The Quantum Carnot engine

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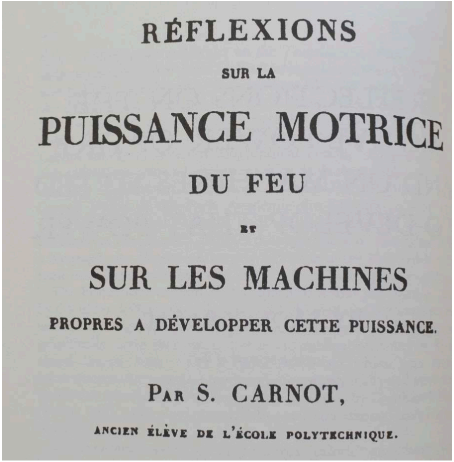
Carnot Workshop on Quantum Thermodynamics and Open Quantum Systems

Monday, November 25 2024

THE FRITZ HABER RESEARCH CENTER
FOR MOLECULAR DYNAMICS

Learning from Example


RÉFLEXIONS

SUR LA

PUISSANCE MOTRICE

DU FEU

ET

SUR LES MACHINES

PROPRES A DÉVELOPPER CETTE PUISSANCE.

PAR S. CARNOT,

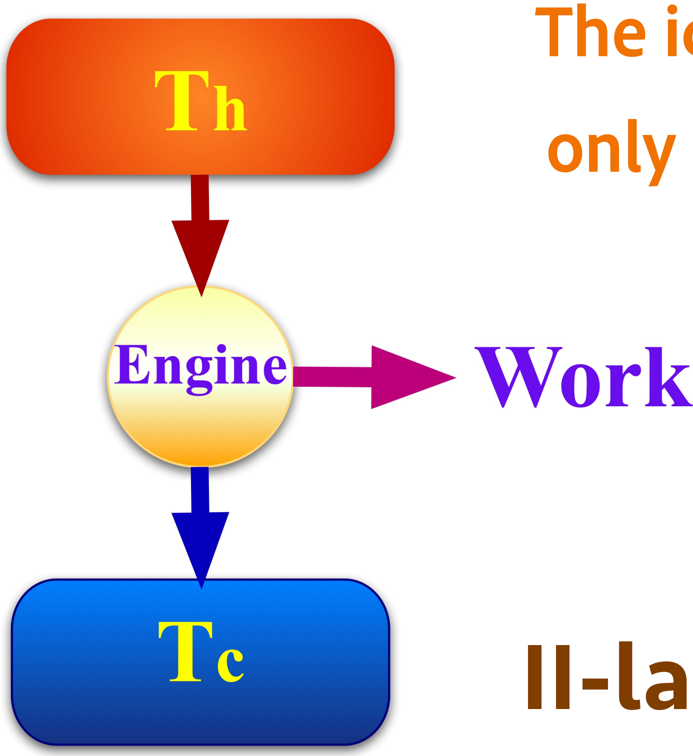
ANCIEN ÉLÈVE DE L'ÉCOLE POLYTECHNIQUE.

Universality

From the impossibility of perpetual motion, S. Carnot deduced that the maximum effciency of an engine exchanging heat with two sources only depends on the temperatures of these sources. He wrote:

"La puissance motrice de la chaleur est *indépendente des agents mis en œuvre pour* la réaliser; sa quantité est fixée uniquement par les températures des corps entre lesquels se fait, en dernier résultat, le transport du calorique" [25], or " The driving power of heat is independent of the agents used to realize it; its value is uniquely fixed by the temperatures of the bodies between which the transfer of caloric is made".

Carnot: The ideal heat engine is universal



The ideal engin's efficiency depends only on the temperatre ratio

$$\eta_C = 1 - \frac{T_c}{T_h}$$

II-law of thermodynamics

Entropy $\Delta S > 0$

For an ideal reversible transformation the entropy change is zero

Quantum Thermodynamics

Any theory should be consistent with Thermodynamics

6. ***Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt;***
von A. Einstein.

Zwischen den theoretischen Vorstellungen, welche sich die Physiker über die Gase und andere ponderable Körper gebildet haben, und der Maxwellschen Theorie der elektro-

Doc. 14

ON A HEURISTIC POINT OF VIEW CONCERNING THE PRODUCTION AND TRANSFORMATION OF LIGHT

by A. Einstein

[*Annalen der Physik* 17 (1905): 132-148]

Einstein 1905

$E=h\nu$

If we restrict ourselves to investigating the dependence of the entropy on th volume occupied by the radiation and denote the entropy of radiation by S_0 when the latter occupies the volume v_0, we obtain

$$S - S_0 = \frac{E}{\beta\nu} \lg \left[\frac{v}{v_0}\right] .$$

This equation shows that the entropy of a monochromatic radiation of sufficiently low density varies with the volume according to the same law as the entropy of an ideal gas or that of a dilute solution. The equation just

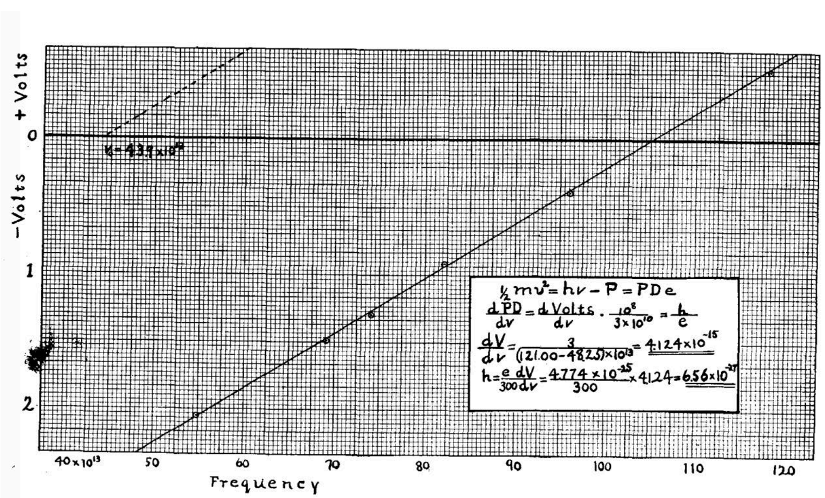
If monochromatic radiation of frequency ν and energy E is enclosed (by reflecting walls) in the volume v_0, the probability that at a randomly chosen instant the entire radiation energy will be contained in the portion v of the volume v_0 is

$$W = \left[\frac{v}{v_0}\right]^{\frac{N}{R}\frac{E}{\beta\nu}}$$

From this we further conclude:

Monochromatic radiation of low density (within the range of validity of Wien's radiation formula) behaves thermodynamically as if it consisted of mutually independent energy quanta of magnitude $R\beta\nu/N$.

The paper is wrongly interpreted as the photoelectric effect

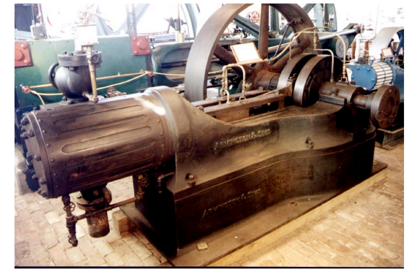


Einstein 1905

$E=h\nu$

.Millikan, R. (1914). "A Direct Determination of "h."". Physical Review. 4 (1): 73–75

Quantum Thermodynamics



Consistency

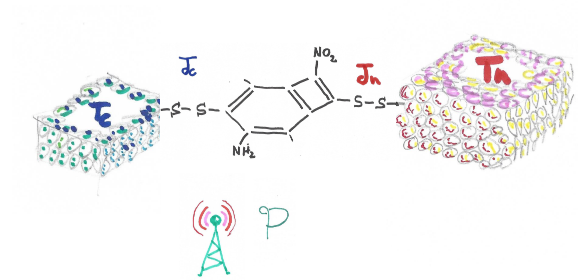
Emergence of Thermodynamics from quantum mechanics

Learning from example

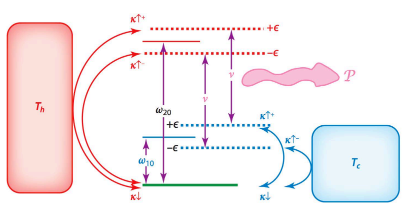
Thermodynamic ideals

Can a Thermodynamical viwpoint be relavant to a single device at the quantum limit ?

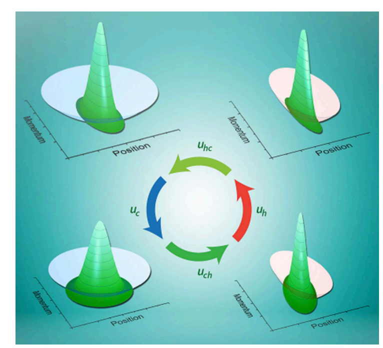
What is the limit of minaturization of a quantum heat engine ?



single molecular refrigerator



What is quantum in a quantum heat engine?



