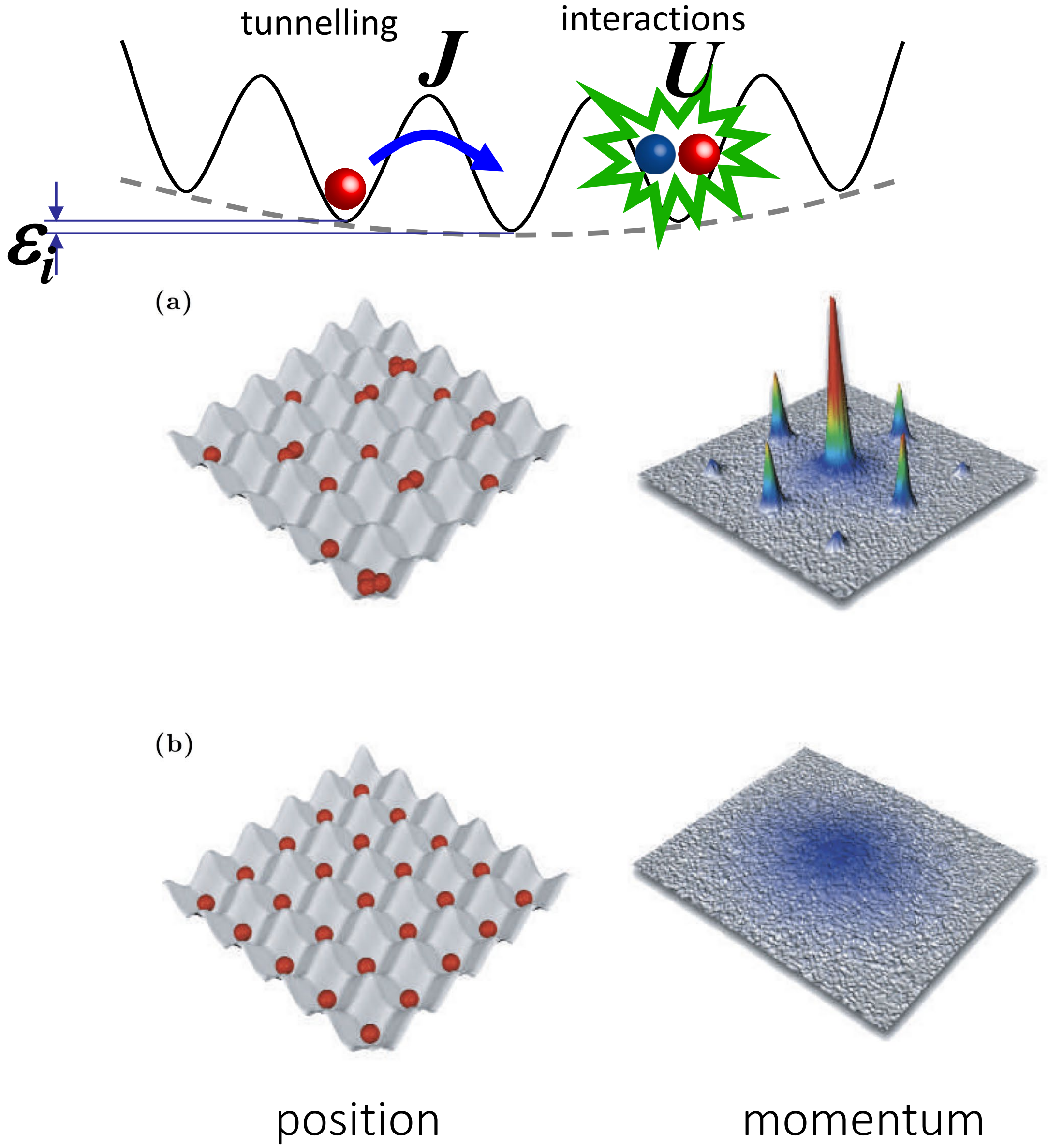


Strong interactions: the system can be described by the **Hubbard Hamiltonian**:

$$\hat{H} = -J \sum_{<i,j>,\sigma} \hat{c}^{+}_{i,\sigma} \hat{c}_{j,\sigma} + U \sum_{i} \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} - \sum_{i,\sigma} \left(\mu - \varepsilon_{i,\sigma} \right) \hat{n}_{i,\sigma}$$

Competition between *delocalization* from tunneling and increased *interaction*

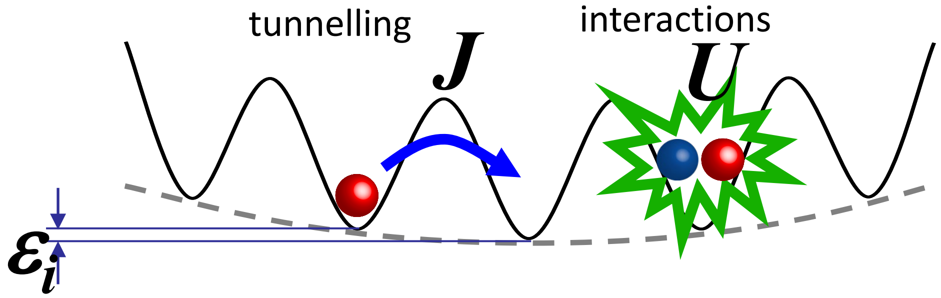
Many-body physics : the Mott transition



Increasing density
→ energetically favorable for atoms to localize on the sites of the lattice

Quantum phase transition
between a *delocalized superfluid* (BEC)
→ peaks in the momentum distribution
and *localized atoms, without global coherence*
→ momentum distribution of a “single well”

Many-body physics : the Mott transition



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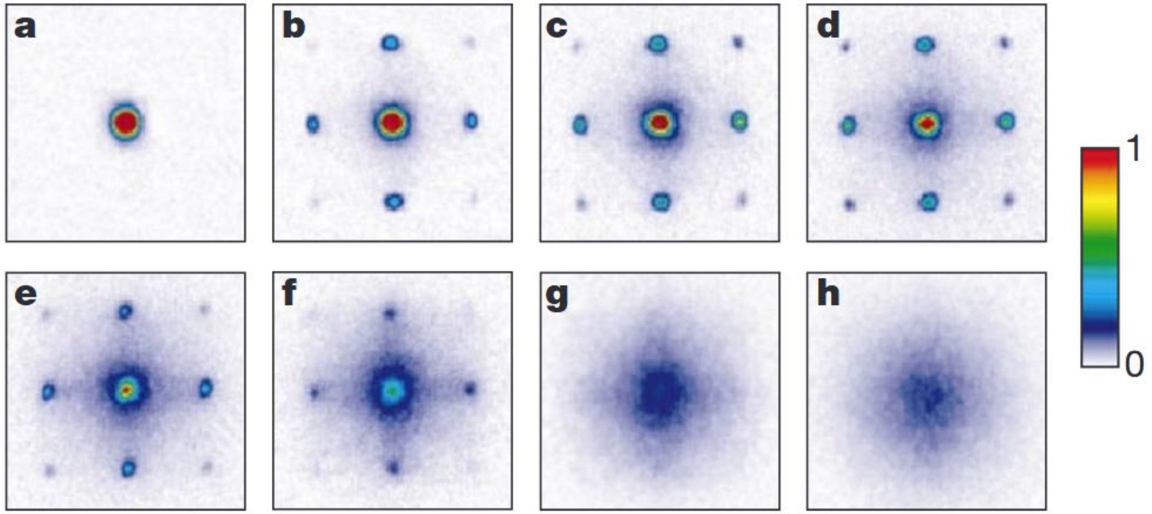
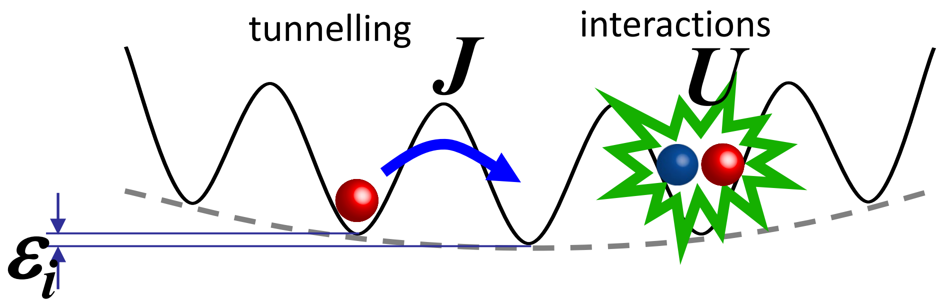


Quantum phase transition

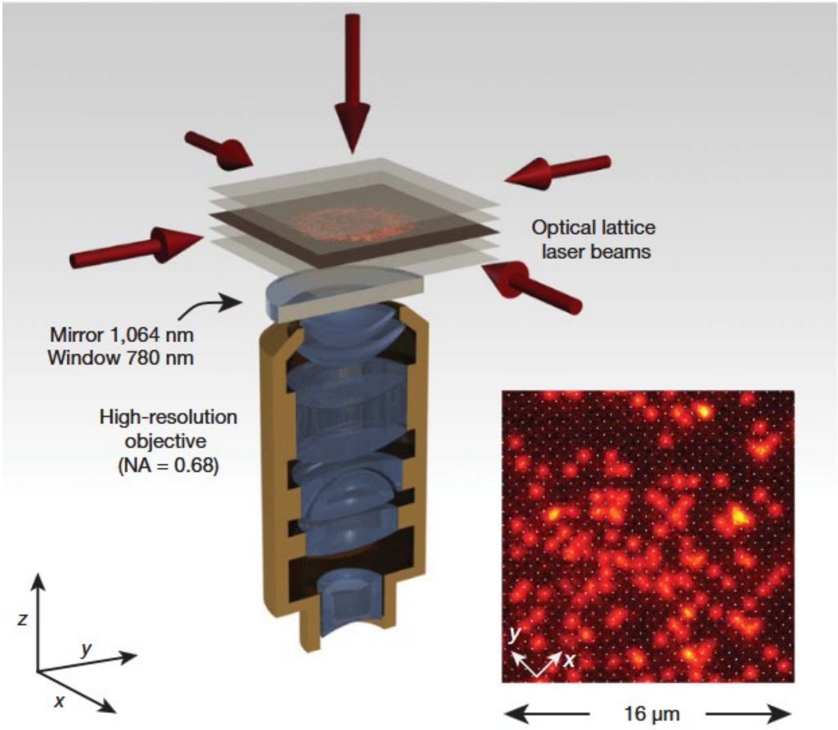
between a *delocalized superfluid* (BEC)
→ peaks in the momentum distribution
and *localized atoms, without global coherence*
→ momentum distribution of a "single well"

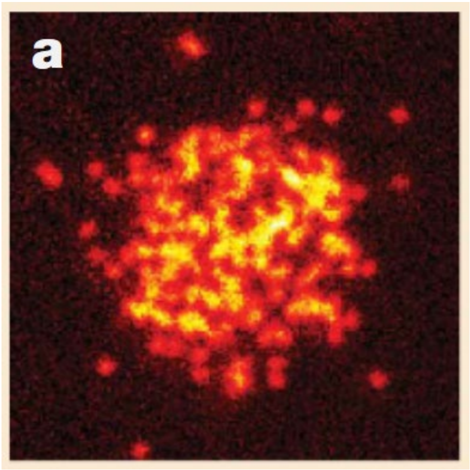
First experimental observation:
Greiner et al., Nature **415**, 39 (2002)
The birth of quantum simulation

Many-body physics : the Mott transition

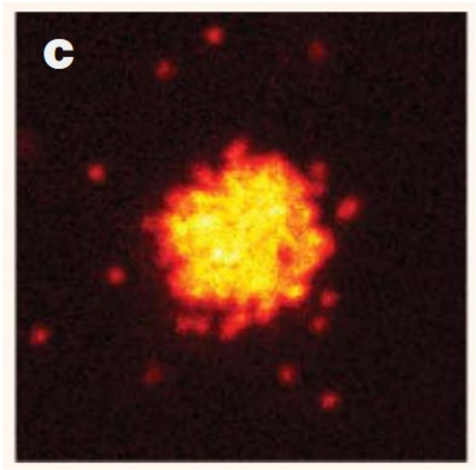


A few years later: quantum gas microscope allows to look at the spatial distribution of atoms.