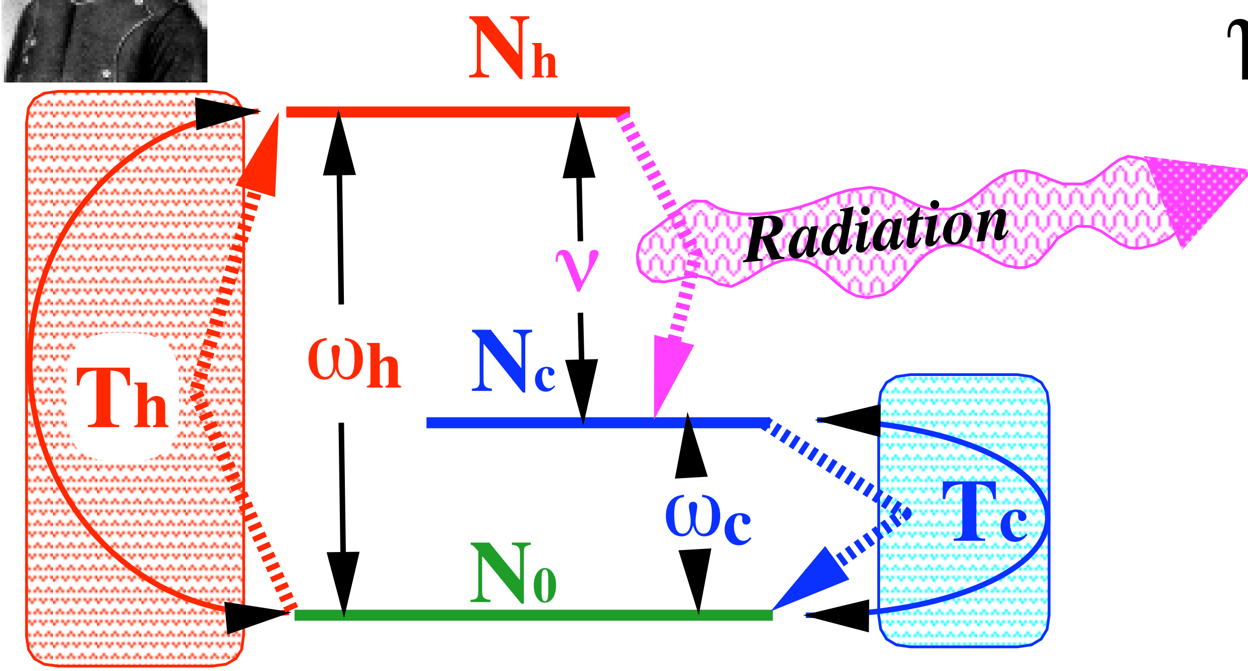
Is there quantum supremacy ?

Is a small quantum engine usefull?

How small a quantum engine can be

What is quantum in quantum heat devices

Carnot efficiency of a 3–level amplifier



$$\eta = \frac{\nu}{\omega_h} = \frac{\omega_h - \omega_c}{\omega_h}$$

$$G = N_h - N_c \geq 0$$

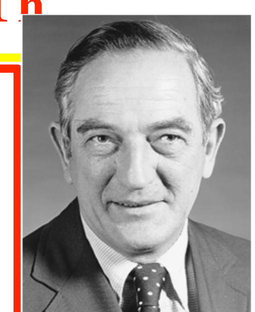
$$\frac{\hbar\omega_h}{k\,T_h} - \frac{\hbar\omega_c}{k\,T_c} \leq 0$$

$$\frac{\omega_c}{\omega_h} \leq \frac{T_c}{T_h}$$

$$N_c = N_0\, e^{-\frac{\hbar\omega_c}{k\,T_c}}$$

$$N_h = N_0\, e^{-\frac{\hbar\omega_h}{k\,T_h}}$$

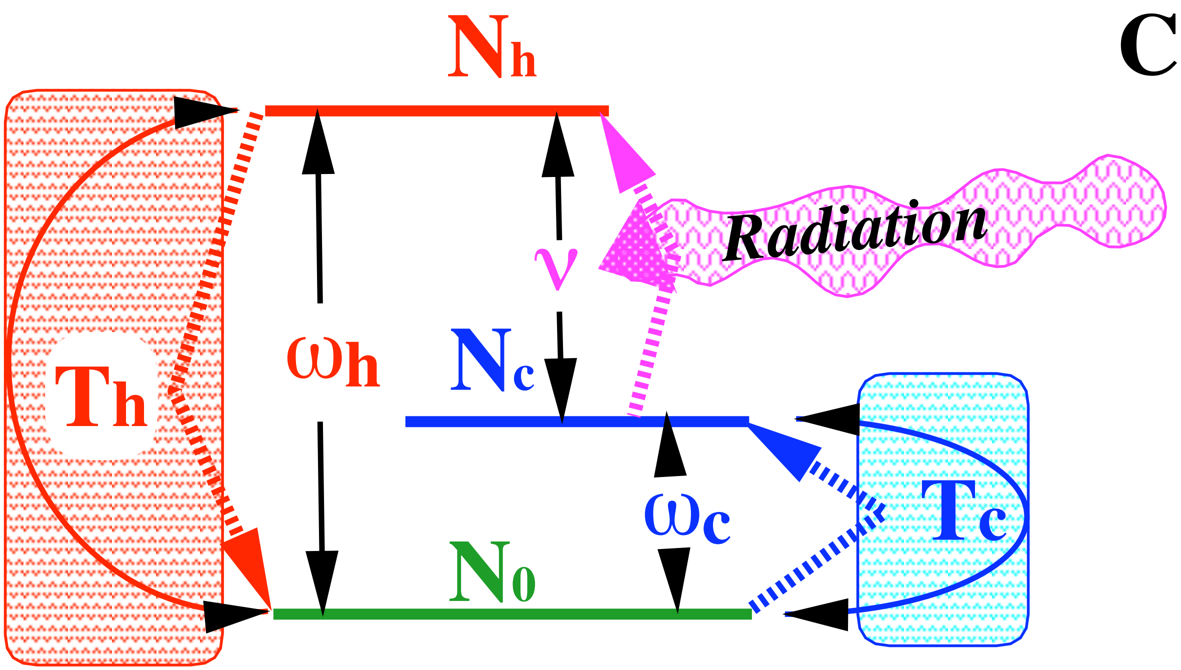
$$\eta = 1 - \frac{\omega_c}{\omega_h} \leq 1 - \frac{T_c}{T_h}$$



Scovil & Shultz de Bois PRL 2 262 (1959)

Laser Cooling reversing the 3–level amplifier

Geusic J, Bois E, De Grasse R, Scovil H. J. App. Phys. 30:1113(1959)



$$COP = \frac{\omega_c}{\nu} = \frac{\omega_c}{\omega_h - \omega_c}$$

$$G = N_h - N_c \leq 0$$

$$\frac{\hbar\omega_h}{k\,T_h} - \frac{\hbar\omega_c}{k\,T_c} \leq 0$$

$$\frac{\omega_c}{\omega_h} \geq \frac{T_c}{T_h}$$

$$N_c = N_0\, e^{-\frac{\hbar\omega_c}{k\,T_c}}$$

$$T_c > \frac{\omega_c}{\omega_h} T_h$$

$$N_h = N_0\, e^{-\frac{\hbar\omega_h}{k\,T_h}}$$

$$COP \approx \frac{T_c}{T_h}$$

D. J. Wineland and H. Dehmelt, Bull. Am. Phys. Soc. 20, 637 (1975); T. W. Hänsch and A. L. Schawlow, "Cooling of Gases by Laser Radiation," Opt. Commun. 13, 68 (1975).

Quantum Thermodynamics

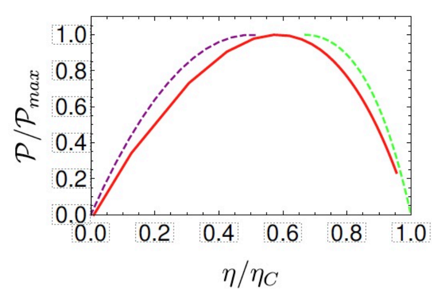
Inserting Dynamics into Thermodynamics

Power or efficiency?

$$\eta_{CA} = 1 - \sqrt{\frac{T_c}{T_h}}$$

Efficiency at maximum power

$\Delta S^u > 0$



$$\eta_c = 1 - \frac{T_c}{T_h}$$

Maximum efficiency

$\Delta S^u = 0$

Reciprocating heat engines

Thermodynamic analysis of quantum error correcting engines

Gabriel T. Landi,[1,*] André L. Fonseca de Oliveira,[2] and Efrain Buksman[2]

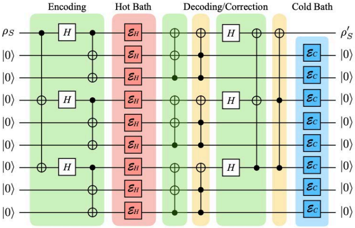


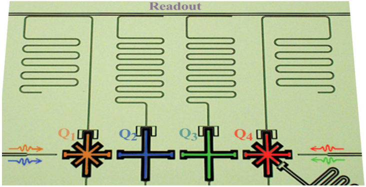
FIG. 4. Shor's -qubit code, capable of correcting both the diagonal as well as the coherent parts of ρ_S against any kind of noise.

Learning from example

How small can an engine be?

What is the role of coherence?

Coherent contol by interference of pahthways?



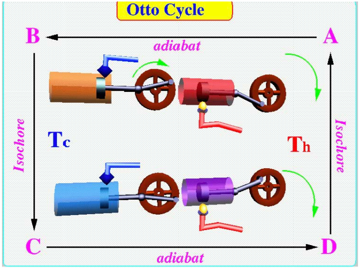
Open system description of a heat engine

Propagator $\hat{\rho}_f = \Lambda \hat{\rho}_i$

1. Hot to cold adiabatic stroke Λ_{hc}
2. Cold isotherm Λ_c Cold isochore $\bar{\Lambda}_c$
3. Cold to hot adiabatic stroke Λ_{ch}
4. Hot isotherm Λ_h Hot isochore $\bar{\Lambda}_h$

$$\Lambda_{cyc} = \Lambda_h \Lambda_{ch} \Lambda_c \Lambda_{hc}$$

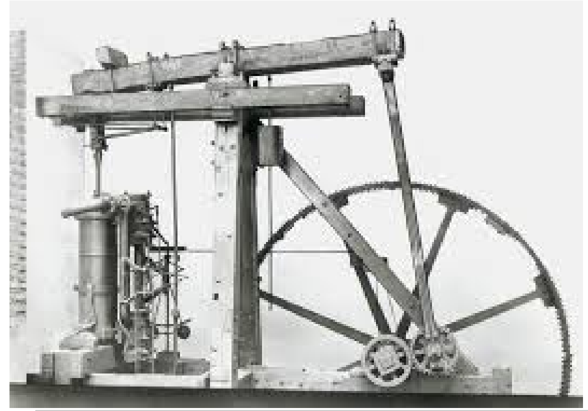
Order matters $[\Lambda_h, \Lambda_{ch}] \neq 0$

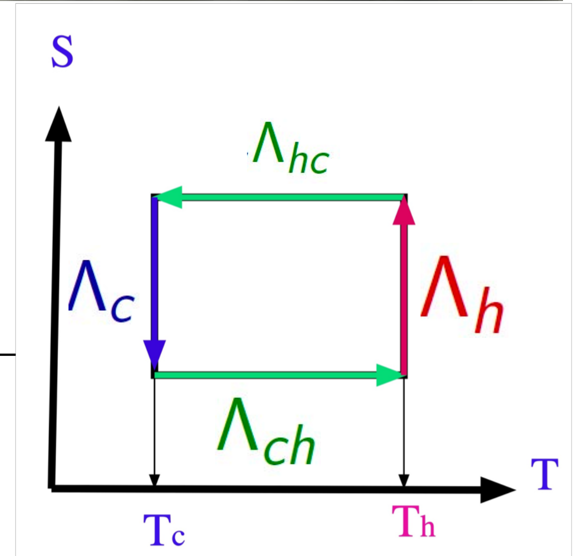


Carnot cycle

1. Hot to cold adiabatic stroke Λ_{hc}
2. Cold isotherm Λ_c
3. Cold to hot adiabatic stroke Λ_{ch}
4. Hot isotherm Λ_h

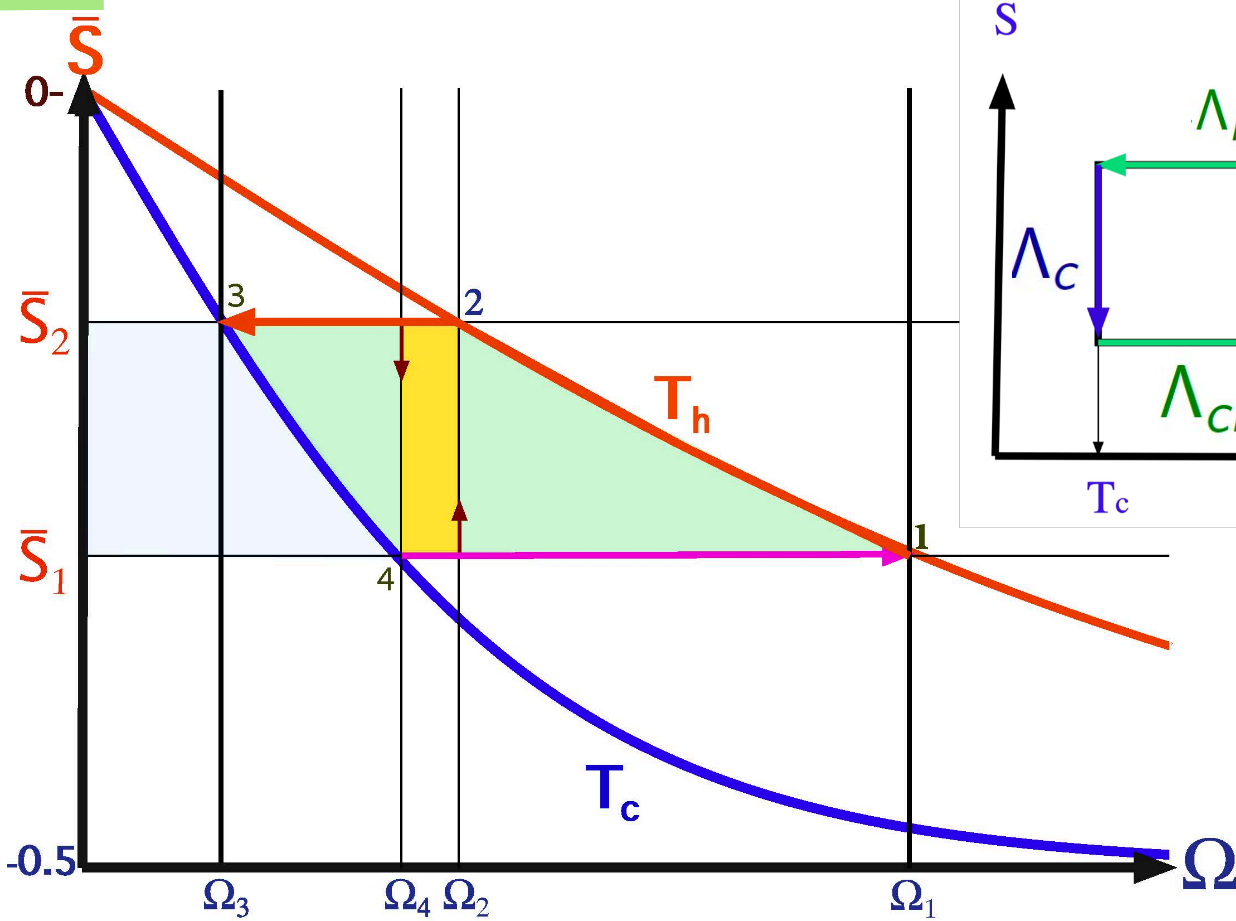
$$\eta_C = 1 - \frac{T_c}{T_h}$$





Carnot cycle:

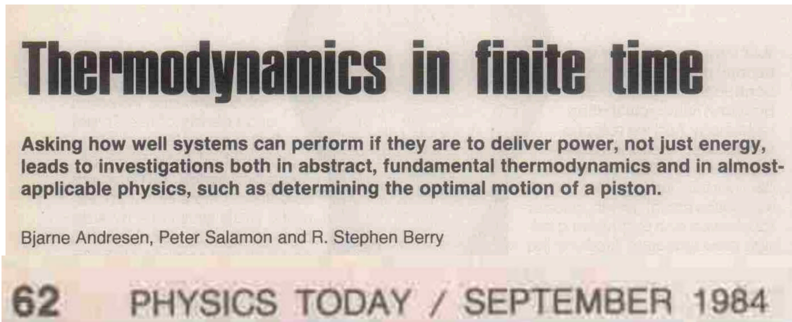
$$\Lambda_{cyc} = \Lambda_h \Lambda_{ch} \Lambda_c \Lambda_{hc}$$



$$\Lambda_{cyc} \hat{\rho}_S = 1 \hat{\rho}_S$$

Operating conditions fixed point of CPTP map

Performance characteristics of real life heat engines

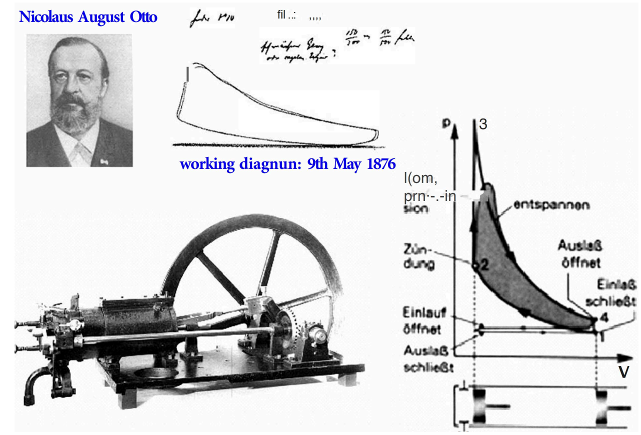


Thermodynamics in finite time

Asking how well systems can perform if they are to deliver power, not just energy, leads to investigations both in abstract, fundamental thermodynamics and in almost-applicable physics, such as determining the optimal motion of a piston.

Bjarne Andresen, Peter Salamon and R. Stephen Berry

62 PHYSICS TODAY / SEPTEMBER 1984



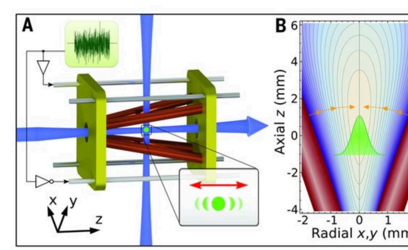
1) An engine operates at finite power.
2) The engine is limited by heat leaks.
3) The motion is subject to friction.

The engine operates under irreversible conditions

A single-atom heat engine

Johannes Roßnagel,[1*] Samuel T. Dawkins,[1] Karl N. Tolazzi,[2] Obinna Abah,[3] Eric Lutz,[3] Ferdinand Schmidt-Kaler,[1] Kilian Singer[1,4*]

Do quantum engines have the same performance characteristics?