BEC



Mott insulator

J. Sherson et al., Nature **467,** 68 (2010)

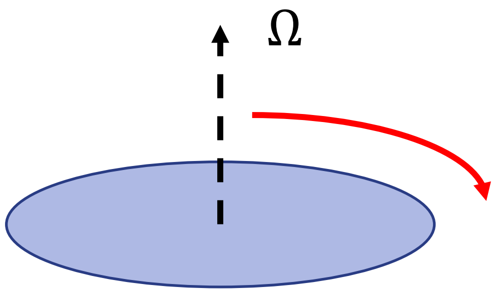
Artificial magnetic fields

Lorentz force on charged particle in a magnetic field:

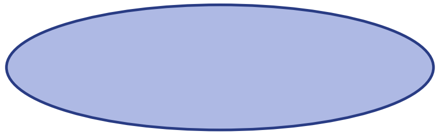
$$F = q\mathbf{v} \times \mathbf{B}$$

Neutral atoms have no charge!

But:



$\Leftrightarrow$



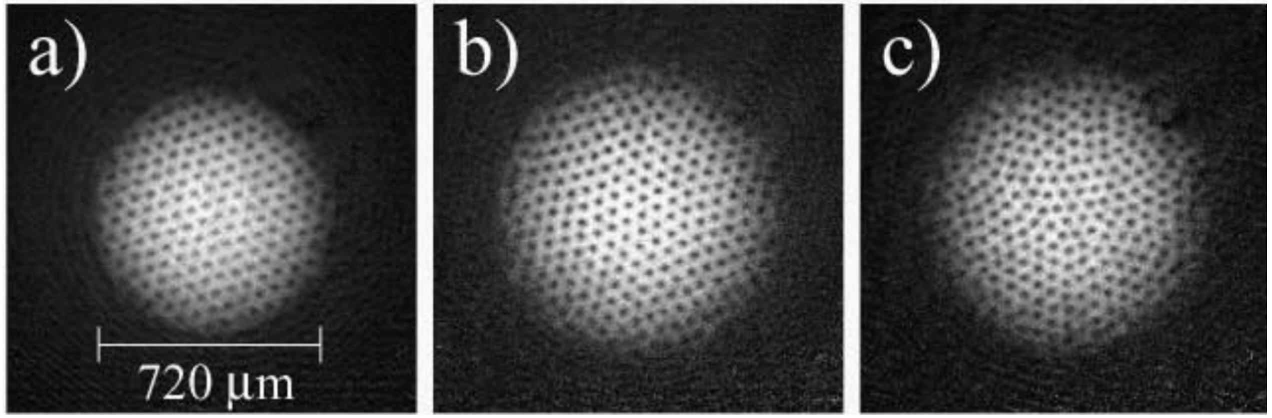
Rotating trap
in the lab frame

Inertial force in
the trap frame
(Coriolis)

$$F = 2m\mathbf{v} \times \mathbf{\Omega}$$

Effective magnetic fields can be implemented by setting BECs in rotation

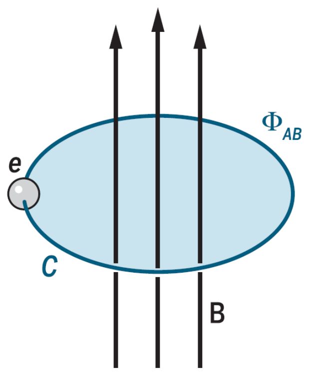
Artificial magnetic fields



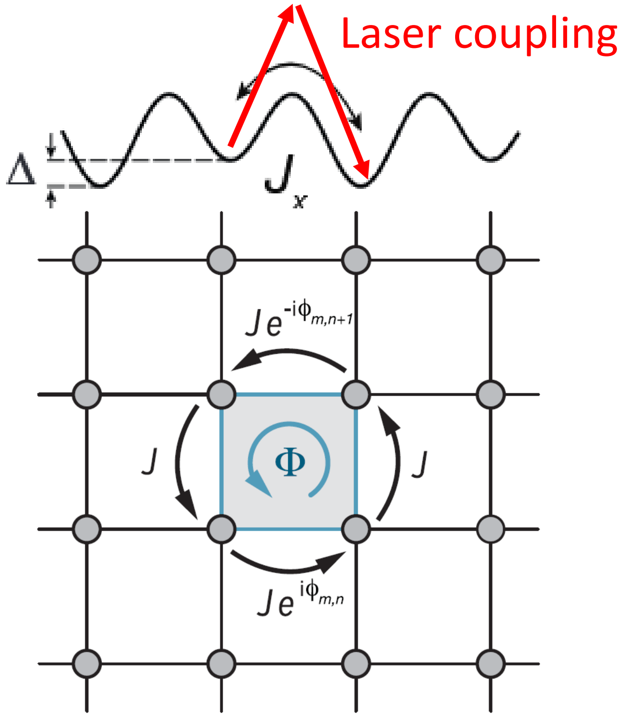
Engels *et al. Phys. Rev. Lett.* **90,** 170405 (2003) (Cornell group/JILA)

The way the condensate reacts to the rotation is to include *vortices,* zeroes in density around which the phase does a full turn: vortex=quantum of rotation

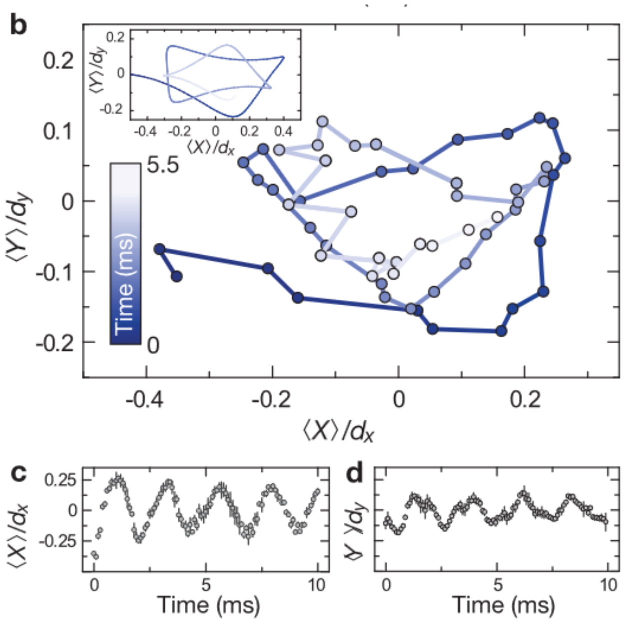
Artificial magnetic fields



Charged particles in a magnetic field acquire a *phase* around a closed trajectory C : *Aharonov-Bohm phase* $\Phi_{AB} = \int_C \boldsymbol{A}.d\boldsymbol{x}$



Engineer *complex-valued tunneling* with laser-induced transitions in a lattice
→ effective magnetic flux: controllable, staggered

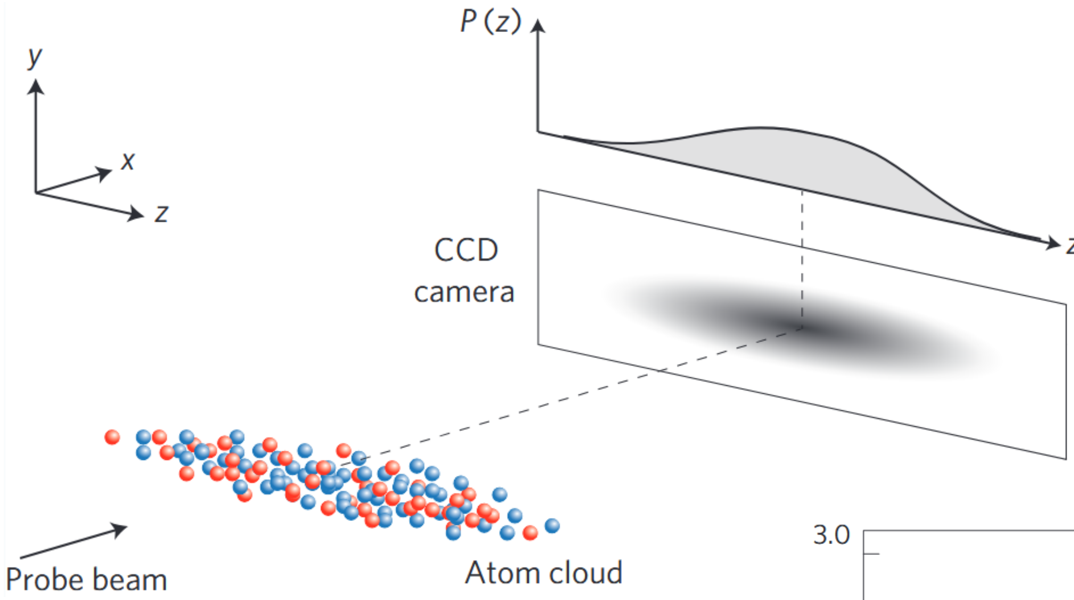


Physics of Hall effect:

Cyclotron-like orbits of cold atoms in a lattice with artificial magnetic field

M. Aidelsburger et al.,
Phys. Rev. Lett. **107**, 255301 (2011)

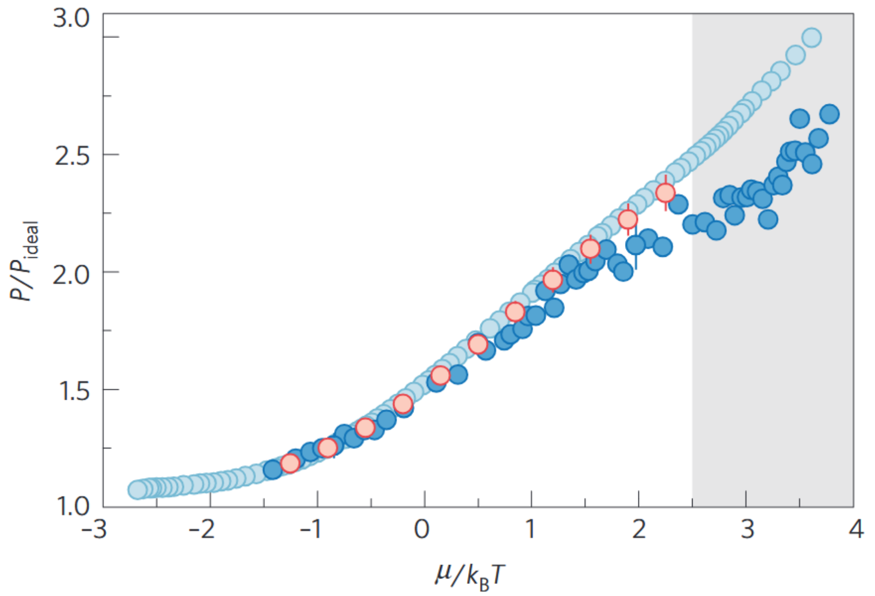
Fermionic systems



Ultracold *fermions* can also be prepared mixture of two states $|\uparrow\rangle$ and $|\downarrow\rangle$