Science 15 Apr 2016

Endoreversible qubit engine

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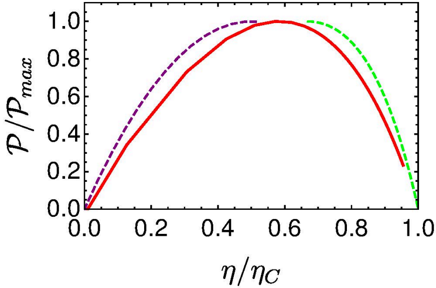
Efficiency at Maximum power

$$\eta_{CA} = 1 - \sqrt{\frac{T_c}{T_h}}$$



Carnot cycle:

$$\eta_C = 1 - \frac{T_c}{T_h}$$

The work per reversible:

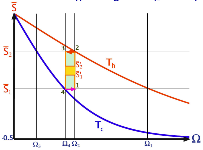
$$\mathscr{W}_C = k_B \Delta T \Delta \bar{S} \text{ v.n}$$

where $\Delta T = T_h - T_c$

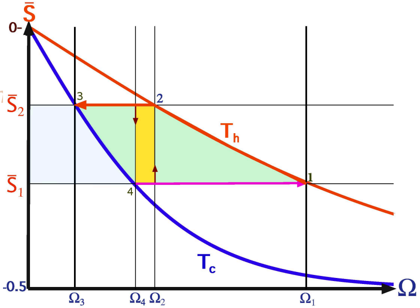
Otto cycle:

$$\mathscr{W}_{Otto} = \Delta\Omega \Delta\bar{S}$$

where $\Delta\Omega = \Omega_h - \Omega_c = \Omega_2 - \Omega_4$ and $\Delta S = \bar{S}_2 - \bar{S}_1$.



$$\eta_{Otto} = 1 - \frac{\Omega_c}{\Omega_h}$$



Carnot cycle

A quantum-mechanical heat engine operating in finite time. A model consisting of spin-1/2 systems as the working fluid

Eitan Geva and Ronnie Kosloff
Department of Physical Chemistry and The Fritz Haber Research Center for Molecular Dynamics, The Hebrew University, Jerusalem 91904, Israel

(Received 28 August 1991; accepted 21 October 1991)

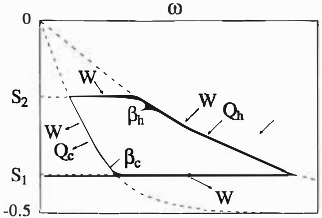


FIG. 1. The reversible Carnotcycle (solid line) in the (ω,S) plane (ω is the field and S the polarization). The cycle is composed of two reversible isotherms corresponding to the temperatures β_h and β_c $(\beta_c > \beta_h)$ and of two adiabats corresponding to the polarizations S_1 and S_2 $(S_1 < S_2;\ S_1, S_2 < 0)$. Positive net work production is obtained by going anticlockwise. The directions of work and heat flows along each branch are indicated.

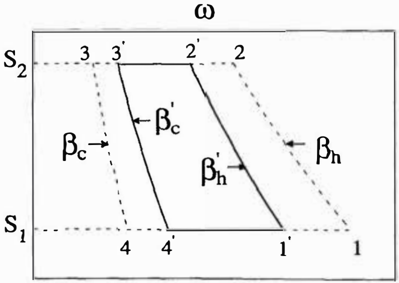


FIG. 5. The cycle $1' \to 2' \to 3' \to 4' \to 1'$ is of the Curzon–Ahlborn

Heisenberg picture, reads as follows:

$$\dot{\mathbf{X}} = i[\mathbf{H},\mathbf{X}] + \frac{\partial \mathbf{X}}{\partial t} + \mathscr{L}_D(\mathbf{X}),$$

$$\mathscr{L}_D(\mathbf{X}) = \sum_\alpha \gamma_\alpha (\mathbf{V}_\alpha^\dagger [\mathbf{X},\mathbf{V}_\alpha] + [\mathbf{V}_\alpha^\dagger,\mathbf{X}]\mathbf{V}_\alpha).$$

Adiabatic approach

No coherence considered

J. Chem. Phys., Vol. 96, No. 4, 15 February 1992

Carnot cycle: The isotherms

The Problem:

Derive a dynamical description for a driven system coupled to a bath beyond the adiabatic limit:

$$\hat{H} = \hat{H}_S(t) + \hat{H}_B + \hat{H}_{SB}$$

Peter Salamon

Carnot cycle: The isotherms

$$\left[\hat{H}_S(t), \hat{H}_S(t')\right] \neq 0$$

The task: Isothermal Dynamics

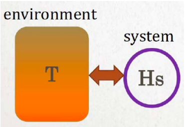
Starting from a thermal initial state $\hat{\rho}_i = e^{-\beta \hat{H}_i}$
Transform as fast and accurate to the state: $\hat{\rho}_f = e^{-\beta \hat{H}_f}$

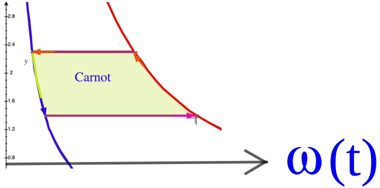
while the system is in contact with a bath of temperature $T = 1/k\beta$

The protocol: $\hat{H}_S(t)$ with $\hat{H}_S(0) = \hat{H}_i$ and $\hat{H}_S(t_f) = \hat{H}_f$

The Problem

We can control directly $\hat{H}_S(t)$ but only indirectly the relaxation rate.
We need the dissipative equation of motion with a time dependent $\hat{H}_S(t)$ with a time dependent protocol.





The adiabatic stroke: Quantum version

The propagator Λ_{hc} is Unitary

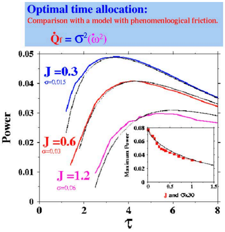
$$\frac{\partial}{\partial t}\hat{\rho} = -\frac{i}{\hbar}[\hat{\mathbf{H}}, \hat{\rho}]$$

Control Hamiltonian of the working medium

$$\hat{\mathbf{H}}(\mathbf{t}) = \hat{\mathbf{H}}_{\mathbf{int}} + \hat{\mathbf{H}}_{\mathbf{cont}}(t)$$

Then: $[\hat{\mathbf{H}}(\mathbf{t}), \hat{\mathbf{H}}(\mathbf{t}')] \neq 0$ and **coherence** is generated.

Generating **coherence** cost **work**.



What is required for a realistic model?

Quantum origin of **friction**: generating **coherence**

Qubit basics

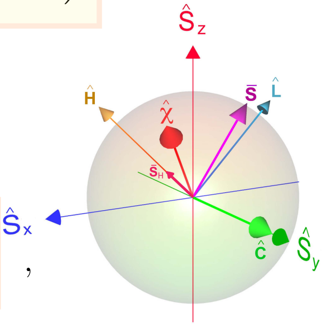
HHamiltonian $\hat{\mathbf{H}}_S(t) = \omega(t)\hat{\mathbf{S}}_z + \varepsilon(t)\hat{\mathbf{S}}_x \quad ,$

$$\hbar\Omega(t) = \hbar\sqrt{\omega^2 + \varepsilon^2} \quad ,$$

State $\hat{\rho} = \frac{1}{2}\hat{\mathbf{I}} + \frac{2}{\hbar^2}\left(\langle\hat{S}_x\rangle\hat{\mathbf{S}}_x + \langle\hat{S}_y\rangle\hat{\mathbf{S}}_y + \langle\hat{S}_z\rangle\hat{\mathbf{S}}_z\right) \quad .$

time dependent set

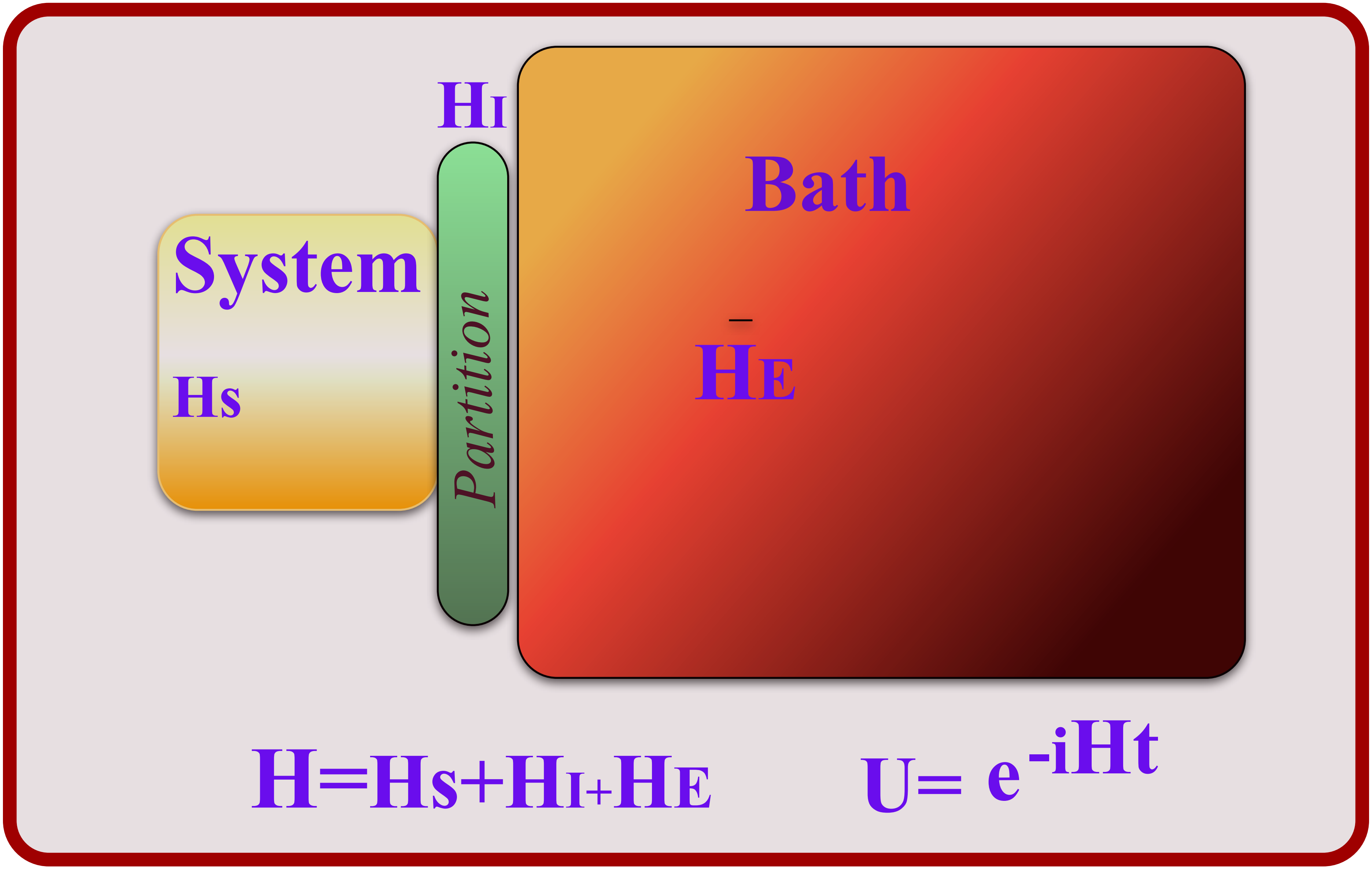
$$\hat{\mathbf{H}} = \omega(t)\hat{\mathbf{S}}_z + \varepsilon(t)\hat{\mathbf{S}}_x$$
$$\hat{\mathbf{L}} = \varepsilon(t)\hat{\mathbf{S}}_z - \omega(t)\hat{\mathbf{S}}_x$$
$$\hat{\mathbf{C}} = \Omega(t)\hat{\mathbf{S}}_y \quad .$$