Interactions (scattering length a_S) can be tuned via a ***Feshbach resonance*** $a_S(B)$

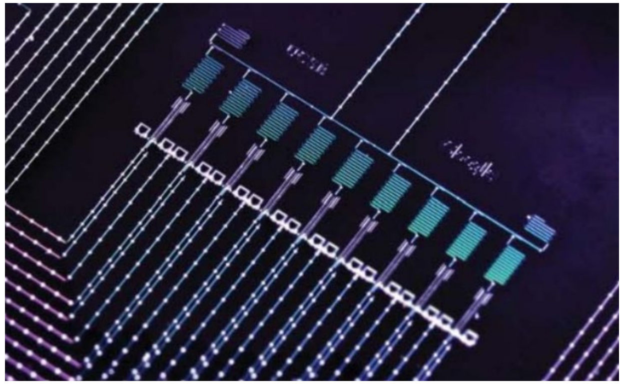
One can measure the equation of state $P(\mu, T)$ of the Fermi gas in the unitary limit $a \to \infty$



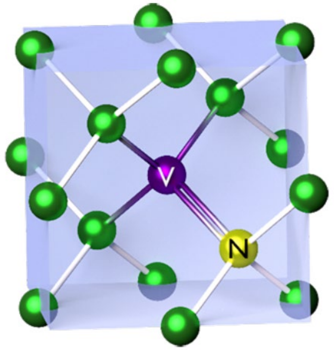
Data from Ku et al., Science **335**, 563 (2012)

A regime encountered in neutron stars

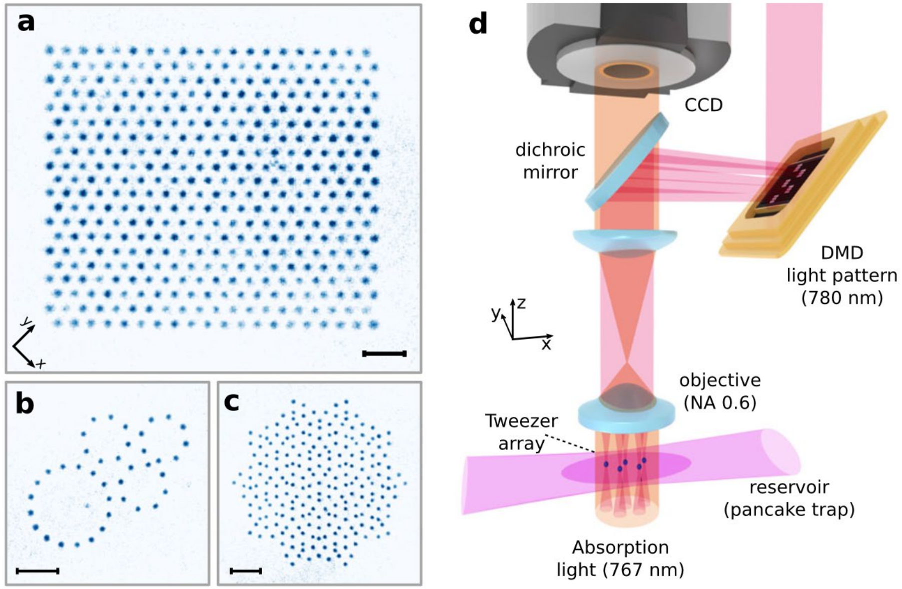
Many other systems



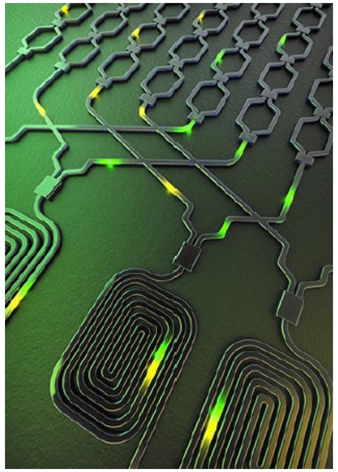
Superconducting qubits



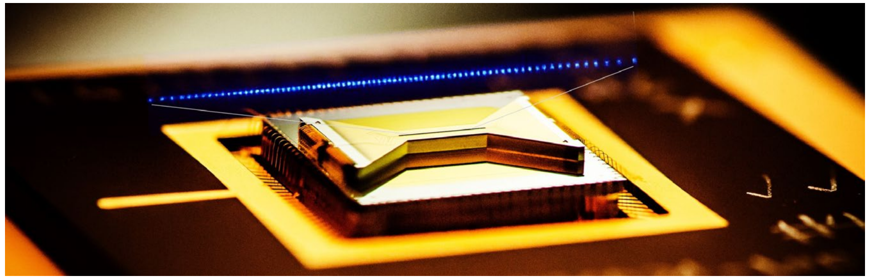
NV centers



Neutral-atom tweezer arrays



Photonic circuits



Trapped ions

Etc.

Localization effects

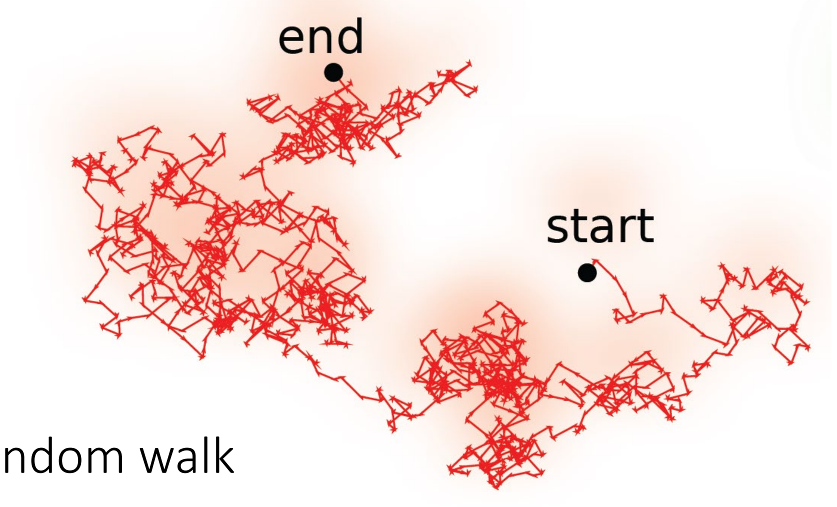
▪ A detour through condensed matter physics

How do we understand conduction in solids?

Drude model:

- Electrons feel the force of the electric field $F = -e\boldsymbol{E}$
- Electrons **scatter** randomly in **impurities/defects** in the solid with mean free path ℓ_s

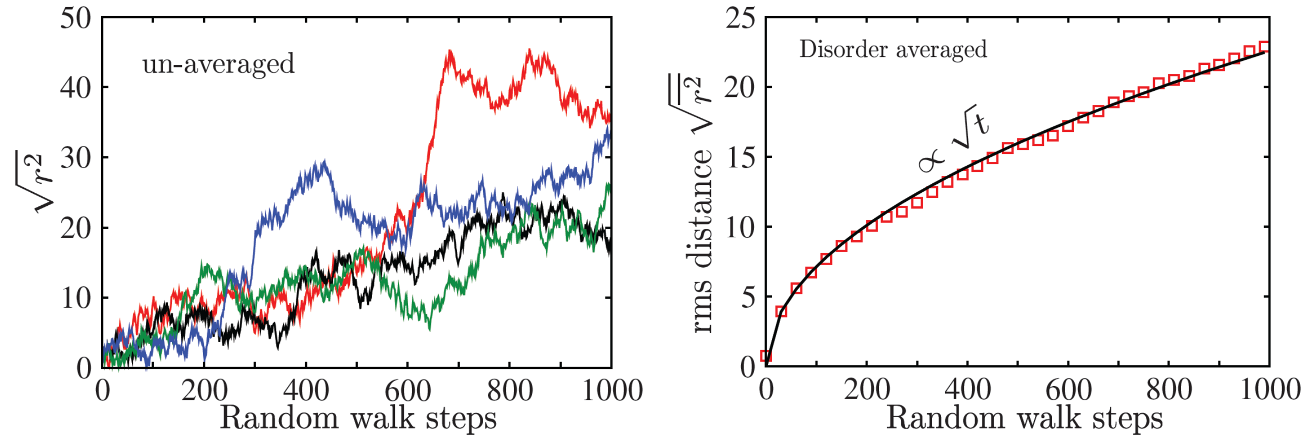
The electrons effectively perform a **random walk**:



Example of a 2D random walk

Diffusion contribution to conductivity

- **The random walk is associated to a diffusion process:**



In d dimensions, the average distance travelled is

$$\overline{r^2}(t) = 2d\mathbf{D}t$$

Diffusion constant

- The diffusion constant is a **key quantity for conductivity**

In general, $\boldsymbol{j} = \sigma\boldsymbol{E}$ with $\sigma = e^2 \times \rho(E) \times D(E)$ (E = typical energy of charge carriers)

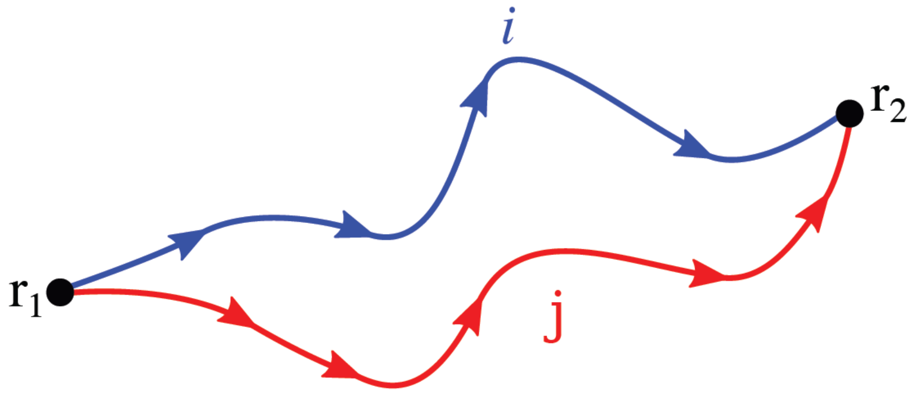
$\rho(E)$ = density of states of charge carriers
= **0** for an **insulator**,
high for a good conductor

$D(E)$ =Diffusion constant

$$D = \frac{v\ell_s}{d}$$

Diffusion with quantum particles

- If we think of the diffusion of **quantum** particles, we have to consider all **amplitudes of probability** connecting two points:



Transfer probability:

$$p_{1\to 2} = \left| \sum_i a_i(1 \to 2) \right|^2 \qquad (i = \text{possible paths})$$

$$p_{1\to 2} = \sum_i |a_i|^2 + \sum_{i\neq j} |a_i||a_j| e^{\iota(\theta_i - \theta_j)}$$

- To describe **typical** properties of solids, theorists perform an **average over disorder** (statistical average over position of scatterers)

$$\bar{p}_{1\to 2} = \sum_i \overline{|a_i|^2} + \sum_{i\neq j} \overline{|a_i||a_j| e^{\iota(\theta_i - \theta_j)}}$$

Interference term with (random) phase differences that depend on disorder

average $\simeq 0$

Classical term
(sum of probabilities)

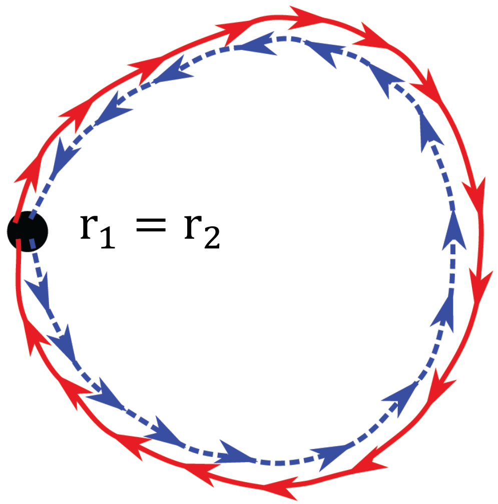
Weak localization

$$\bar{p}_{1\to 2} = \sum_i \overline{|a_i|^2} + \sum_{i\neq j} \overline{|a_i||a_j|e^{\iota(\theta_i-\theta_j)}}$$

$p_{\mathrm{class},1\to 2}$ average $\simeq 0$

Is there no difference between **particles** and **waves**?

There is, for looped trajectories:



If the dynamics is **time-reversible,** the reverse path has exactly the **same phase** as the direct path:

→ **non-zero interference contribution** (positive)

→ Enhanced **return probability:**

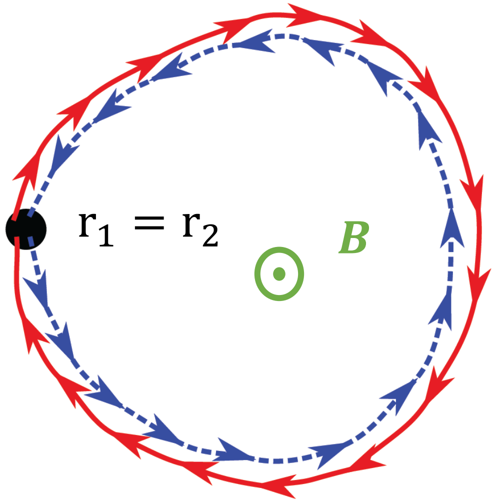
Weak localization

→ **Reduced diffusion**

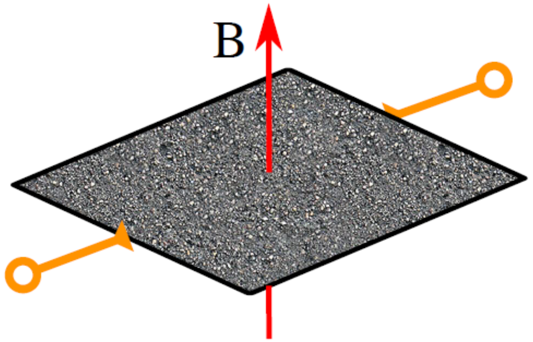
→ **Conductivity is decreased!** $\sigma = e^2\rho D$

Weak localization

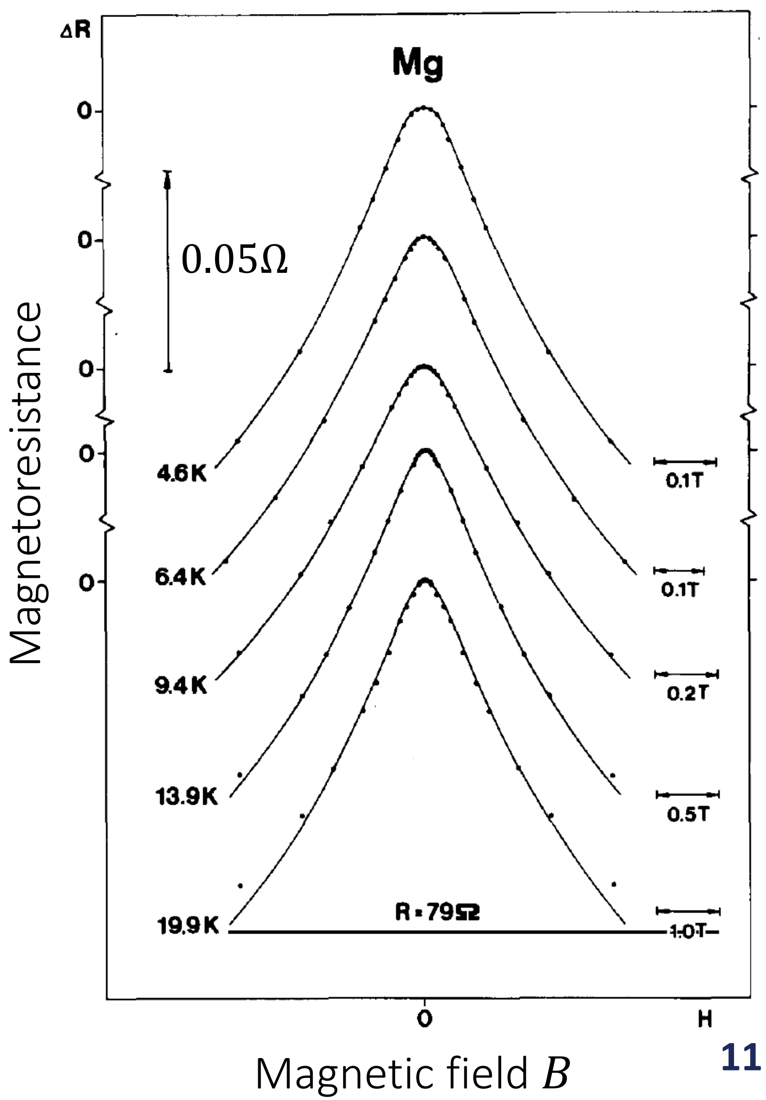
- The quantum effect of weak localization should decrease conductivity/increase resistance
- Experimental evidence? Place conductor in a magnetic field:break time-reversal symmetry



Experiment on a magnesium film (Bergman, Phys. Rep. 107, 1984)

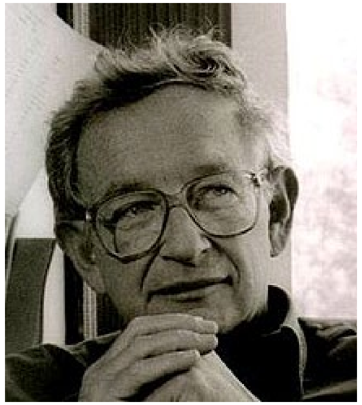


Breaking of weak localization reduces resistance!



Strong localization

- **Philip W. Anderson (1923-2020)**



In the presence of "strong enough" disorder,
total suppression of diffusion:

$$D = 0$$

A **cumulative effect of many loops:**
the probability to diffuse is **destroyed by destructive interference**

$$\sigma = e^2 \rho D = 0$$

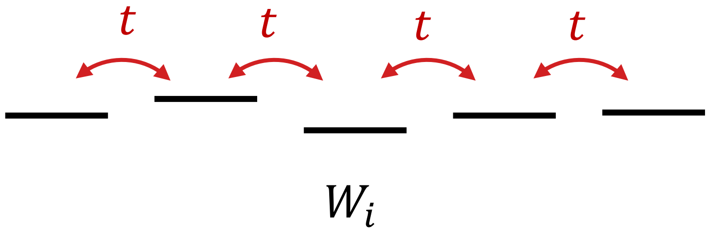
A peculiar **insulator**: charge carriers are available ($\rho \neq 0$), but cannot diffuse! **Anderson insulator**

Anderson model:

$$H = \sum_i \left[W_i |i\rangle\langle i| + t \left(|i+1\rangle\langle i| + |i\rangle\langle i+1| \right) \right]$$

Random on-site energy

Uniform coupling constant between sites



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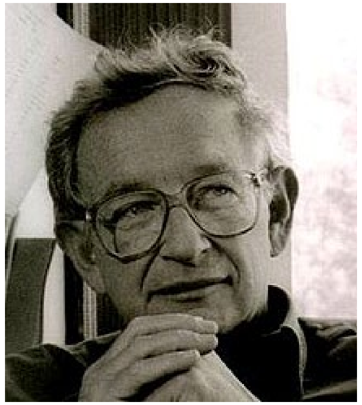

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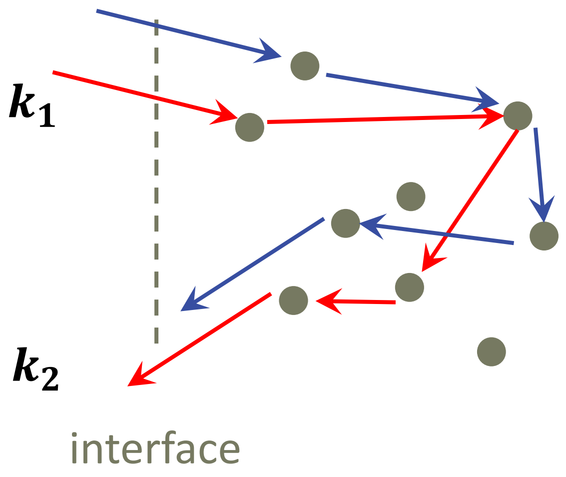
Theory predicts Anderson localization:
- **always** in true 1D and 2D systems
- Above a certain energy (mobility edge) in 3D

Signatures in reciprocal space

- We have considered transition probabilities between **positions** (diffusion) with scatterers randomly distributed in **space**
- What about **momentum**?
  Consider a plane wave, incident on the same kind of disordered medium



Two scattering paths contributing to diffusion from plane wave with $\mathbf{k_1}$ to $\mathbf{k_2}$

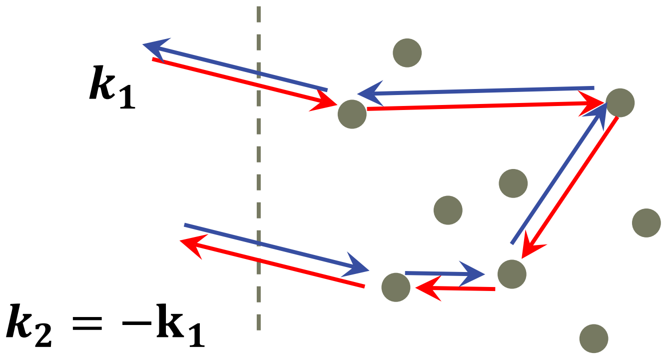
As before, to get the probability of transfer from 1 to 2, we need to sum amplitudes

$$p_{1\to 2} = \sum_i |a_i|^2 + \sum_{i\neq j} |a_i||a_j| e^{\iota(\theta_i - \theta_j)}$$

In general, the disorder average washes out the interference terms

$$\bar{p}_{1\to 2} \simeq \ p_{\text{class},1\to 2}$$

Coherent backwards scattering



Does plane wave scattering always behave classically? No!

$$p_{1\to2} = \sum_i |a_i|^2 + \sum_{i\neq j} |a_i||a_j|e^{\iota(\theta_i-\theta_j)}$$

For a **time-reversible system**, if $\boldsymbol{k_2} = -\boldsymbol{k_1}$, the reverse path contributes **constructively**