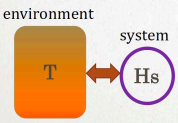
sstate $\hat{\rho} = \frac{1}{2}\hat{I} + \frac{2}{(\hbar\Omega)^2}\left(\langle\hat{H}\rangle\hat{\mathbf{H}} + \langle\hat{L}\rangle\hat{\mathbf{L}} + \langle\hat{C}\rangle\hat{\mathbf{C}}\right) \quad ,$

coherence $\mathscr{C} = \frac{1}{\hbar\Omega}\sqrt{\langle\hat{\mathbf{L}}\rangle^2 + \langle\hat{\mathbf{C}}\rangle^2} \quad ,$

Quantum partition



Inserting dynamics into thermodynamics



The global Hamiltonian:

$$\hat{\mathbf{H}} = \hat{\mathbf{H}}_S + \hat{\mathbf{H}}_E + \hat{\mathbf{H}}_{SE}$$

$$\hat{\rho}_f = \hat{\mathbf{U}}\hat{\rho}_i\hat{\mathbf{U}}^\dagger = \mathscr{U}_t\hat{\rho}_i \quad \mathscr{U}_t = e^{-i[\hat{\mathbf{H}},\bullet]t}$$

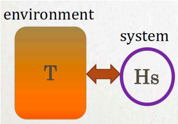
Reduced description:

$$\hat{\rho}_S(t) = \Lambda_t\hat{\rho}_S(0) \quad \Lambda_t = e^{\mathscr{L}t}$$

In a differential form:

$$\frac{\partial}{\partial t}\hat{\rho}_S = \mathscr{L}\hat{\rho}_S$$

The theory of open quantum systems



The quantum Markovian Master Equation .

A completely positive map:

Kraus 1971

G. Lindblad

$$\Lambda\hat{\rho} = \sum_j \hat{W}_j^\dagger \hat{\rho} \hat{W}_j,$$

where $\sum_j \hat{W}_j^\dagger \hat{W}_j = \hat{I}$

The Gorini-Kossakowski-Lindblad-Sudarshan (GKLS) quantum Master equation 1975

$$\frac{d}{dt}\hat{\rho} = -\frac{i}{\hbar}[\hat{H},\hat{\rho}] + \frac{1}{2}\sum_j([\hat{V}_j\hat{\rho},\hat{V}_j^\dagger] + [\hat{V}_j,\hat{\rho}\hat{V}_j^\dagger]) \equiv -\frac{i}{\hbar}[\hat{H},\hat{\rho}] + \mathscr{L}\hat{\rho}\,.$$

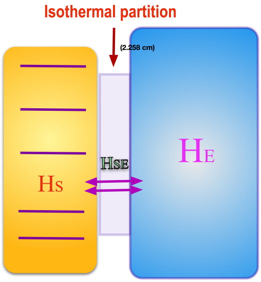
System and bath are in tensor product form in all times Lindblad 1996

Thermodynamic restriction

1. The dynamical map Λ_t is Markovian, satisfying the semi-group property.
2. The environment remains in a stationary state with respect to the environment's free Hamiltonian $\hat{\mathbf{H}}_E$.
3. The fixed point of the dynamical map is a thermal state $\hat{\rho}_S^{eq} = \hat{\rho}_S^{th} = Z_S^{-1} e^{-\beta \hat{\mathbf{H}}_S}$, where Z_S is the partition function and β is the inverse temperature of the environment.
4. The composite system satisfies *strict energy conservation* between the system and environment: $\left[\hat{\mathbf{H}}_{SE}, \hat{\mathbf{H}}_S + \hat{\mathbf{H}}_E\right] = 0.$

Roie Dann and Ronnie Kosloff,
Open system dynamics from thermodynamic compatibility,
Phys. Rev. Research, ***3***, 023006 *(2021)*. [pdf]

Isothermal partition



Energy is not accumulated on the interface

Strict energy conservation

$$\left[\hat{H}_{SE}, \hat{H}_S + \hat{H}_E\right] = 0$$

The free propagator and the dynamical map commute

$$[\mathcal{H}_S, \mathcal{L}] = 0$$

Time translation

Dynamical symmetry

Emmy Noether

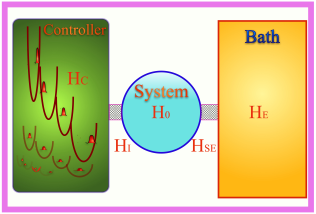
$$\mathcal{D}[\bullet] = \sum_k \gamma_k \left(\hat{L}_k \bullet \hat{L}_k^\dagger - \frac{1}{2} \left\{ \hat{L}_k^\dagger \hat{L}_k, \bullet \right\} \right),$$

Lindblad jump operators are eigenoperators of the free dynamics

Open driven quantum system dynamics

Unitary evolution of the Universe.

$$\hat{\mathbf{H}} = \hat{\mathbf{H}}_S + \hat{\mathbf{H}}_C + \hat{\mathbf{H}}_{SC} + \hat{\mathbf{H}}_{SE} + \hat{\mathbf{H}}_E \quad ,$$



Generating a propagator $\mathscr{U} = \exp(-i[\hat{\mathbf{H}}, \bullet]t)$

.

Initial tensor product: $\hat{\rho}(0) = \hat{\rho}_S(0) \otimes \hat{\rho}_E(0) \otimes \hat{\rho}_C(0)$

Reduced description:

$$\hat{\rho}_S(t) = \Lambda_t[\hat{\rho}_S(0)] = \mathrm{tr}_{E,C}\left(\hat{\mathbf{U}}(t,0)\hat{\rho}_S(0) \otimes \hat{\rho}_E(0) \otimes \hat{\rho}_C(0)\hat{\mathbf{U}}^{\dagger}(t,0)\right)$$

Imposing semigroup property

The controller is not stationary

$$\Lambda_t = \Lambda_{t-s}\Lambda_s$$

Semi-group generator $\mathscr{L}(t)$

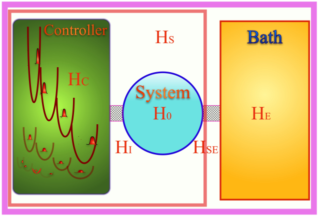
$$\mathscr{L}(t) = \lim_{\Delta t \to 0} \frac{\Lambda(t+\Delta t) - \mathscr{I}}{\Delta t}$$

, Roie Dann and Ronnie Kosloff
.Quantum 5, 590 2021

Open driven quantum system dynamics

Unitary evolution of the Universe.

$$\hat{H} = \hat{H}_S + \hat{H}_C + \hat{H}_{SC} + \hat{H}_{SE} + \hat{H}_E \quad ,$$



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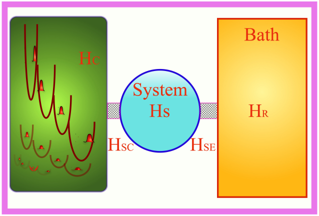
$$\mathscr{L}(t) = \lim_{\Delta t \to 0} \frac{\Lambda(t+\Delta t) - \mathscr{I}}{\Delta t}$$

Autonomous drive

.

Which means that

$$[\tilde{\mathscr{U}}_{SC}, \mathscr{U}] = 0$$



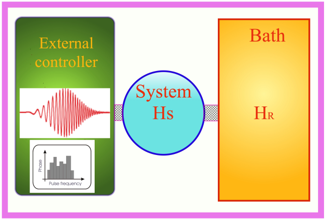
and

$$[\tilde{\mathscr{U}}_{SC}, \Lambda] = 0$$

External drive

Which also leads to:

$$[\tilde{\mathscr{U}}_{S}(t), \Lambda] = 0$$



The eigenoperators of $\tilde{\mathscr{U}}_{S}(t)$ will constitue the Lindblad jump operators when the system is externally driven. $\hat{\mathbf{H}}_{S}(t)$

The Non-Adiabatic Master Equation (NAME)

The system is driven:

$$\hat{\mathbf{H}}(t) = \hat{\mathbf{H}}_S(t) + \hat{\mathbf{H}}_B + \hat{\mathbf{H}}_{SB} \quad . \qquad [\hat{\mathbf{H}}_{SB}, \hat{\mathbf{H}}(t)] = 0$$

We still obtain from **I-law**:

$$[\Lambda, \mathscr{U}_S(t)] = 0$$

.

where $\mathscr{U}_S(t) = \mathscr{T} e^{-\frac{i}{\hbar}\int^t [\hat{\mathbf{H}}_S(t'), \bullet] dt'}$

.