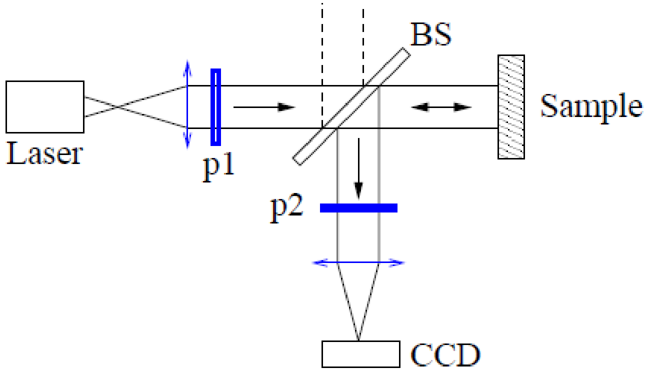
Doubling of the probability to scatter backwards:

Coherent Backwards Scattering (CBS)

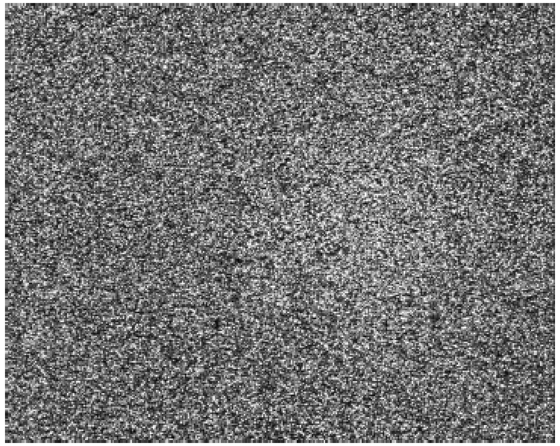
A **signature in reciprocal space** of **Weak Localization**

Coherent backwards scattering

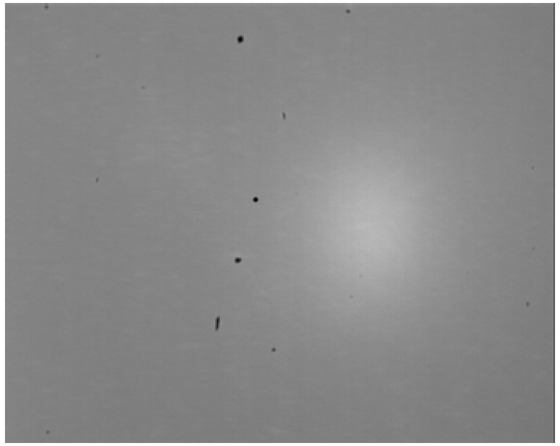
▪ A simple experiment:



From: "Disorder and interference",
C. Müller and D. Delande, Lecture at Les Houches, 2009

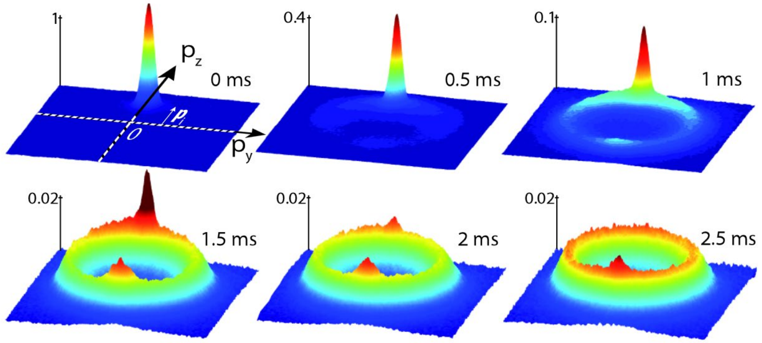


Single pattern:
speckle from multiple scattering



Average pattern:
CBS peak

Has been observed with multiple waves and scatterers:
light, acoustic, seismic, **matter waves**...

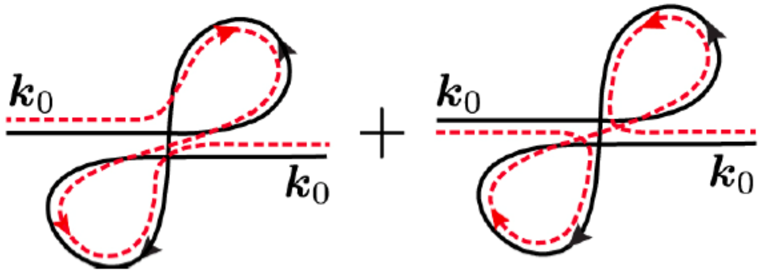


F. Jendrzejewski et al, PRL **109**, 195302 (2025)

Coherent forward scattering

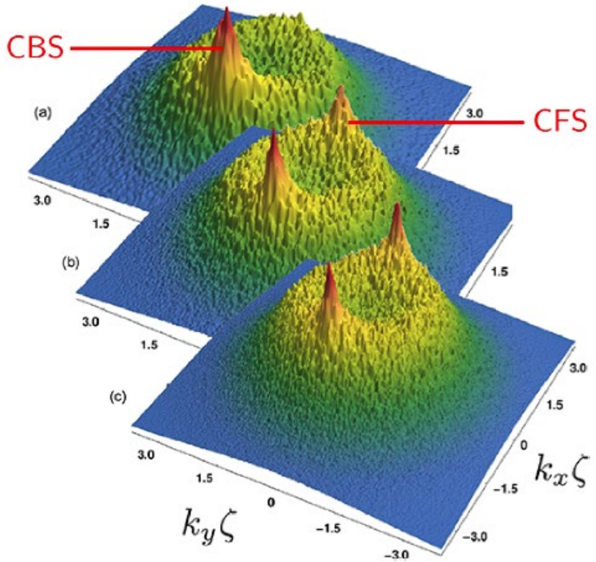
- Is there a signature in the scattering of plane waves of **strong localization?**

Yes, **Coherent Forward Scattering (CFS)**



Can be seen as a sort of **"double backscattering"**

Is associated with **strong (Anderson) localization**



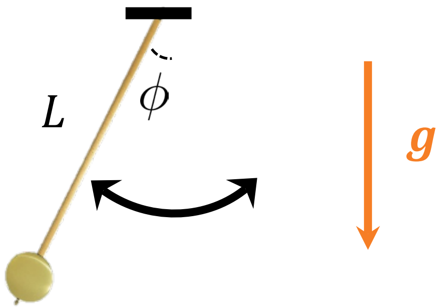
Has only been observed numerically, or *indirectly in experiments*

Karpiuk et al., PRL 109, 190601 (2012)

The quantum pendulum

- Dynamics of a pendulum:

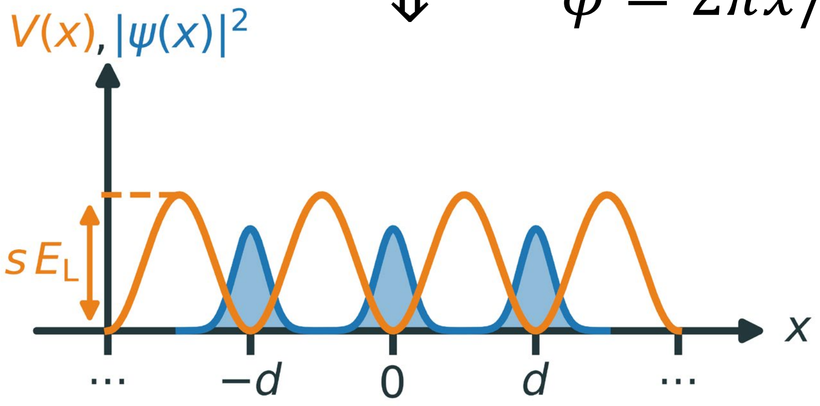
$$\hat{H} = -\frac{\hbar^2}{2mL^2}\frac{d^2}{d\phi^2} + mgL(1-\cos\phi)$$



$\Updownarrow \qquad \phi = 2\pi x/d$

Exactly analogous to the lattice potential

$$x \leftrightarrow \phi$$
$$s \leftrightarrow g$$



A "depth" variation $s(t)$ corresponds to moving the pivot point up and down

A "phase" variation $\cos(\phi + \varphi(t))$ corresponds to adding a forced rotation

Our system is also called
"shaken pendulum"
"shaken rotor"

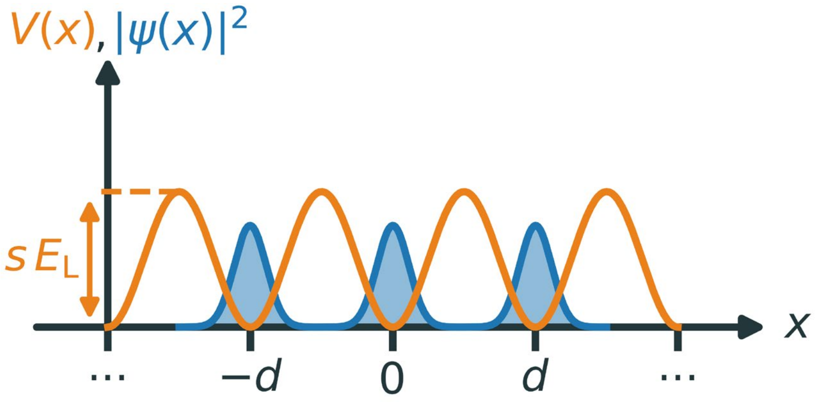
The quantum pendulum

Special connection between the **shaken pendulum** and **disorder physics**

Kicked rotor model:

$$\hat{H}(t) = \frac{\hat{p}^2}{2} - K\cos(\hat{x}) \sum_n \delta(t-n)$$

Periodic "kick" of infinite strength with the potential



(units chosen so that period $T \equiv 1$, spatial period $d \equiv 2\pi$

- What is the classical dynamics?

Between kicks, momentum is constant: call (x_n, p_n) the coordinates just before the kick at $t = n$:

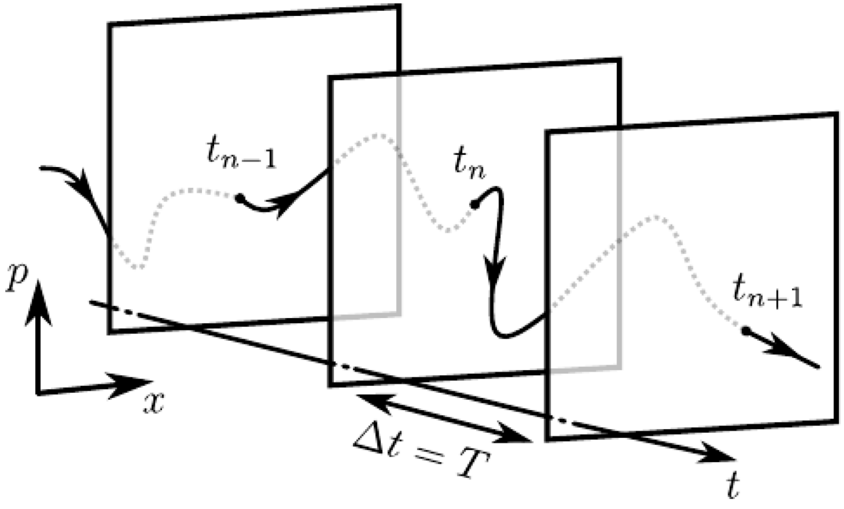
$$p_{n+1} = p_n - K\sin(x_n)$$
$$x_{n+1} = x_n + p_{n+1}$$

Standard map

Emblematic model of **chaotic dynamics**

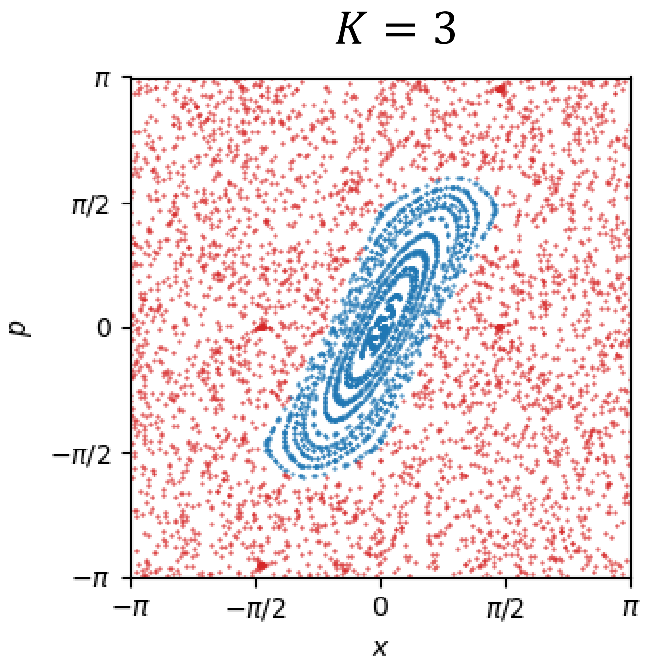
Classical dynamics of kicked rotor

- *Periodically modulated system*:
  represent the trajectory in phase space at each period



M. Martinez (2021)

$$p_{n+1} = p_n - K\sin(x_n)$$
$$x_{n+1} = x_n + p_{n+1}$$



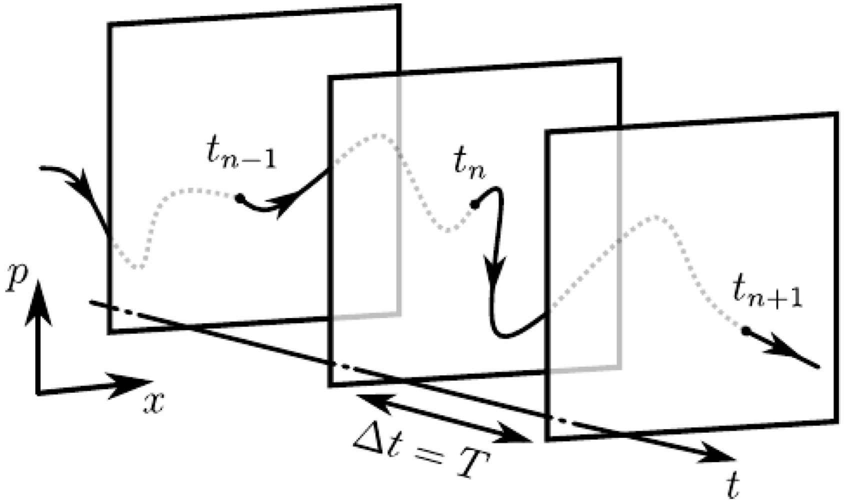
Stroboscopic phase portrait

- some trajectories remain close to an orbit: regular trajectories

- Some trajectories explore all of the phase space at random: chaotic trajectories

Classical dynamics of kicked rotor

- Periodically modulated system:
  represent the trajectory in phase space at each period



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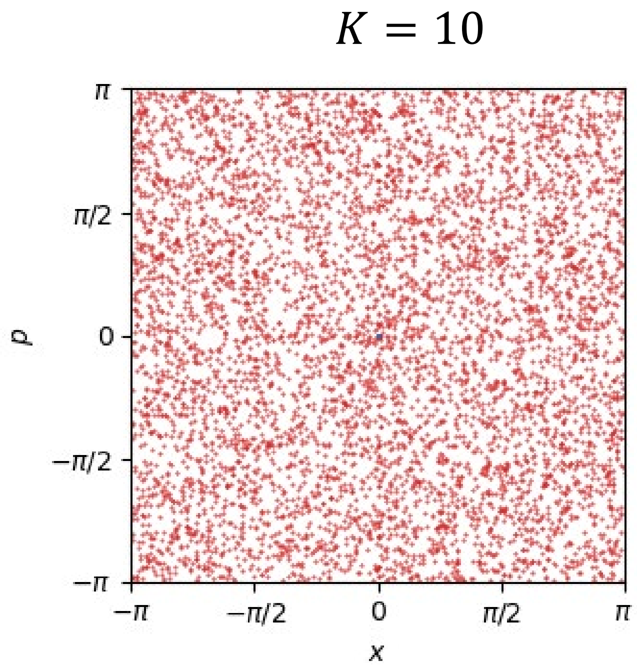


For a strong enough kick,
all trajectories are chaotic

Momentum evolves diffusively:

$$\overline{p^2} \simeq \frac{K^2}{2} t$$

Quantum dynamics

$\hat{H}(t) = \frac{\hat{p}^2}{2} - K\cos(\hat{x})\sum_n \delta(t-n)$

The **evolution operator** over 1 period is always the same:

$$U_T = \exp\left(-\frac{i\hat{p}^2}{2\hbar_e}\right)\exp\left(\frac{iK\cos(\hat{x})}{\hbar_e}\right)$$

Here $\hbar_e = -i[x,p]$ is an **effective Planck constant**:

$$\hbar_e = \frac{4\pi E_L T}{h}$$

State evolution : $|\psi(t=n)\rangle = U_T^n|\psi(0)\rangle$

Quantum dynamics

- **What does the evolution over one period do?**

$$U_T = \exp\left(-\frac{i\hat{p}^2}{2\hbar_e}\right)\exp\left(\frac{iK\cos(\hat{x})}{\hbar_e}\right)$$

Momentum state $|p = \ell\hbar_e\rangle$ acquires a phase during a period:

$$\phi_\ell = -\frac{\ell^2\hbar}{2}$$

The **free evolution operator** gives **pseudo-random phases** to diffusive trajectories ($\hbar_e/4\pi \notin \mathbb{Q}$)

$$\cos(\hat{x}) = \frac{1}{2}\left(e^{\frac{i\hat{x}\hbar_e}{\hbar_e}} + e^{-\frac{i\hat{x}\hbar_e}{\hbar_e}}\right)$$

→Translations in momentum of $\pm\hbar_e$

The **kick operator** $\exp\left(-\frac{K\cos(\hat{x})}{\hbar_e}\right)$ connects states on a ladder of momenta $|p = \ell\hbar_e\rangle$ ($\ell \in \mathbb{N}$):
Diffusion on momentum ladder

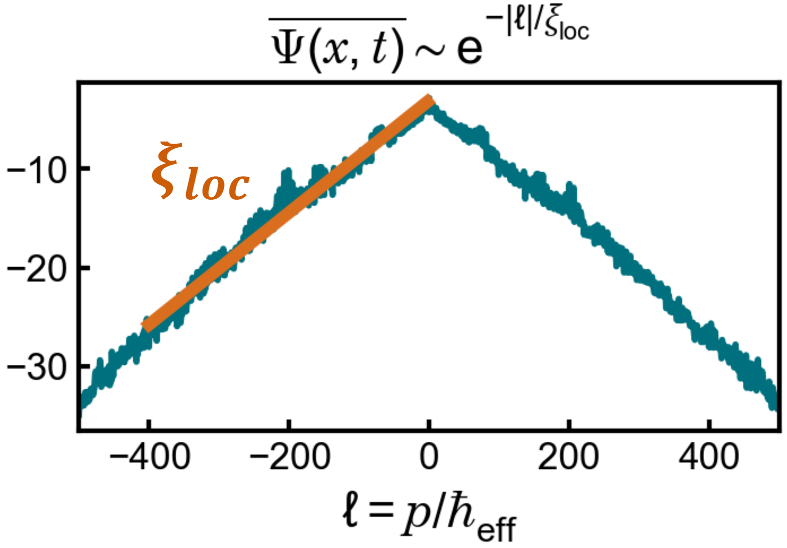
Diffusion + Random phases = ingredients for localization!

Dynamical localization

$$U_T = \exp\left(-\frac{i\hat{p}^2}{2\hbar_e}\right)\exp\left(\frac{iK\cos(\hat{x})}{\hbar_e}\right)$$

A state initially peaked in momentum experiences **dynamical localization** (localization in momentum space)

After many periods of evolution
Characterized by a localization length $\boldsymbol{\xi_{loc}}$



And in reciprocal space?
Can we see CBS and CFS with the kicked rotor?

Quantum dynamics

- **Can we see CBS and CFS with the kicked rotor?**

Solid state physics:

- Diffusion/localization in x
- Reciprocal space p hosts CBS and CFS
- CBS for *time-reversal symmetry:*

$$x \to x$$
$$p \to -p$$
$$t \to -t$$

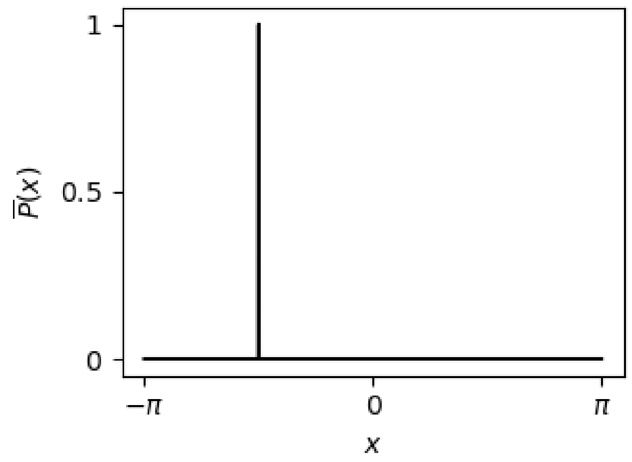
Kicked rotor:

- Diffusion/localization in p
- Reciprocal space x hosts CBS and CFS
- CBS for *time-parity symmetry*

$$p \to p$$
$$x \to -x$$
$$t \to -t$$

peaked initial condition in momentum → localization,
peaked initial condition in position → CBS and CFS

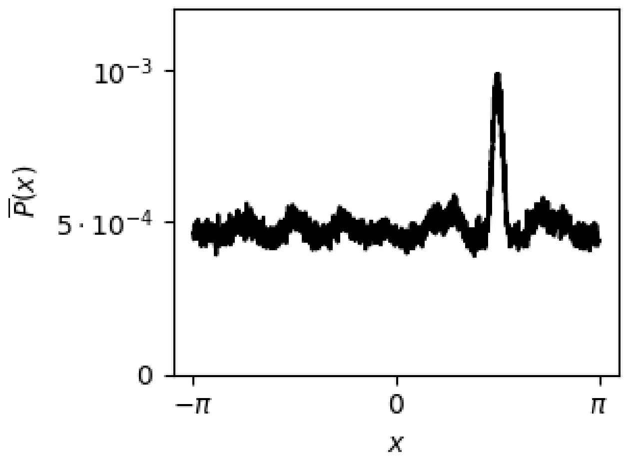
CBS and CFS in the kicked rotor



$t = 0$, peaked probability
at $x = -\pi/2$

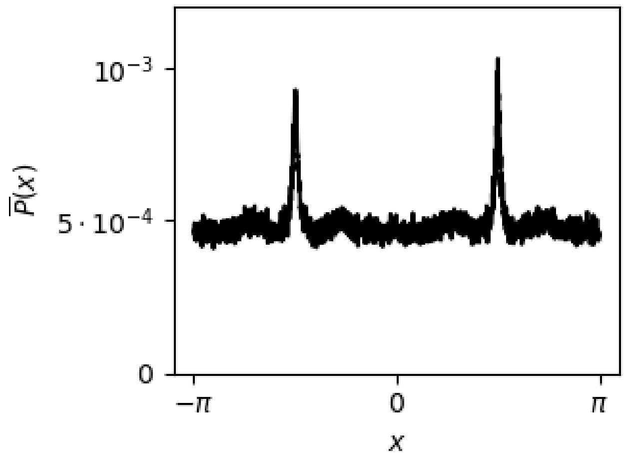
Simulations with the kicked rotor

$K = 10, \hbar_e = 1,$



$t = 4$, probability has flattened
a peak reappears at $x = -\pi/2$
Coherent Backward Scattering!