The time dependent dissipator:

$$\mathscr{L}_D(\hat{\rho}_S) = \sum_k \gamma_k(t) \left(\hat{\mathbf{F}}_j(t) \hat{\rho}_S \hat{\mathbf{F}}_j^\dagger(t) - \frac{1}{2}\{\hat{\mathbf{F}}_j^\dagger(t) \hat{\mathbf{F}}_j(t), \rho_S\} \right) \quad .$$

The Lindblad jump operators are eigenoperators

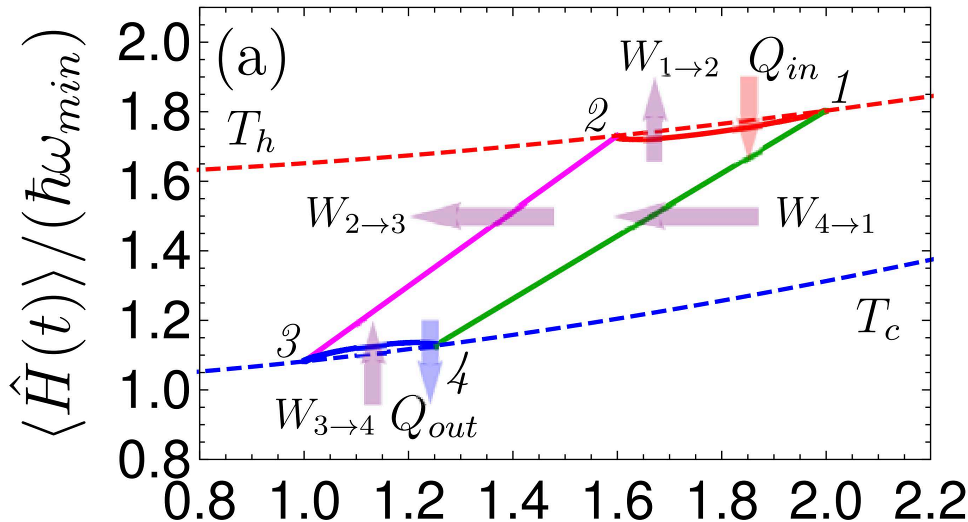
of the free dynamics $\mathscr{U}_S(t) \hat{\mathbf{F}}_k(t) = e^{-i\theta_k(t)} \hat{\mathbf{F}}_k(t).$

.

The fixed point becomes an instantaneous attractor:

$$\mathscr{L}_D(t)(\hat{\mathbf{F}}_l(t)) = 0$$

At last: Shortcut to four stroke Carnot cycle

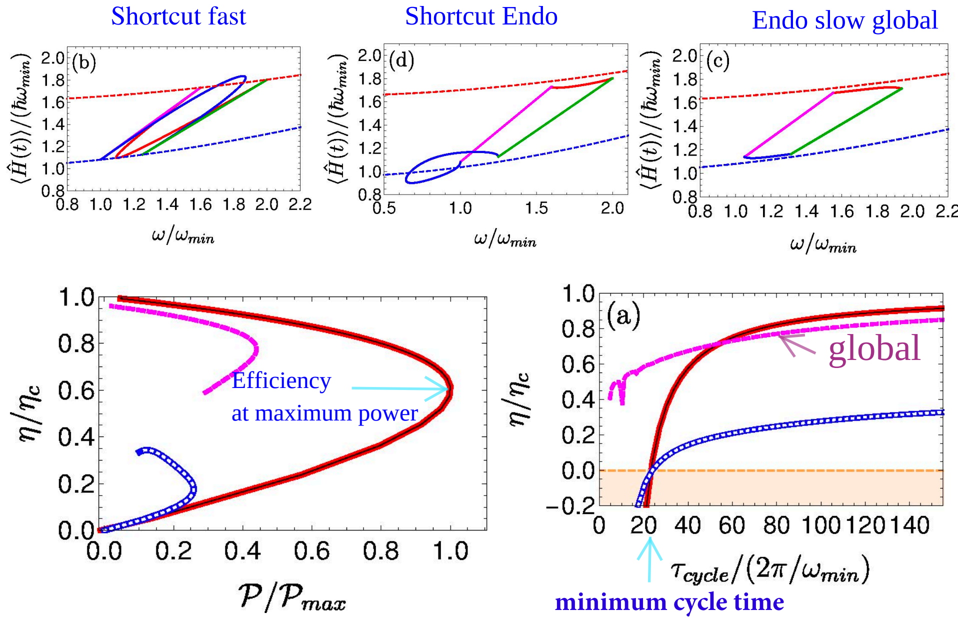


Carnot cycle:

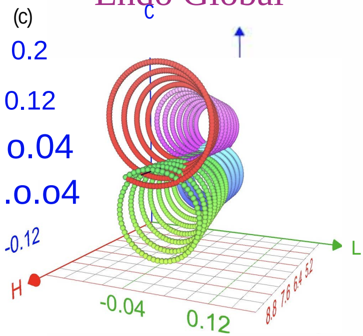
$$\Lambda_{cyc} = \Lambda_h \Lambda_{ch} \Lambda_c \Lambda_{hc}$$

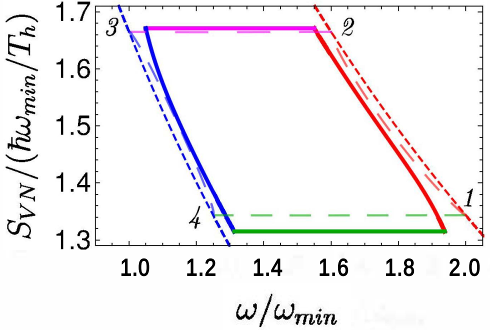
Quantum Signatures in the Quantum Carnot Cycle
Roie Dann, Ronnie Kosloff NJP 22, 013055, (2020)

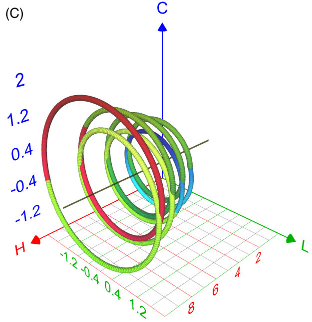
Performance of Shortcut to Carnot



Endo Global

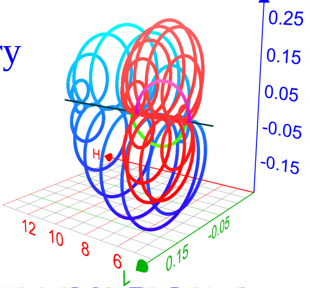






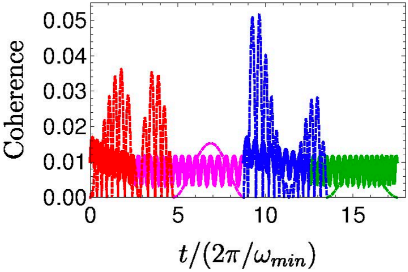
Cycle trajectory

shortcut



Quantum eqivalence

The propagator: $\mathscr{U} = e^{\mathscr{L} t}$



Four stroke cycle propagator:

$$\mathscr{U}_{cyc} = \mathscr{U}_c \mathscr{U}_{hc} \mathscr{U}_h \mathscr{U}_{ch} = e^{\mathscr{L}_c t} e^{\mathscr{L}_{hc} t} e^{\mathscr{L}_h t} e^{\mathscr{L}_{ch} t}$$

In the limit of small action: $s = ||\mathscr{L} t|| \ll \hbar$

$$\mathscr{U}_{cyc} = e^{\mathscr{L}_c t/2} e^{\mathscr{L}_{hc} t} e^{\mathscr{L}_h t} e^{\mathscr{L}_{ch} t} e^{\mathscr{L}_c t/2}$$

$$\mathscr{U}_{cyc} \approx e^{(\mathscr{L}_c + \mathscr{L}_{hc} + \mathscr{L}_h + \mathscr{L}_{ch}) t} + O(s^3)$$

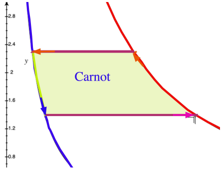
Raam Uzdin, Amikam Levy, and Ronnie Kosloff
Equivalence of Quantum Heat Machines, and Quantum-Thermodynamic
.(Phys. Rev. X 5, 031044 2015

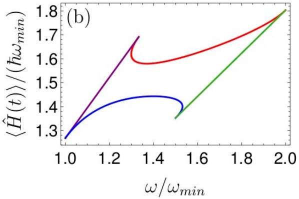
The Voyage:

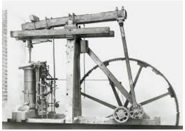
Seeking for quantum open system description of the Carnot cycle

- Non Adiabatic Master Equation **NAME**.
- The **inertial theorem**.
- Shortcuts to non unitary maps with **entropy change**.
- Finite time quantum **Carnot cycle**.