$t = 200$, another peak!
return of probability at $x = \pi/2$
Coherent Forward Scattering!



Can we do experiments?

An infinitely short/strong kick is not realistic.

- Shaken-rotor model :

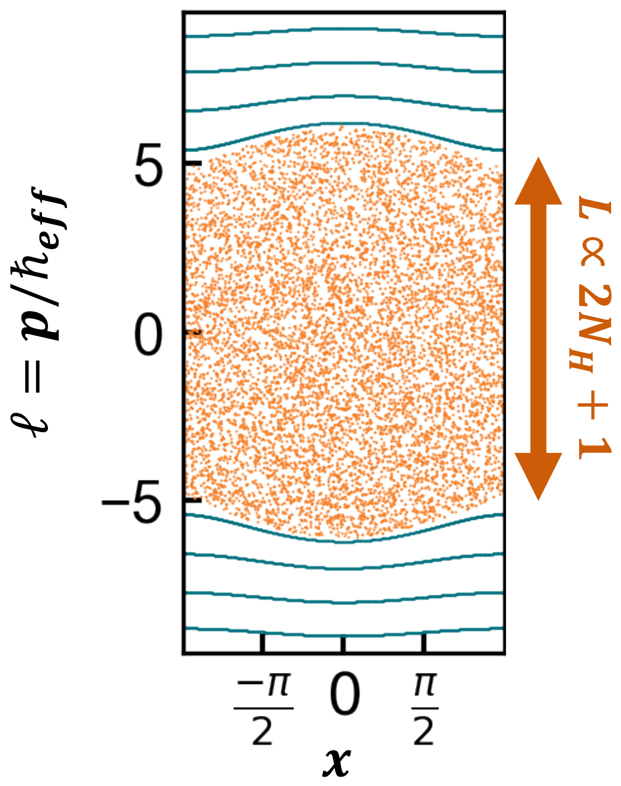
$$\begin{cases} \widehat{\mathcal{H}}_{SR}(\hat{x},\hat{p},\tau) = \dfrac{\hat{p}^2}{2} + K\cos(\hat{x} + \varphi(\tau))\,F(\tau) \\ [\hat{x},\hat{p}] = i\hbar_{eff} \end{cases}$$

Kick strength

periodic modulation:

$$F(\tau) = 1 + 2\sum_{n=1}^{N_H} \cos(2\pi n\tau + \varphi_n)$$

Stroboscopic phase-space



- Generalization of QKR : equivalent if $\varphi(t) = 0$ and $N_H \to \infty$
- **Finite** and **tunable** **chaotic sea** ($L \propto N_H$): control of localization regime
- **Disorder average** using **different** $F(\tau)$ inducing differents dynamics
- Additional **phase modulation**

Can we do experiments?

An infinitely short/strong kick is not realistic.

- Shaken-rotor model :

$$\begin{cases} \hat{\mathcal{H}}_{SR}(\hat{x},\hat{p},\tau) = \dfrac{\hat{p}^2}{2} + K\cos(\hat{x}+\varphi(\tau))\,F(\tau) \\ [\hat{x},\hat{p}] = i\hbar_{eff} \end{cases}$$

Kick strength

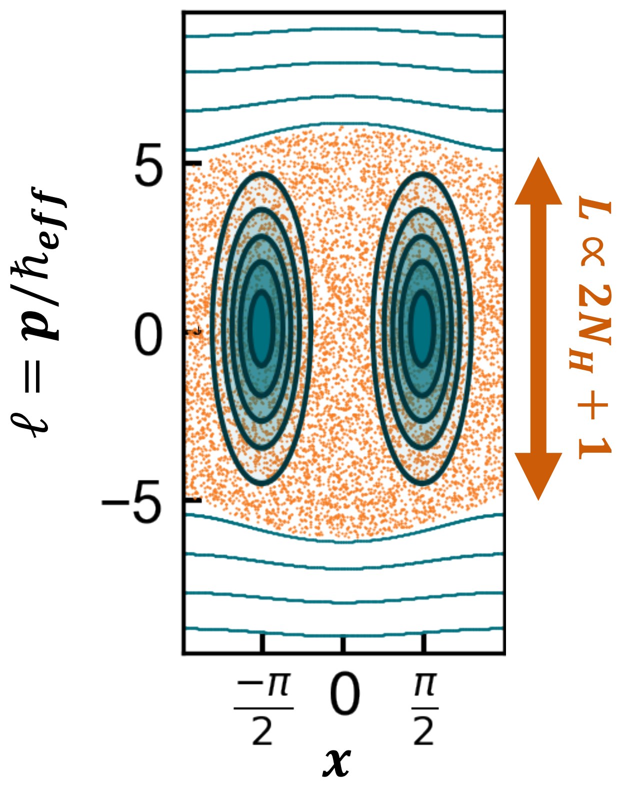
periodic modulation:

$$F(\tau) = 1 + 2\sum_{n=1}^{N_H} \cos(2\pi n\tau + \varphi_n)$$

- Same behaviour as quantum kicked rotor

CBS & CFS peaks expected at $\pm\frac{\pi}{2}$ for initial peaked state at $-\frac{\pi}{2}$

Stroboscopic phase-space



Phase space rotation

- The CBS and CFS peaks occur in each lattice cell : $\boldsymbol{P(x)}$ **reflects the presence of the peaks**
- We can use a **static lattice dynamics** to **transfer information from** $\boldsymbol{P(x)}$ **onto** $\boldsymbol{P(\ell)}$

$$t_{rot} \sim T_{rot}/4$$

T_{rot} *period of harmonic oscillator*

x_r *center of harmonic oscillator w.r.t. the center of the cell*

Phase space rotation

- The CBS and CFS peaks occur in each lattice cell : $\boldsymbol{P(x)}$ **reflects the presence of the peaks**
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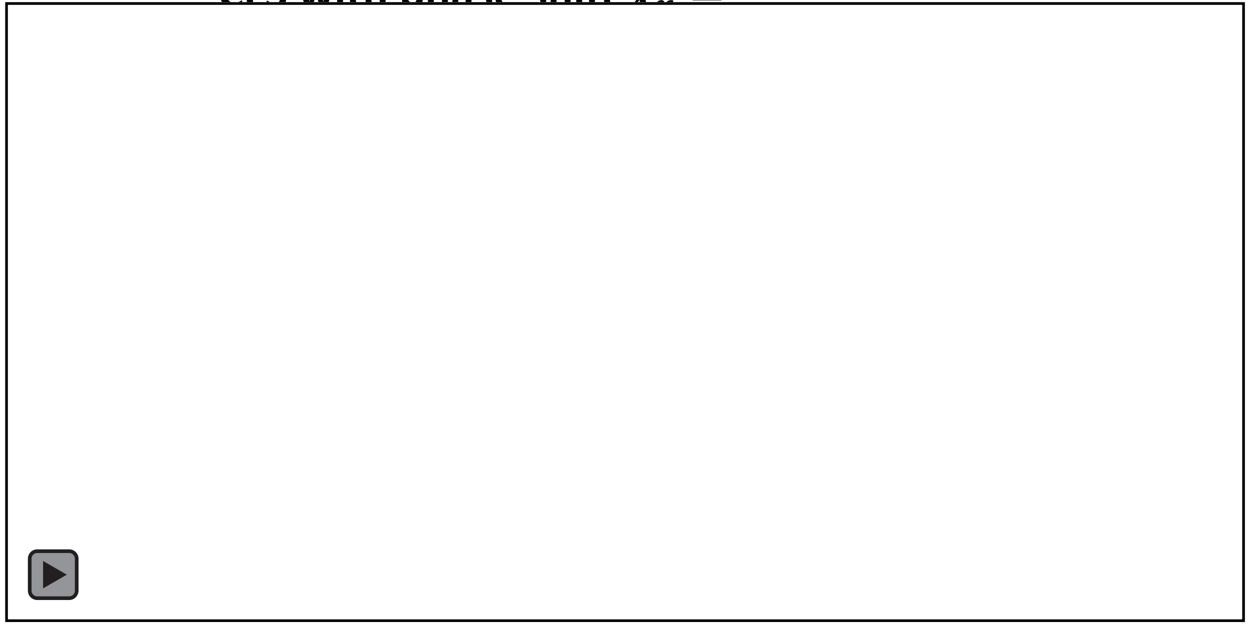
$$t_{rot} \sim T_{rot}/4$$