Quantum signature!







uantum control of open systems

Roie Dann

Shimshon Kallush

Amikam Levy

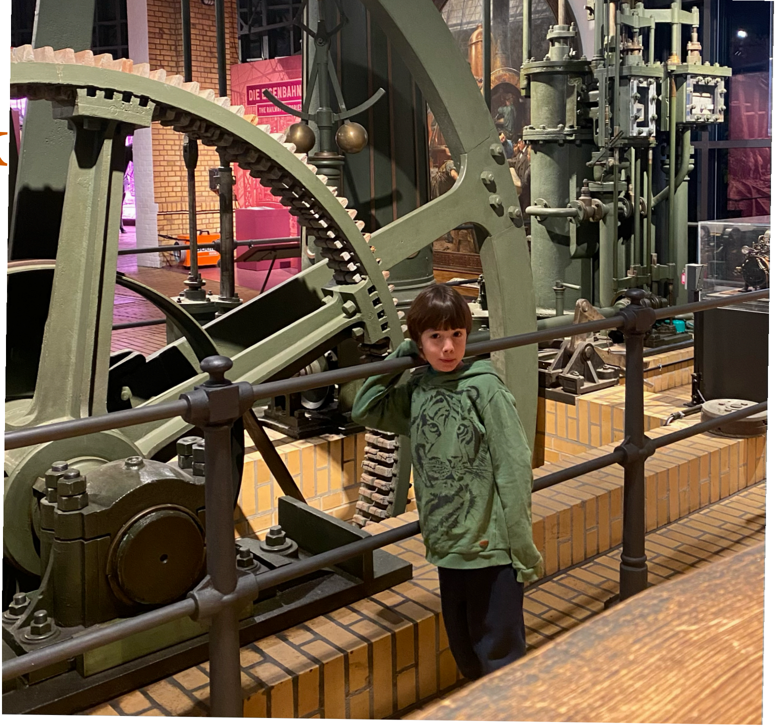
David annor

Allon artana

Ander obalina

Christiane Koch

Thank you



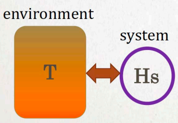
The end

Thank you

Dynamical equations consistent with Thermodynamics .

Isothermal Partition .

$$\hat{\mathbf{H}} = \hat{\mathbf{H}}_S + \hat{\mathbf{H}}_B + \hat{\mathbf{H}}_{SB} \quad . \qquad [\hat{\mathbf{H}}_{SB}, \hat{\mathbf{H}}] = 0$$



Reduced description: $\Lambda_s \hat{\rho}_S = \sum_j \hat{\mathbf{K}}_j \hat{\rho}_S \hat{\mathbf{K}}_j^\dagger$

.

0- law: The fixed point of the map is a Gibbs state:

$\Lambda_s \hat{\rho}_S(eq) = \hat{\rho}_S(eq) = \frac{1}{Z} e^{-\beta H_S}$ where $\beta = 1/kT_B$

.

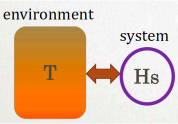
1-law conservation of energy $dE_S = -dE_B$

This implies: $[\Lambda_s, \mathscr{U}_S] = 0$ where $\mathscr{U}_S = e^{-\frac{i}{\hbar}[\hat{\mathbf{H}}_S, \bullet]t}$.

2-law Contraction:

$$\mathscr{D}(\Lambda(\hat{\rho}_S) | \Lambda(\hat{\rho}_S(eq))) \leq \mathscr{D}(\hat{\rho}_S | \hat{\rho}_S(eq))$$

The GKLS Master Equation



.

$$\frac{d}{dt}\hat{\rho}_S(t) = -i[\hat{\mathbf{H}}_S(t), \hat{\rho}_S] + \mathscr{L}_D(\hat{\rho}_S)$$

$$\mathscr{L}_H(\hat{\rho}_S) = -i[\hat{\mathbf{H}}_S(t), \hat{\rho}_S]$$

$$\mathscr{L}_D(\hat{\rho}_S) = \sum_k \gamma_k \left(\hat{\mathbf{F}}_j \hat{\rho}_S \hat{\mathbf{F}}_j^\dagger - \frac{1}{2}\{\hat{\mathbf{F}}_j^\dagger \hat{\mathbf{F}}_j, \rho_S\} \right)$$

.

G. Lindblad

From the **I-law**: $[\mathscr{L}_H, \mathscr{L}_D] = 0$, this implies that $\hat{\mathbf{F}}_k$ are common eigenoperators of $\mathscr{L}_H$ and $\mathscr{L}_D$.

$$\mathscr{L}_H(\hat{\mathbf{F}}_k) = i\omega_k \hat{\mathbf{F}}_k \quad \mathscr{L}_{\mathrm{D}}(\hat{\mathbf{F}}_k) = \gamma_k \hat{\mathbf{F}}_k$$

when $\omega_k = 0$ $\hat{\mathbf{F}}_k$ is an invariant of the unitary dynamics.

.

For $\gamma_l = 0$ $\hat{\mathbf{F}}_l$ is an invariant a fixed point $\hat{\mathbf{F}}_l = \frac{1}{Z} e^{-\beta \hat{\mathbf{H}}_S}$

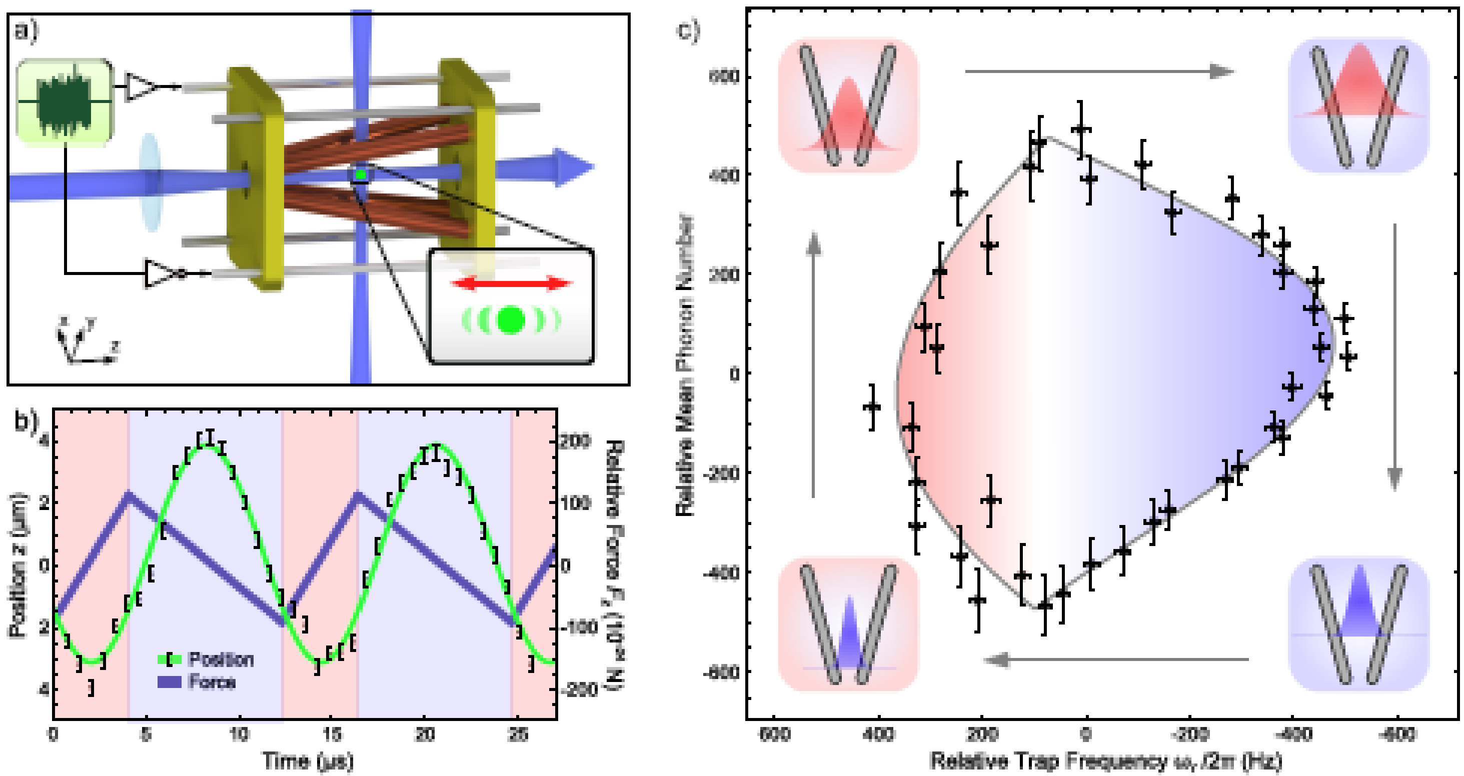
for $k \neq l$, $\hat{\mathbf{F}}_k$ are Lindblad jump operators.

Realizations 2016

Science 352 , 325 (2016)

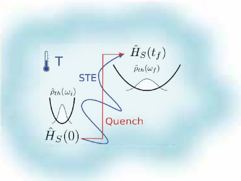
A single-atom heat engine

Johannes Roßnagel,[1,*] Samuel Thomas Dawkins,[1] Karl Nicolas Tolazzi,[1]
Obinna Abah,[2] Eric Lutz,[2] Ferdinand Schmidt-Kaler,[1] and Kilian Singer[1,3]



Shortcut to Equilibration (STE)

State to state control



The task: Isothermal Dynamics

Starting from a thermal initial state $\hat{\rho}_i = e^{-\beta \hat{H}_i}$

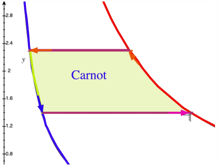
Transform as fast and accurate to the state: $\hat{\rho}_f = e^{-\beta \hat{H}_f}$

while the system is in contact with a bath of temperature $T = 1/k\beta$

The protocol: $\hat{H}_S(t)$ with $\hat{H}_S(0) = \hat{H}_i$ and $\hat{H}_S(t_f) = \hat{H}_f$

Entropy change

$\Delta S \neq 0$

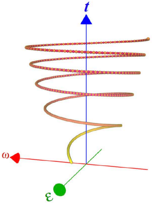


Shortcut to Equilibration of an Open Quantum System,
R. Dann, A. Tobalina, and R. Kosloff, *PRL* 122, 250402 (2019)

GKLS master equation of a driven qubit

The Hamiltonian:

$$\hat{\mathbf{H}}_S(t) = \omega(t)\hat{\mathbf{S}}_z + \varepsilon(t)\hat{\mathbf{S}}_x \quad ,$$



Define the Liouville time dependent vector

$$\vec{v}(t) = \{\hat{\mathbf{H}}_S(t), \hat{\mathbf{L}}(t), \hat{\mathbf{C}}(t)\}^T \quad .$$

$\hat{\mathbf{L}}(t) = \varepsilon(t)\hat{S}_z - \omega(t)\hat{S}_x$ **ime dependent operator base**

and $\hat{\mathbf{C}}(t) = \bar{\Omega}(t)\hat{\mathbf{S}}_y$,

.

Raby frequency $\bar{\Omega}(t) = \sqrt{\omega^2(t) + \varepsilon^2(t)}$

.

The adiabatic parameter

$$\mu = \frac{\dot{\omega}\varepsilon - \omega\dot{\varepsilon}}{\Omega^3}$$

Inertial solution

Solving for the free propagator $\mathcal{U}_S(t)$

$$\frac{1}{\Omega}\frac{d}{dt}\begin{pmatrix}\hat{\mathbf{H}}(t)\\ \hat{\mathbf{L}}(t)\\ \hat{\mathbf{C}}(t)\end{pmatrix} = \left(\begin{pmatrix}0 & \mu & 0\\ -\mu & 0 & 1\\ 0 & -1 & 0\end{pmatrix} + \frac{\dot{\Omega}}{\Omega^2}\hat{I}\right)\begin{pmatrix}\hat{\mathbf{H}}(t)\\ \hat{\mathbf{L}}(t)\\ \hat{\mathbf{C}}(t)\end{pmatrix},$$

$\mathcal{U}_S(t) = \mathcal{U}_1(t)\mathcal{U}_2(t)$, where $\mathcal{U}_1(t) = \frac{\Omega(t)}{\Omega(0)}\hat{I}$

$$\mathcal{U}_2(t) = \frac{1}{\kappa^2}\begin{pmatrix}1+\mu^2 c & \kappa\mu s & \mu(1-c)\\ -\kappa\mu s & \kappa^2 c & \kappa s\\ \mu(1-c) & -\kappa s & \mu^2 + c\end{pmatrix},$$

where $\kappa = \sqrt{1+\mu^2}$ and $s = \sin(\kappa\theta)$, $c = \cos(\kappa\theta)$
and $\theta(t) = \int_0^t \Omega(t')dt'$.

Roie Dann and Ronnie Kosloff,
Inertial Theorem: Overcoming the quantum adiabatic limit,
Phys. Rev. Research, **3**, 013064 *(2021)*. [pdf]

The eigenoperators of $\mathscr{U}_S(t)$

.

The eigenoperators of the free propagators which constitute the eigenoperators of $\mathscr{D}(t)$:
$\hat{\mathbf{F}}_k = \{\hat{\chi}, \hat{\sigma}, \hat{\sigma}^\dagger\}$

$$\hat{\chi}(\mu, 0) = \frac{1}{\kappa \bar{\Omega}(0)} \left(\hat{\mathbf{H}}_S(0) + \mu \hat{\mathbf{C}}(0) \right)$$

$$\hat{\sigma}(\mu, 0) = \frac{1}{2\kappa^2 \bar{\Omega}(0)} \left(-\mu \hat{\mathbf{H}}_S(0) - i\kappa \hat{\mathbf{L}}(0) + \hat{\mathbf{C}}(0) \right)$$

and $\sigma^\dagger(\mu, 0)$,
with corresponding eigenvalues
$\lambda_1 = 0, \lambda_2(t) = i\Omega\kappa(\mu(t))$ and $\lambda_3(t) = -i\Omega\kappa(\mu(t))$,
where

$$\kappa(\mu(t)) = \sqrt{1 + \mu^2(t)} \ .$$

The Non-Adiabatic Master Equation (NAME) for the qubit

.

Exigenoperators of $\mathscr{U}_S(t)$ in the $\{\hat{\mathbf{H}}, \hat{\mathbf{L}}, \hat{\mathbf{C}}\}$ basis:

$$\hat{\chi}(t) = \frac{\sqrt{2}}{\kappa\hbar\Omega}\left(\hat{\mathbf{H}} + \mu\hat{\mathbf{C}}\right)$$

$$\hat{\sigma}(t) = \frac{1}{\kappa\hbar\Omega}\left(-\mu\hat{\mathbf{H}} - i\kappa\hat{\mathbf{L}} + \hat{\mathbf{C}}\right) \quad ,$$

where $\kappa = \sqrt{1+\mu^2}$

$$\frac{d}{dt}\tilde{\rho} = \tilde{\mathscr{L}}(t)[\tilde{\rho}] = k_{\downarrow}(\alpha(t))\left(\hat{\sigma}\tilde{\rho}(t)\hat{\sigma}^{\dagger} - \frac{1}{2}\{\hat{\sigma}^{\dagger}\hat{\sigma}, \tilde{\rho}(t)\}\right) + k_{\uparrow}(\alpha(t))\left(\hat{\sigma}^{\dagger}\tilde{\rho}(t)\hat{\sigma} - \frac{1}{2}\{\hat{\sigma}\hat{\sigma}^{\dagger}, \tilde{\rho}(t)\}\right) \quad .$$

GKLS form

$$\alpha(t) = \kappa(t)\Omega(t) = \sqrt{1+\mu(t)^2}\,\Omega(t) \quad .$$

$$\frac{k\uparrow}{k\downarrow} = e^{-\frac{\hbar\alpha(t)}{kT}}$$

effective frequency

The end

Thank you

The entropy production

The attractor can be expressed in the Gibbs form

ΔS≠0

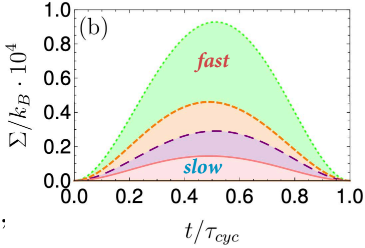
$$\tilde{\rho}_{i.a}(t) = Z^{-1} e^{-\frac{\hbar\alpha(t)\hat{\chi}}{\sqrt{2}k_B T}} \quad ,$$

where $Z = \mathrm{tr}\left(e^{-\hbar\alpha\hat{\chi}/\sqrt{2}k_B T}\right)$

$$\Sigma^u \equiv -\frac{d}{dt}\mathscr{D}(\hat{\rho}|\hat{\rho}_{i.a}) = -k_B \mathrm{tr}\left(\tilde{\mathscr{L}}[\tilde{\rho}]\ln\tilde{\rho}\right) + k_B tr\left(\tilde{\mathscr{L}}[\tilde{\rho}]\ln\tilde{\rho}_{i.a}\right)$$

Thermodynamic forces $\mathscr{F}_l$ and : $\mathscr{F}_\chi = \frac{1}{T_\chi} - \frac{1}{T}$, where $T_\chi = \frac{\hbar\alpha}{\sqrt{2}k_B\bar{\beta}}$, $\mathscr{F}_{\sigma_x} = \frac{k_B\bar{\gamma}_x}{\hbar\alpha}$ and $\mathscr{F}_{\sigma_y} = \frac{k_B\bar{\gamma}_y}{\hbar\alpha}$ then:

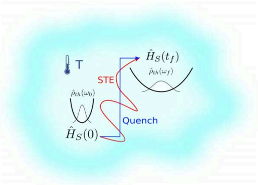
$$\Sigma^u = \sum_{l=\chi,\sigma_x,\sigma_y} \mathscr{F}_l \mathscr{J}_l \quad ,$$



. $\mathscr{J}_\chi = -\frac{\hbar\alpha\Gamma}{\sqrt{2}}\left(\langle\hat{\chi}\rangle - \langle\hat{\chi}_{i.a}\rangle\right)$

$\mathscr{J}_{\sigma_x} = -\frac{\hbar\alpha\Gamma}{2}\langle\hat{\sigma}_x\rangle$; $\mathscr{J}_{\sigma_y} = -\frac{\hbar\alpha\Gamma}{2}\langle\hat{\sigma}_y\rangle$;

The cost of shortcuts $\mathscr{W}$, S_u



Shortcuts to Equilibrium (STE)

Starting on the energy shell:

$$\langle \hat{\mathbf{H}} \rangle \neq 0 \, , \langle \hat{\mathbf{L}} \rangle = 0 \, , \langle \hat{\mathbf{C}} \rangle = 0$$

Nonadiabtic dynamics generates coherence and requires extra work.
The coherence is dissipated generating **uantum friction**
The system entropy changes $\Delta S_{sys} \neq 0$.
Irreversibility i s i nherent $\Delta S_U > 0$.

he speedup cost work and entropy production

Dynamics for any squeezed thermal state.

$$\dot{\beta} = k_{\downarrow}\left(e^{\beta} - 1\right) + k_{\uparrow}\left(e^{-\beta} - 1 + 4e^{\beta}|\gamma|^2\right)$$
$$\dot{\gamma} = \left(k_{\downarrow} + k_{\uparrow}\right)\gamma - 2k_{\downarrow}\gamma e^{-\beta} \quad ,$$

We assume that the system is in a thermal state at initial time, which infers $\gamma(0) = 0$. This simplifies to

$$\tilde{\rho}_S\left(\beta\left(t\right), \mu\left(t\right)\right) = \frac{1}{Z} e^{\beta\, \hat{b}^{\dagger}\hat{b}(\mu)} \, .$$

The system dynamics are described by

$$\dot{\beta} = k_{\downarrow}(t)\left(e^{\beta} - 1\right) + k_{\uparrow}(t)\left(e^{-\beta} - 1\right)$$

with initial conditions $\beta\left(0\right) = \frac{\hbar\omega(0)}{k_B T}$ and $\mu\left(0\right) = 0$.