T_{rot} period of harmonic oscillator

x_r center of harmonic oscillator w.r.t. the center of the cell

CFS with phase shift $x_r = -\frac{\pi}{}$

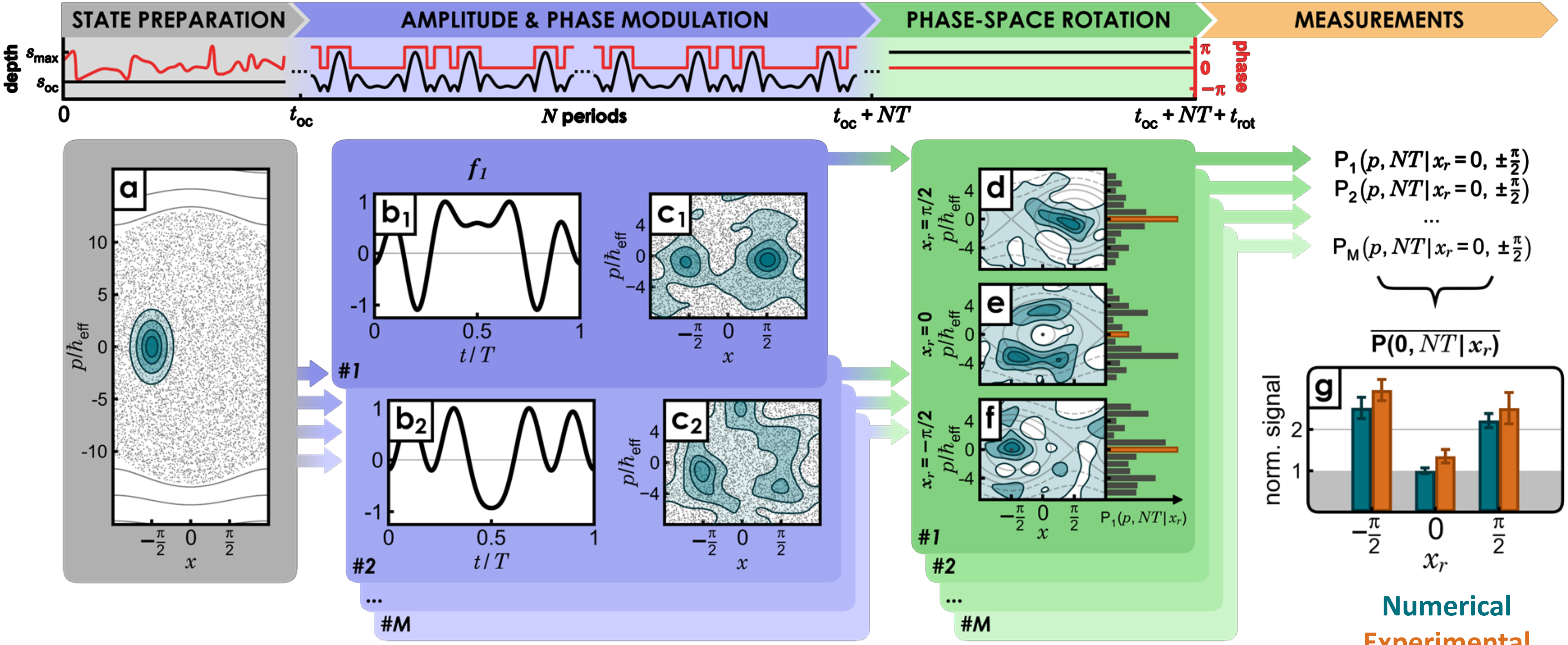


x_r : **Phase shift** to center the well **on the peak**

Full measurement

Initial state : prepared with Optimal Control

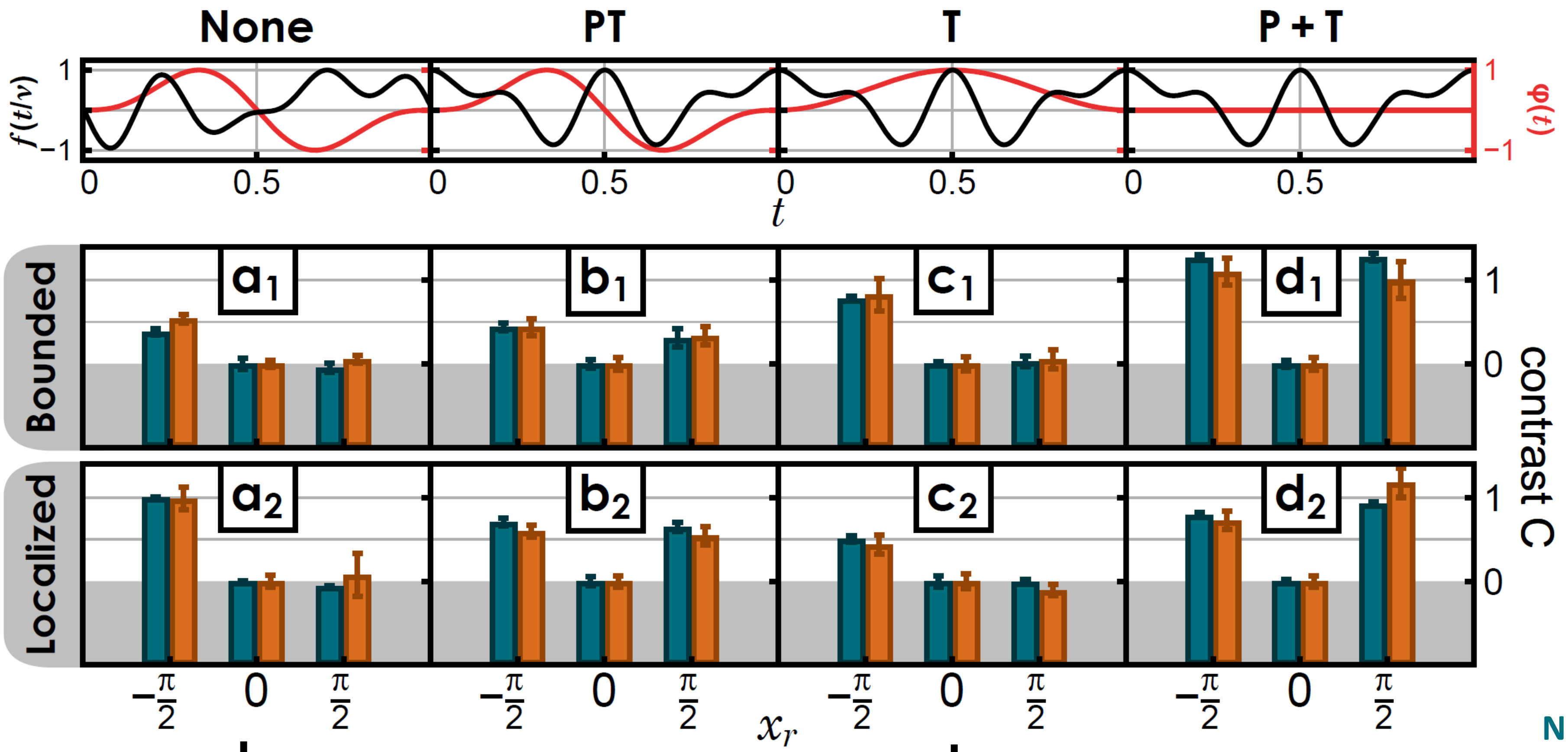
$N_H = 4, \hbar_{eff} = 2.89, L \sim 24, \xi_{loc} \sim 4.5$



Numerical
Experimental

Signature of weak and strong localization transfered in momentum space !

Results



$\xi_{loc} \gg L$

$L \gg \xi_{loc}$

Numerical
Experimental

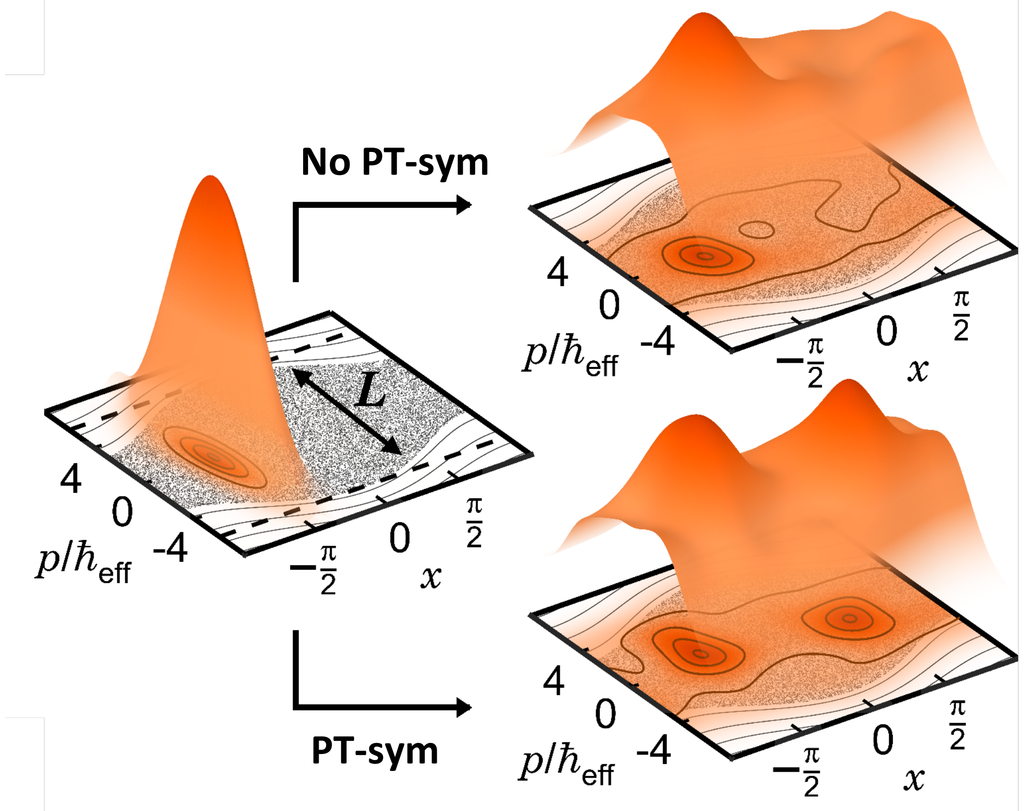
When PT-symmetry is broken : no CBS !

CFS: robust to symmetry-breaking

State reconstruction

With the quantum state tomography → reconstruction of the **disorder-averaged Husimi :**

Initial state



No-PT symmetry
No CBS
& CFS expected

PT symmetry
CBS & CFS expected

Conclusion

- Cold atoms (BEC) are a good platform for *quantum simulation*

- Control of parameters, preparation and detection allow to *implement* and *study* quantum models

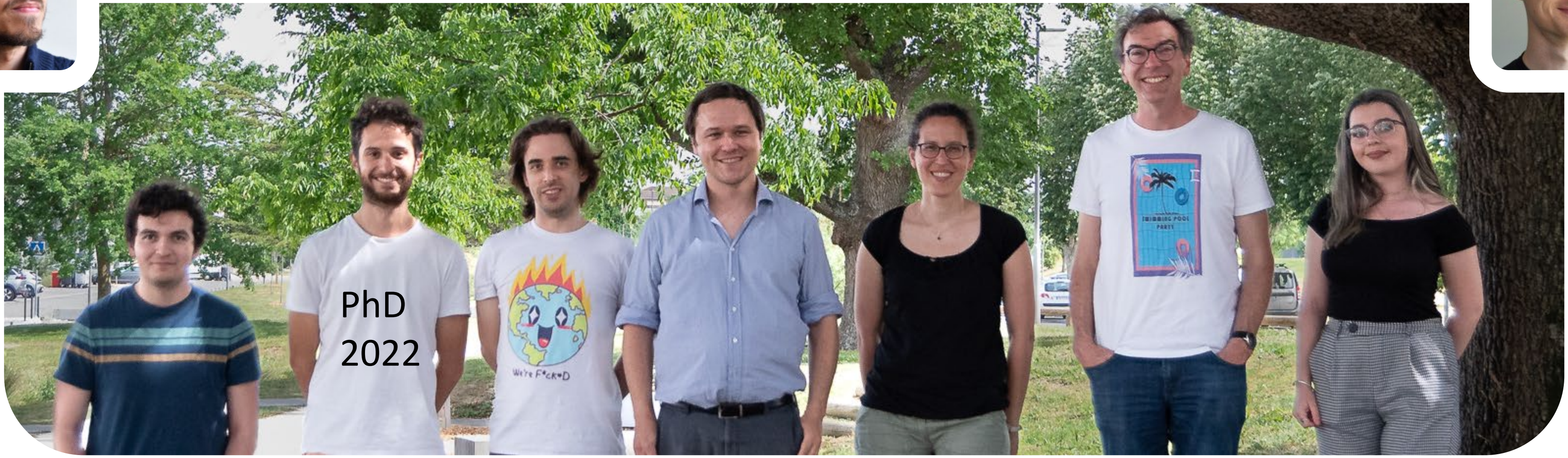
- One example of *periodic driving* (Floquet engineering): the kicked/shaken rotor

- Allowed us to realize a first direct observation of CBS and CFS peaks together using phase-space rotation or density matrix reconstruction
  (should go on the arxiv this week)

Thanks – Cold Atoms team & Collaborators

PhD
2020

May 2022



PhD
2022

PhD
2021

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Postdoc
2020-2022

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Carnot de Bourgogne

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October 2024

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